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Mathematical Optimisation of Smart Irrigation Scheduling Using Climate and Soil-moisture Data: A Simulation Study for Potato Production

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Authors' contributions

This work was carried out in collaboration among all authors. Author SK designed the study, performed the mathematical modelling and numerical simulations, and prepared the original manuscript. Authors JB and RT supervised the study and reviewed the methodology. All authors read and approved the final manuscript.

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Abstract

Background: Efficient irrigation scheduling requires simultaneous consideration of soil-water status, atmospheric demand and crop response. Sensor-based monitoring and optimisation methods can support this integration, but model outputs must be interpreted according to the quality and provenance of their inputs.
Objective: This study develops a five-state compartmental framework for potato irrigation and evaluates threshold-based and optimisation-based irrigation scenarios using a 2025 rainfall forcing series associated with Elgeyo-Marakwet County, Kenya.
Methods: The model represents soil water, root-zone water, plant tissue water, biomass and an environmental water compartment through ordinary differential equations. Soil-water retention follows the van Genuchten relation, reference evapotranspiration is represented with the Penman-Monteith formulation, and irrigation is formulated as a constrained quadratic tracking problem. Numerical integration was performed with MATLAB ode45 and Python solve_ivp; constrained optimisation

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was represented by an SLSQP-based implementation. The analysis is treated as a simulation study rather than as a field-validation study because the underlying station metadata, raw meteorological file and independent crop-calibration dataset were not available for verification.

Results: For the reported loam-soil scenario, total available water was 120 mm m^{-1}, rooting depth was 0.6 m and the adopted depletion fraction was 0.35, giving root-zone total available water of 72 mm and readily available water of 25.2 mm. The supplied simulations produced a rain-fed yield-equivalent output of 3.1 kg m^{-2} and a threshold-controlled output of 6.3 kg m^{-2}, corresponding to a 103% increase within the model scenario.

Conclusion: The simulations illustrate how climate and soil-moisture information can be incorporated into an irrigation-control framework, but the quantitative outputs should be interpreted as scenario results pending calibration and independent field validation.

Keywords: Smart irrigation; mathematical optimisation; soil moisture; optimal control; potato; water balance; simulation.

1. Introduction

Irrigation scheduling seeks to determine when irrigation should occur and how much water should be applied while maintaining crop water status and avoiding unnecessary losses. Established approaches include evapotranspiration-water-balance methods, direct soil-moisture monitoring, plant-based indicators and model-based decision support (Gu *et al*., 2020; Jones, 2004). These approaches differ in data requirements, spatial representativeness, responsiveness and operational complexity. Modern smart-irrigation systems increasingly combine sensor observations, weather information and algorithmic control so that irrigation decisions can respond to changing field conditions rather than follow fixed schedules (Bwambale *et al*., 2022; Datta & Taghvaeian, 2023).

The agronomic value of soil-moisture sensing depends strongly on sensor calibration, installation and interpretation. A systematic review by Datta and Taghvaeian (2023) found that sensor-based irrigation scheduling commonly reduced water use relative to traditional scheduling while maintaining or improving yield, but also showed that measurement error was sensitive to calibration. Automated scheduling studies likewise demonstrate that a soil-water-balance model can be strengthened by feedback from capacitance-type soil-moisture sensors (Domínguez-Niño *et al*., 2020). These findings support closed-loop control concepts in which irrigation decisions are updated using information about soil status and atmospheric demand.

Model-based irrigation has also developed from rule-based scheduling towards state-space and optimisation methods. Root-zone soil-moisture prediction can be represented through identified dynamic models that incorporate rainfall, irrigation and evapotranspiration, with field validation being essential for judging model adequacy (Delgoda *et al*., 2016). Model predictive control and related optimisation strategies can use such state estimates to regulate soil moisture under water or energy constraints (Bwambale *et al*., 2023). Earlier work demonstrated the feasibility of real-time irrigation control using physically based state-space representations (Protopapas & Georgakakos, 1990), while more recent research has coupled agrohydrological models with optimal control to improve water productivity at field or plantation scales (Kassing *et al*., 2020). Analytical studies also show that constrained crop-irrigation problems can exhibit non-trivial optimal-control structures when water supply is limited (Boumaza *et al*., 2021).

Potato is an appropriate crop for evaluating irrigation scheduling because tuber yield is sensitive to water availability and the timing of water stress. A global meta-analysis reported that supplementary irrigation increased potato yield by approximately 55% on average, although responses varied with irrigation amount, method, rainfall, soil characteristics and fertilisation (Niu *et al*., 2025). Field research has similarly shown strong interactions between irrigation and fertilisation in determining potato yield and water productivity (Ierna & Mauromicale, 2018). In north-western Kenya, potato is an important food and cash crop, including in Lelan, Elgeyo-Marakwet County, where production is predominantly rain-season-based but some farmers have access to irrigation (Kwambai *et al*., 2023). This regional context motivates decision-support approaches that can combine rainfall, soil-water status and crop demand.

The present study develops a compact five-compartment mathematical framework for irrigation scheduling and evaluates two simulation scenarios: rainfall-driven production without artificial irrigation and threshold-based smart irrigation. A constrained optimisation formulation is also presented for a 30-day control window. In contrast to a field-validation study, the numerical results are interpreted explicitly as simulation outputs because the underlying meteorological station metadata, full calibration dataset and independent validation observations were not available within the study material. The contribution is therefore methodological, demonstrating a coherent mathematical structure and identifying the conditions that must be met before the model can support site-specific agronomic recommendations.

Accordingly, a methodological gap remains in demonstrating, within the supplied scenario data, a coherent framework that links climate forcing, soil-moisture dynamics and constrained irrigation control while clearly distinguishing simulation outputs from field validation.

1.1 Objectives

The objectives were to (1) formulate a compartmental dynamic model linking soil water, root-zone water, plant tissue water, biomass and environmental forcing; (2) represent irrigation scheduling as a constrained optimisation problem using soil-moisture and climatic information; (3) examine rain-fed and threshold-controlled potato scenarios using the supplied 2025 rainfall forcing; and (4) evaluate the interpretation and limitations of the simulated yield and economic outputs against relevant irrigation literature.

2. Literature Review

2.1 Irrigation Scheduling, Sensors and Decision Support

Irrigation scheduling can be based on estimated crop evapotranspiration, measured soil-water status, crop response or predictive models. Gu *et al*. (2020) emphasised that no single scheduling method is universally superior; performance depends on crop, soil, climate, sensor quality and management objectives. Plant-based methods can directly reflect crop stress, but they may require careful interpretation and may be difficult to scale spatially (Jones, 2004). Soil-water sensors offer a more direct link to root-zone conditions and are particularly useful for automated irrigation when calibration is adequate (Datta & Taghvaeian, 2023).

Closed-loop irrigation systems integrate monitoring and control. Bwambale *et al*. (2022) reviewed smart-irrigation monitoring and control strategies and highlighted the value of combining soil-, plant- and weather-based information. Domínguez-Niño *et al*. (2020) demonstrated that a water-balance algorithm adjusted by soil-moisture sensors could autonomously schedule irrigation in an orchard and produce irrigation totals consistent with independently measured evapotranspiration. These studies support the use of a model as the organising layer of a smart-irrigation system, provided that sensor and weather inputs are validated.

Decision-support systems must also communicate uncertainty. Ara *et al*. (2021) found that many irrigation decision-support systems insufficiently represent uncertainty and may be developed without adequate end-user involvement. Consequently, model outputs should not be presented as deterministic prescriptions when input uncertainty, calibration uncertainty or economic assumptions are substantial.

2.2 Dynamic Models and Optimisation

Dynamic irrigation models can represent the soil-crop-atmosphere system as a state-space process in which rainfall and irrigation act as inputs and soil water and crop state variables evolve through time. Protopapas and Georgakakos (1990) provided an early optimal-control formulation for real-time irrigation scheduling using a physically based dynamic representation of soil-crop-climate interactions. Delgoda *et al*. (2016) later showed that root-zone soil-moisture dynamics can be identified from rainfall, irrigation and evapotranspiration data and validated using both synthetic and field datasets.

Optimisation-based feedback control extends these ideas by explicitly balancing competing objectives such as moisture regulation, crop production and water use. Kassing *et al*. (2020) combined seasonal allocation and model predictive control for precision irrigation and demonstrated improved water productivity in simulation relative to local heuristics. Bwambale *et al*. (2023) reviewed data-driven model predictive control for precision irrigation and identified system identification, feedback and disturbance prediction as central components of robust irrigation control. Boumaza *et al*. (2021) further demonstrated that optimal irrigation under water scarcity can involve singular-control structures rather than simple on-off strategies. Collectively, this literature indicates that mathematical optimisation is appropriate for irrigation, but also that model identification, validation and constraint specification are necessary before operational deployment.

2.3 Soil-water Retention and Evapotranspiration

Soil-water dynamics depend on the nonlinear relationship between water content and matric pressure. The van Genuchten (1980) retention equation remains a widely used representation because its parameters provide a compact description of the soil-water retention curve and can be linked to hydraulic conductivity. Evapotranspiration estimation is another critical component of irrigation modelling. Allen *et al*. (2011) documented the measurement and reporting challenges associated with evapotranspiration and emphasised the importance of transparent data quality and methodological description. In this study, the Penman-Monteith form is used as a reference climatic forcing relation; its output is treated as an external evapotranspiration input to the dynamic model rather than as a directly measured plant-water flux.

2.4 Potato Response to Irrigation

Potato yield response to irrigation is strongly context dependent. Ierna and Mauromicale (2018) found that irrigation and fertilisation jointly influenced tuber yield and water productivity, illustrating the need to avoid attributing yield differences to water alone. Niu *et al*. (2025), synthesising 132 field studies, reported an average yield increase of approximately 55% under supplementary irrigation, while deficit irrigation could improve water-productivity metrics under suitable conditions. The 103% increase generated by the present simulation is therefore larger than the global average response reported in that meta-analysis and should be regarded as a model-scenario result rather than an expected field effect.

For Kenya, Kwambai *et al*. (2023) documented potato-production practices in north-western counties, including Elgeyo-Marakwet, and showed that productivity is shaped by seed, fertiliser, crop protection, crop rotation, altitude and access to irrigation. This reinforces the need to interpret irrigation simulations within a broader agronomic context.

3. Materials and Methods

3.1 Study Design and Data Interpretation

The study is a numerical simulation of a smart-irrigation framework. The numerical forcing includes a rainfall time series labelled for Elgeyo-Marakwet County in 2025 and agronomic scenario values reported in the original model configuration. The underlying meteorological station identifier, raw rainfall file, sensor-calibration records and independent potato field dataset were not available for verification. Accordingly, the rainfall sequence is treated as a scenario forcing and the crop outputs as simulated yield-equivalent values, not as independently validated observations.

The numerical system was implemented using MATLAB ode45 and Python scipy.integrate.solve_ivp for ordinary differential equation integration. Constrained optimisation was represented using sequential least-squares programming (SLSQP). Because software version numbers, solver tolerances and the complete original parameter vector were not recorded in the source material, the analysis focuses on the mathematical structure and explicitly reported scenario inputs rather than claiming independent computational reproduction.

3.2 Conceptual Compartment Structure

The model describes five interacting state variables: soil water, root-zone water, plant tissue water, biomass and an environmental water compartment. Fig. 1 summarises the broader water-cycle processes, while Fig. 2 shows the model compartments and the principal pathways between them.

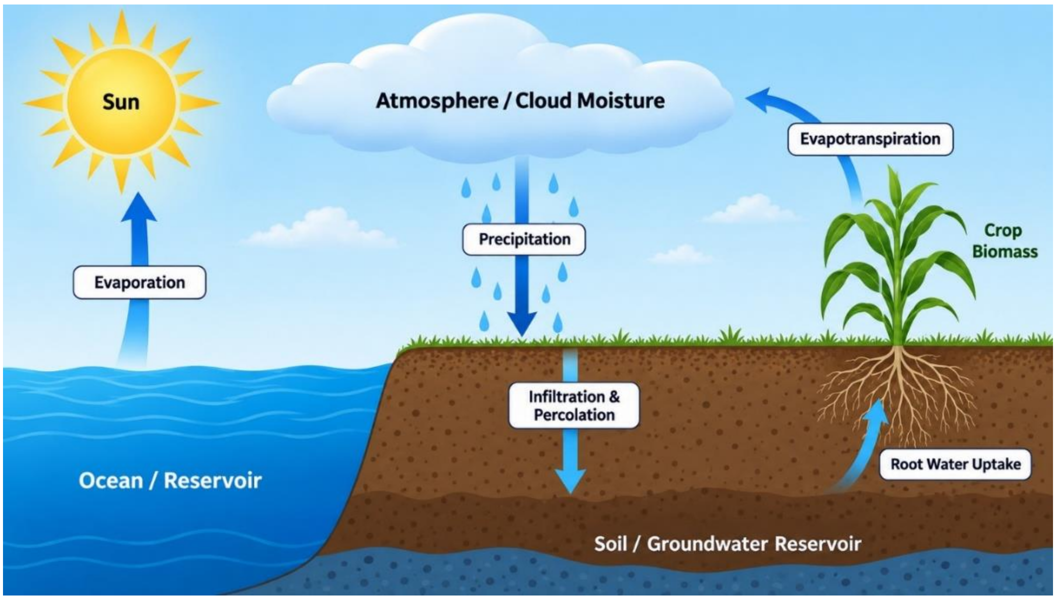

Fig. 1. Water-cycle diagram showing the basic processes represented in the irrigation framework
(Source: Author)

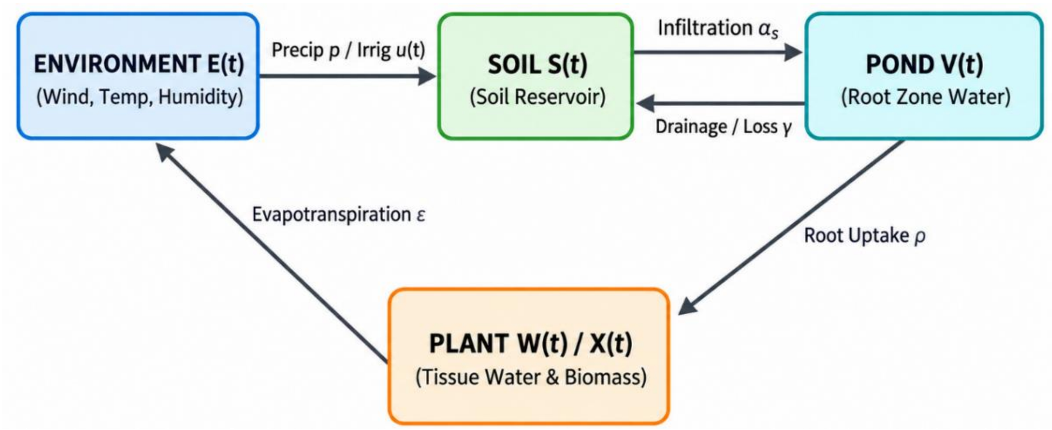

Fig. 2. Diagrammatic representation of the interacting model compartments
(Source: Author)

3.3 Soil-water Retention Relation

The soil-water retention relation is represented by the standard van Genuchten form (van Genuchten, 1980):

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{[1 + (\alpha|h|)^n]^m}, \qquad m = 1 - \frac{1}{n},$$

where $\theta(h)$ is the volumetric water content at matric pressure head h, θ_r and θ_s are residual and saturated water contents, respectively, and α, n and m are shape parameters. The relation is used conceptually to link soil hydraulic state to available water; the present simulation does not claim independent estimation of these parameters from local retention-curve measurements.

3.4 Five-state Ordinary Differential Equation Model

The model state is $z(t) = [S(t), V(t), W(t), X(t), E(t)]^T$, where S denotes soil water, V denotes root-zone water, W denotes plant tissue water, X denotes biomass and E denotes an environmental water state. Irrigation input is denoted by $u(t)$. The supplied model structure is:

$$\frac{dS}{dt} = u(t) + pE - \epsilon SE - (\alpha_s + \vartheta)S$$

$$\frac{dV}{dt} = \alpha_s S - \rho\left(\frac{V}{1 + rV}\right)S - \gamma V$$

$$\frac{dW}{dt} = \rho\left(\frac{V}{1 + rV}\right)\left(\frac{X}{X + K_x}\right) - (\beta + \varepsilon_T)W - \mu GW$$

$$\frac{dX}{dt} = \eta X\left(1 - \frac{X}{X_{max}}\right)\left(\frac{W}{W + K_w}\right) - \mu X$$

$$\frac{dE}{dt} = \varepsilon_T W - pE + \gamma V + \epsilon S.$$

The terms are interpreted as phenomenological fluxes rather than direct measurements of individual physical processes. Table 1 defines the principal model symbols. Parameters not numerically reported in the source configuration are retained as model coefficients; no unsupported numerical values are introduced.

Table 1. Principal model variables and coefficients

Symbol	Interpretation	Role in model
$S(t)$	Soil-water state	Receives irrigation and environmental input; loses water through transfer and loss terms
$V(t)$	Root-zone water state	Receives soil transfer and supplies plant water uptake
$W(t)$	Plant tissue-water state	Represents plant-accessible water and evapotranspiration-related loss
$X(t)$	Biomass/yield-equivalent state	Logistic growth modulated by plant water availability
$E(t)$	Environmental water state	Aggregates precipitation-related input and return/loss pathways
$u(t)$	Irrigation control	Decision variable for irrigation application
$p, \alpha_s, \vartheta, \rho, r, \gamma$	Transfer/loss coefficients	Regulate water exchange among environmental, soil and root-zone states
$\beta, \varepsilon_T, \mu, G$	Plant-water loss coefficients	Regulate tissue-water depletion as specified in the original model
$\eta, X_{\max}, K_w, \mu$	Biomass-growth coefficients	Regulate growth, water limitation and biomass loss
K_x	Saturation constant	Modulates root-zone to plant-water transfer

3.4.1 Existence, Uniqueness and Non-negativity

For bounded non-negative forcing $u(t)$ and non-negative parameters, and for a state domain satisfying $1 + rV > 0$, $X + K_x > 0$ and $W + K_w > 0$, the right-hand side of the five-state system is continuous and locally Lipschitz in the state variables. The Picard-Lindelöf conditions therefore provide local existence and uniqueness of solutions. Moreover, evaluation of each state derivative on the corresponding zero boundary shows that the non-negative orthant is forward invariant under non-negative forcing. No general Lyapunov-stability claim is made because a stability would require a specified equilibrium, complete parameter set and explicit Lyapunov function.

3.5 Evapotranspiration Forcing

Reference evapotranspiration is represented with the Penman-Monteith form:

$$\varepsilon_T(t) = \frac{0.408\,\Delta(R_n - G) + \gamma\,\frac{900}{T + 273}\,v_2(e_s - e_a)}{\Delta + \gamma(1 + 0.34v_2)}$$

where R_n is net radiation, G is soil heat flux, T is mean air temperature, v_2 is wind speed at 2 m, $e_s - e_a$ is the vapour-pressure deficit, Δ is the slope of the saturation vapour-pressure curve and γ is the psychrometric constant. Evapotranspiration measurement and parameterisation are important sources of uncertainty in irrigation models (Allen *et al*., 2011). In the ODE framework, $\varepsilon_T(t)$ represents the evapotranspiration-related loss rate after conversion of climatic forcing into the model's time and storage scale. Because the complete original conversion and crop-coefficient parameterisation were not available, the present study does not claim site-specific calibration of $\varepsilon_T(t)$.

3.6 Irrigation-control Formulation

The irrigation-control problem is formulated as a finite-horizon quadratic tracking problem. Let $e(t) = z(t) - z_d(t)$ denote the deviation of the state from a desired trajectory $z_d(t)$. The corrected objective functional is

$$J = \frac{1}{2}e^T(t_1)He(t_1) + \frac{1}{2}\int_{t_0}^{t_1}[e^T(t)Qe(t) + u^T(t)Ru(t)]\,dt$$

where H and Q are non-negative state-tracking weights and R is a positive irrigation-cost weight. This formulation is consistent with the broader use of optimisation-based irrigation control in the literature (Kassing *et al*., 2020; Protopapas & Georgakakos, 1990). The supplied computational implementation used numerical constrained optimisation with SLSQP. Accordingly, the study does not claim a closed-form solution derived from Pontryagin's Maximum Principle. Pontryagin-based analyses remain theoretically relevant to irrigation-control problems (Boumaza *et al*., 2021), but a formal Hamiltonian, adjoint system and transversality derivation is beyond what is supported by the available model record.

3.7 Scenario Inputs and Irrigation Threshold

The explicitly reported scenario values are summarised in Table 2. Total available water (TAW) of 120 mm m^{-1} combined with a 0.6 m rooting depth gives a root-zone TAW of 72 mm. Applying the adopted depletion fraction $d_f = 0.35$ gives a readily available water threshold of 25.2 mm:

$$\text{TAW}_{\text{root}} = 120 \text{ mm m}^{-1} \times 0.6 \text{ m} = 72 \text{ mm},$$

$$\text{RAW} = \text{d}_\text{f} \times \text{TAW}_{\text{root}} = 0.35 \times 72 = 25.2 \text{ mm}.$$

Threshold-based scheduling is conceptually consistent with soil-moisture and water-balance irrigation methods reviewed by Gu *et al*. (2020), but the value $d_f = 0.35$ is treated here as a scenario assumption rather than a locally validated potato parameter.

Table 2. Explicitly reported inputs used in the numerical scenarios

Input	Value	Interpretation
Soil total available water	120 mm m^{-1}	Loam-soil scenario value
Rooting depth	0.6 m	Effective root-zone depth
Root-zone total available water	72 mm	Calculated as 120 x 0.6
Depletion fraction, d_f	0.35	Adopted scenario threshold parameter
Readily available water	25.2 mm	Calculated as $d_f \times TAW_{root}$
Rain-fed yield-equivalent output	3.1 kg m^{-2}	Supplied simulation result
Smart-irrigation yield-equivalent output	6.3 kg m^{-2}	Supplied simulation result
Optimisation horizon	30 days	Reported control window
Potato selling-price assumption	KSh 50 kg^{-1}	Scenario value used in the original economic comparison
Pumping-cost proxy	KSh 0.05 mm^{-1}	Incomplete proxy; not sufficient for a farm-level net-margin calculation

4. Results

4.1 Rainfall Forcing Scenario

Fig. 3 presents the rainfall series supplied for the 2025 Elgeyo-Marakwet simulation. The sequence contains periods of frequent rainfall separated by prolonged low-rainfall intervals. Because the station metadata and raw meteorological file were unavailable, the figure is interpreted as the forcing series used by the model rather than as an independently verified county-wide rainfall record.

4.2 Rain-fed Simulation

In the supplied rain-fed simulation, the yield-equivalent biomass output varied with the rainfall-driven soil-water state and reached approximately 3.1 kg m^{-2}. Fig. 4 shows the simulated soil-moisture, rainfall and biomass trajectories. The graph should be read as a model-state illustration rather than an observed crop-response dataset because no independent field measurements were available for comparison.

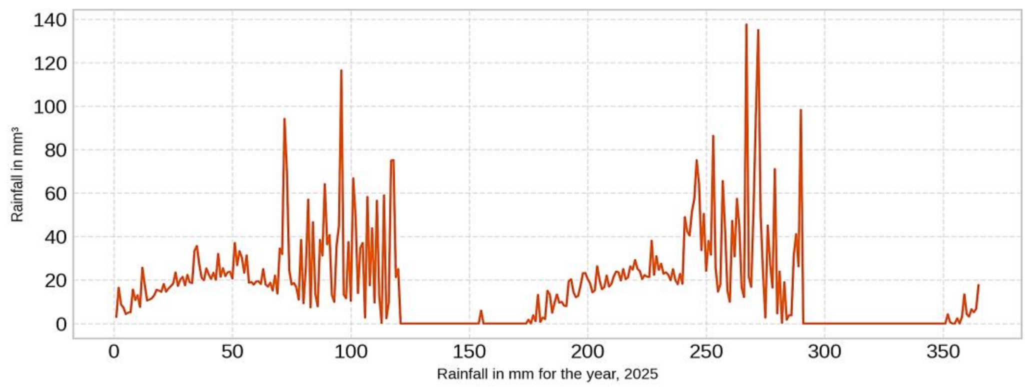

Fig. 3. Rainfall forcing series used in the 2025 Elgeyo-Marakwet simulation scenario

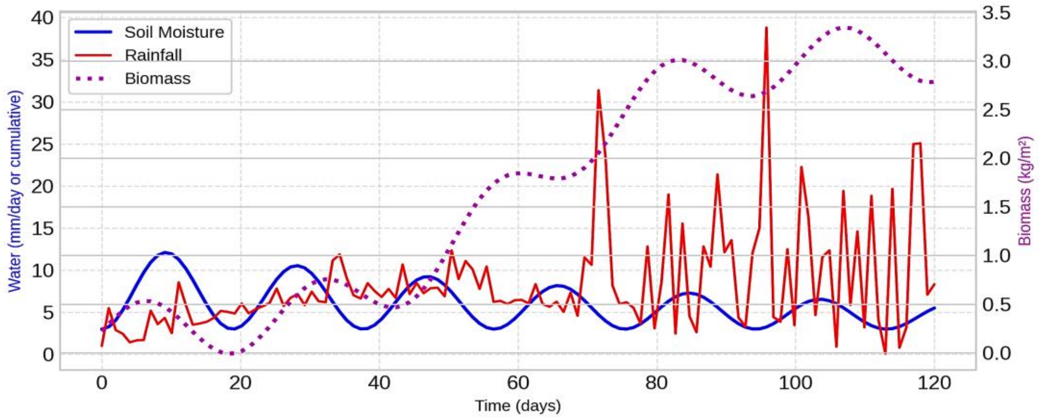

Fig. 4. Simulated crop response under rainfall-only conditions without artificial irrigation

4.3 Threshold-based Smart-irrigation Simulation

The threshold-controlled scenario maintained the soil-water state around the adopted RAW threshold of 25.2 mm and produced a final yield-equivalent output of approximately 6.3 kg m^{-2} (Fig. 5). Relative to the rain-fed output of 3.1 kg m^{-2}, this is an increase of approximately 103%. The magnitude should not be interpreted as a field-effect estimate because it depends on the model parameterisation and does not include an independent calibration or uncertainty interval.

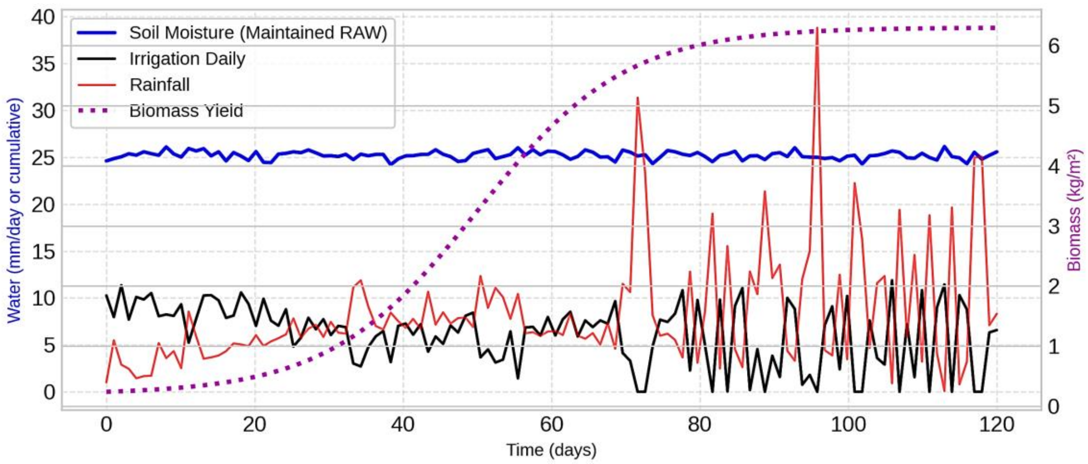

Fig. 5. Simulated crop response under threshold-based smart irrigation

4.4 Thirty-day Optimisation Scenario

Fig. 6 presents the state trajectories generated for the 30-day optimisation window. The figure shows the simulated soil-moisture and biomass states relative to desired trajectories. The irrigation control $u(t)$ itself is not plotted; consequently, the figure is interpreted as evidence of the resulting state response rather than a complete representation of the irrigation schedule. The optimisation framework is therefore best regarded as a proof of concept for state regulation rather than a fully reproducible operational schedule.

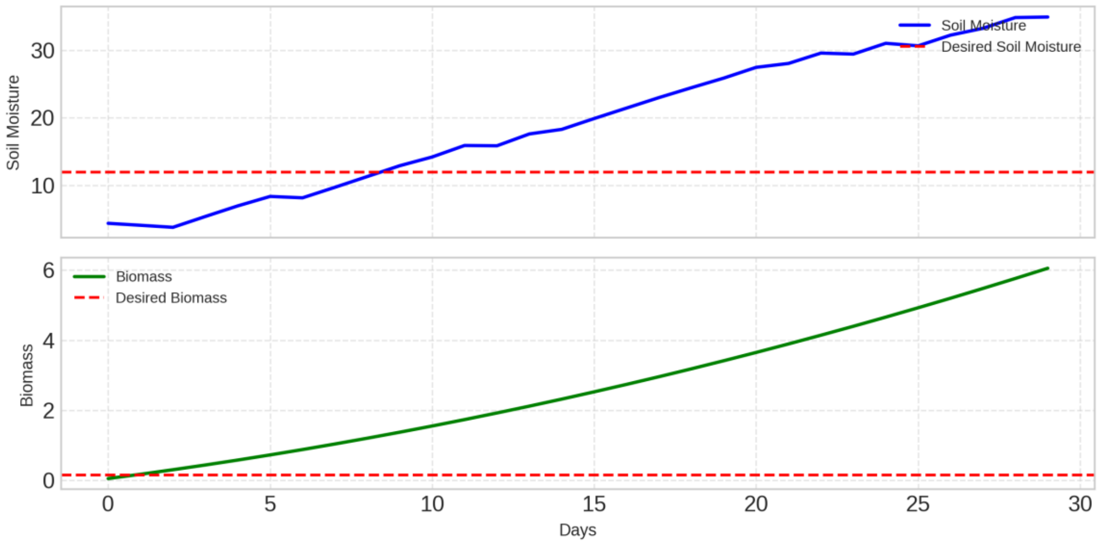

Fig. 6. State trajectories from the 30-day optimisation scenario

4.5 Economic Interpretation

The original simulation used a potato selling-price assumption of KSh 50 kg^{-1} and a pumping-cost proxy of KSh 0.05 mm^{-1}. At the stated selling price, the simulated gross crop value is KSh 155 m^{-2} for the 3.1 kg m^{-2} rain-fed output and KSh 315 m^{-2} for the 6.3 kg m^{-2} smart-irrigation output, before any production or irrigation costs. The difference is therefore KSh 160 m^{-2} in gross crop value within the model scenario.

A farm-level net profit margin cannot be derived defensibly from the available information because cumulative irrigation depth, irrigated area, pump head, pump efficiency, energy tariff and non-water production costs are not reported. The previously stated 64.8%-65% figure is therefore not retained as a verified net-profit result. This treatment is consistent with the broader requirement that irrigation decision-support systems communicate economic and model uncertainty transparently (Ara *et al*., 2021).

5. Discussion

The simulation demonstrates the expected qualitative behaviour of a feedback irrigation framework: when water is added to maintain the soil-water state above a threshold, the model predicts improved crop growth relative to a rainfall-only scenario. This behaviour is compatible with empirical irrigation literature, although the magnitude of the simulated response is comparatively large. Niu *et al*. (2025) reported an average 55% potato-yield increase from supplementary irrigation across a large global evidence base, whereas the present model scenario produced a 103% increase. The difference does not imply contradiction because the present result is a single simulated scenario without a field uncertainty interval; rather, it highlights the need for calibration before the magnitude is interpreted agronomically.

The literature also shows that water response cannot be separated from other production factors. Ierna and Mauromicale (2018) demonstrated important irrigation-by-fertilisation effects on potato growth, yield and water productivity. In north-western Kenya, Kwambai *et al*. (2023) identified seed quality, diseases, fertiliser use, crop rotation and technical knowledge as major determinants of potato productivity. Consequently, even a well-calibrated irrigation model would not by itself explain field-level yield variability. The biomass state in the present model should therefore be considered an irrigation-responsive crop-growth indicator rather than a complete crop-production model.

The model structure has several strengths for future development. First, it separates environmental forcing, soil water, root-zone water and plant water rather than representing irrigation through a single empirical threshold. Second, it allows irrigation to enter as an explicit control variable. Third, the corrected quadratic objective functional provides a clear path towards model predictive control, where state tracking and water-use penalties can be balanced. These elements are consistent with established optimal-control and MPC literature (Bwambale *et al*., 2023; Kassing *et al*., 2020; Protopapas & Georgakakos, 1990).

However, the current model should not yet be used as an operational prescription. Delgoda *et al*. (2016) demonstrated that soil-moisture prediction models should be evaluated using independent data and quantitative goodness-of-fit diagnostics. Datta and Taghvaeian (2023) similarly showed that sensor calibration can materially affect irrigation scheduling. For the present framework, field deployment would therefore require, at minimum, site-specific soil hydraulic calibration, a verified rainfall and weather dataset, crop-stage information, sensor calibration, measured irrigation amounts, and observed soil-moisture and yield data for calibration and validation.

The treatment of evapotranspiration is another important limitation. The Penman-Monteith equation provides a reference climatic estimate, but conversion to crop evapotranspiration and then to a state-loss coefficient requires crop-specific and time-scale information. Allen *et al*. (2011) stressed that evapotranspiration data and modelling can be biased when measurement and reporting details are incomplete. In this study, $\varepsilon_T(t)$ is therefore retained as a model loss term without claiming that it has been locally calibrated from measured evapotranspiration.

The 30-day optimisation result should also be interpreted cautiously. Fig. 6 displays state trajectories but not the irrigation control, so it cannot by itself establish the timing or quantity of the optimal irrigation sequence. Operational optimisation would require reporting the control bounds, complete weighting matrices, time discretisation, cumulative irrigation, constraint satisfaction and sensitivity of the solution to parameter uncertainty. Theoretical studies such as Boumaza *et al*. (2021) show that irrigation-control solutions can change qualitatively with water constraints and crop-water stress functions, reinforcing the need for transparent specification.

Finally, the economic comparison is intentionally restricted to gross crop value because a valid net-margin analysis requires explicit costs and irrigation-energy calculations. This is particularly important for decision support: Ara *et al*. (2021) noted that uncertainty and the economic dimensions of irrigation decisions are often inadequately represented in decision-support systems. Future implementation should therefore link the hydrological model to a partial-budget framework with measured water volume, pumping energy and farm-production costs.

5.1 Limitations

The principal limitations are as follows: (1) the rainfall forcing lacks independently verifiable station metadata in the available study record; (2) the complete numerical parameter vector, initial conditions, solver tolerances and optimisation weights are not available; (3) no independent field dataset is available for calibration or validation; (4) the conversion from biomass state to harvested tuber yield is represented by the supplied simulation output rather than a documented local calibration equation; (5) cumulative irrigation, evapotranspiration and water-productivity metrics are not available for verification; and (6) the economic input set is insufficient for a net-profit calculation. These limitations restrict the study to proof-of-concept simulation inference.

6. Conclusions

A five-state mathematical framework was formulated to integrate environmental forcing, soil water, root-zone water, plant tissue water, biomass and irrigation control. The soil-water retention equation and quadratic objective functional were corrected to standard forms, and the mathematical interpretation was aligned with the numerical SLSQP optimisation used in the simulation. Under the reported loam-soil scenario, the root-zone readily available water threshold is 25.2 mm, not 25.2 mm m^{-1}. The supplied simulations produced yield-equivalent outputs of 3.1 kg m^{-2} under rainfall-only conditions and 6.3 kg m^{-2} under threshold-based irrigation, a 103% increase within the model scenario.

These numerical results demonstrate the potential of combining climate and soil-moisture information in an irrigation-control framework but do not establish field-scale efficacy or economic profitability. Before the framework is used for farm recommendations, it should be calibrated and independently validated using site-specific weather, soil-moisture, irrigation, evapotranspiration and potato-yield observations, and the optimisation should report the complete irrigation control trajectory and uncertainty analysis.

Data and Code Availability

The study is based on a numerical simulation. The governing equations and explicitly reported scenario inputs are provided in the article. The complete original solver configuration, raw meteorological file and independent field-calibration dataset were not available in the study record; therefore, the reported numerical results should be interpreted as proof-of-concept scenario outputs rather than as independently reproducible field validation.

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Declaration of AI Use

This manuscript was prepared through the intellectual contributions of the author(s) to the study design, data analysis, interpretation of results, and scholarly content. Grammarly and ChatGPT were used solely to assist with grammatical refinement and reference formatting during manuscript revision. The author(s) accept full responsibility for the accuracy, integrity, and final content of the manuscript.

Competing Interests

Authors have declared that no competing interests exist.

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