HEAD LOSS ANALYSIS IN PIPE FLOW SYSTEMS

BY

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Declaration

This project is my own work and has not been presented for a degree award in any other institution.

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This project has been submitted for examination with our approval as the university supervisors.

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To my parents Shem Amudavi and late mum Selina. To my wife Mary, daughters Sheila and Govelet.

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Abstract

Fluid flow in pipes is accompanied by major and minor friction losses. Major friction losses are as a result of loss of energy in pipes due to viscous effects of the fluids on pipe surfaces. Minor friction losses do occur and give an account of the energy loss due to obstructions of the fluid as a result of narrower and wider sections of the pipe. Friction loss lead to head loss. The determination of these losses that occur in pipe systems together with effect of additional components in the system such as abrupt expansions, contractions and bends on the overall head loss is investigated. Studies in head loss in pipe systems are important since they give an idea of how long, thick or rough or the kind of fittings should be used in a given pipe system to transmit fluid in pipe optimally. There are in general two methods that are used in head loss prediction; use of tables and formulas in pipes without fittings and the K-factor method together with the equivalent length of pipe in linear fit method for head loss in pipes and pipe fittings and valves. There is an exponential decrease in head loss with the ratio of areas for sudden expansions and sudden contractions. The head loss term increases exponentially with the fluid speed in the pipe ad the corresponding length of the pipe.

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Chapter 1

Introduction

1.1 Background

There are two types of practical methods that are used to describe the variety of fluid flow. These are finite control volume analysis and differential analysis of fluid flow. Distinction is made between the two analyses as done by Morison(1982)[1] as follows: A finite control and volume analysis of the behaviour of the contents to a finite region in space is called control volume. The concept of a control volume and system occupying the same region of space at an instant is a key element in the derivation of the control volume analysis. The use of pipe bends may introduce increased cross-sectional flow velocities thus making the flow more non-uniform at the bend exit and causing flow separation. In fluid mechanics and thermodynamics a control volume is a mathematical abstraction employed in the process of creating mathematical models of physical processes. In an inertial frame of reference it is a volume fixed in space or moving with a constant velocity through which the fluid (gas or liquid) flows. The surface enclosing the control volume is referred to as the control surface Ito (1966)[2]. At a steady state, a control volume can be thought of as an arbitrary volume in which the mass of the fluid remains constant. Consider a bug that is moving through a volume where there is some scalar quantity, for example pressure that varies with time and position.

$$P = P(t, x, y, z) \tag{1.1}$$

If the bug during the time interval from t to t + dt moves from (x, y, z) to $(x + dx, y + dy, z + dz)$, then the bug experiences a change dP in the scalar value given by

$$dP = \frac{\partial P}{\partial t}dt + \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}dz \tag{1.2}$$

If the bug is moving with velocity V then the change in position

$$Vdt = V_x dt + V_y dt + V_z dt \tag{1.3}$$

and we may write

$$dP = (\frac{\partial P}{\partial t} + V.\nabla P)dt \tag{1.4}$$

where ∇P is the gradient of the scalar field P. If the bug is just a fluid particle moving with the fluid's velocity field, the same formula applies, but now the velocity vector is that of the fluid. Friction loss is the loss of energy or "head" that occurs in the pipe flow due to viscous effects generated by the surface of the pipe, Dimmock (2010)[4]. Friction is considered as a "major loss" which includes energy lost due to obstructions.

1.2 Differential Analysis

The need to know how the velocity varies over the cross section of a pipe, or how the pressure and shear stress vary along the surface of an air plane wing. The development of a relationship that apply at a point, or at least in a very small region (infinitesimal volume) within a given flow field is necessary. Involvement of infinitesimal control volume instead of finite control volume in differential analysis gives the governing equations as differential equations. Although differential analysis has the potential for supplying very detailed information about flow fields, the information is not easily extracted. Nevertheless, this approach provides a fundamental basis for the study of fluid flow. We also have some exact solutions for laminar flow that can be obtained which have proved useful. For certain types of flows, the flow field can be conceptually divided into two regions

- A very thin region near the boundaries of the system in which viscous effects are important
- A region away from the boundaries in which the flow is essentially inviscid

Flow fields in which the shearing stresses are zero are said to be inviscid, non-viscous, or frictionless for fluids in which there are no shearing stresses the normal stress at a point is independent of direction for an inviscid flow in which all the shearing stresses are zero and the normal stresses are replaced by $-P$,

$$-P = \sigma_{xx} = \sigma_{yy} = \sigma_{zz} \tag{1.5}$$

The Navier-Stokes equations reduce to Euler's equation.

$$\rho\mathbf{g} - \nabla P = \rho(\frac{\partial P}{\partial t} + (\mathbf{V}.\nabla)\mathbf{V}) \tag{1.6}$$

In Cartesian co-ordinates we have:

$$\rho g_x - \frac{\partial P}{\partial x} = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) \tag{1.7}$$

$$\rho g_y - \frac{\partial P}{\partial y} = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) \tag{1.8}$$

$$\rho g_z - \frac{\partial P}{\partial x} = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) \tag{1.9}$$

Starting from Euler's equation, Bernoulli equations can be derived. For inviscid, incompressible fluids,

$$\frac{P}{\rho} + \frac{v^2}{2} + gz = \text{constant} \tag{1.10}$$

Thus for any two points (1) and (2) along a streamline the equation becomes

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 \tag{1.11}$$

where $\gamma = \rho g$, P is pressure ,V is velocity z is pipe elevation above some datum and r is diameter relation. Equation (1.11) is restricted to inviscid, steady, incompressible flow, flowing along a streamline, where P is the pressure, v the fluid velocity, z is the pipe elevation above some datum and γ is the diameter relation. , , ,

1.2.1 The Velocity Potential

For an irrotational flow:

$$\nabla \times \mathbf{V} = (\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z})\mathbf{i} - (\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z})\mathbf{j} + (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})\mathbf{k} = \mathbf{0} \qquad (1.12)$$

So that $\partial w/\partial y = \partial v/\partial z$,$\partial w/\partial x = \partial u/\partial z$, $\partial v/\partial z = \partial u/\partial y$ This means that;

$$\mathbf{V} = \nabla\phi \qquad (1.13)$$

where ϕ is the velocity potential . The velocity potential is a consequence of the irrotationality of the flow field, whereas the stream function is a consequence of conservation of mass. The velocity potential can be defined for a general three-dimensional flow, whereas the stream function is restricted to two dimensional flows. For an incompressible flow we know from the conservation of mass:

$$\nabla.\mathbf{V} = 0 \qquad (1.14)$$

and therefore for incompressible flow, it follows that

$$\nabla^2\phi = 0 \qquad (1.15)$$

The velocity potential satisfies the Laplace equation. In Cartesian co-ordinates;

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{1.16}$$

and in cylindrical coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{1.17}$$

1.3 Viscous flow in Pipes

Using the one-dimensional Bernoulli equation for viscous flow, the velocity of the fluid is taken into account and the fatal energy head

$$H = \frac{v^2}{2g} + \frac{P}{\rho g} + z \tag{1.18}$$

is no longer constant along the pipe. In the direction of flow, due to friction caused by viscosity of the fluid we have

$$\frac{v_1^2}{2g} + \frac{P_1}{\rho g} + z_1 > \frac{v_2^2}{2g} + \frac{P_2}{\rho g} + z_2 \tag{1.19}$$

So to restore the equality we must add some scalar quantity to the right side of the inequality.

$$\frac{v_1^2}{2g} + \frac{P_1}{\rho g} + z_1 + \frac{v_2^2}{2g} = \frac{P_2}{\rho g} + z_2 + \triangle hls \tag{1.20}$$

The scalar $\triangle hls$ is called the hydraulic loss. The hydraulic loss between two different cross sections along the pipe is equal to the difference of total energy for this cross section.

$$\triangle hls = H_1 - H_2 \tag{1.21}$$

Always $H1 > H2$. In horizontal pipe when $z_1 = z_2$ and the diameter of pipe is constant $v_1 = v_2$ hydraulic loss is equal to the head of pressure drop or head loss.

$$\triangle hls = \frac{P_1 - P_2}{\rho g} \tag{1.22}$$

The head loss h_L is expressed by Darcy-Weisbach equation

$$h_L = f \frac{L}{D} \frac{\nu^2}{2g} \tag{1.23}$$

For horizontal pipe with constant diameter. This loss may be measured by height of the pressure drop $\Delta P/\rho g = h$. In general with $v_1 = v_2$ but $z_1 \neq z_2$ the head loss is given by

$$\frac{P_1 - P_2}{\rho g} = (z_1 - z_2) + f \frac{L}{D} \frac{\nu^2}{2g} \tag{1.24}$$

Part of the pressure change is due to elevation change and and part is due to head loss associated with frictional effects, which are given in terms of the friction factor f that depends on Reynolds number Re and relative roughness $f = \phi(Re\epsilon/D)$. It is not easy to determine the frictional dependence of the friction factor on the Reynolds number and relative roughness (ϵ/D). Much of this information is a result of experiments conducted by Nikuradse(1933)[3] and amplified by many other since then. Nikuradse(1933)[3], used artificially roughened pipes produced by gluing sand grains of known size onto pipe walls to produce pipes with sand paper-type surfaces. In commercially available pipes the roughness is not as uniform and well defined as in the artificially roughness pipes used by Nikuradse (1933)[3]. However, it is possible to obtain a measure of the effective relative roughness of typical pipes and thus to obtain the friction factor.

1.4 Friction Loss

Most pipe systems consist of considerably more than straight pipes. The additional components add to the overall head loss of the system, such losses are generally termed minor losses with the apparent implication being that the majority of the system loss is associated with the friction in the straight portions of the pipes, the major losses or local losses. In many cases this is true. And the minor losses are greater than the major losses. The minor losses may be raised by

1. Pipe entrance or exit
2. Sudden expansion or contraction
3. Bends, elbows, tees and other fittings
4. Valves, open or partially closed
5. Gradual expansion or contractions

The major losses may not be so minor for example a partially closed valve can cause a greater pressure drop than a long pipe. The losses are commonly measured experimentally. The data especially for valves are some what dependent upon the particular manufacturers design. The most common method used to determine the head losses or pressure drop.

$$h_L = \frac{\Delta P}{\rho g} = K_L \frac{\nu^2}{2g} \tag{1.25}$$

where K_L means local loss coefficient. Although KL is dimensionless, it is not cor-related in the literature with the Reynolds number and roughness ratio but rather simply with the raw size of the pipe. Most data are reported for turbulent flow conditions.

Studies carried out early showed that secondary vortices in the stream wise direction downstream of the bend Harlock (1955)[5], the so-called Dean motions, Mohanty (1994)[6], become turbulent. The strength of the secondary motions is often described using the Dean number, which is the ratio of the square root of the product of the inertia and centrifugal forces to the viscous force.

$$D_\epsilon = \left(\sqrt{\frac{D}{2R_c}}\right) \frac{\overline{V} D}{\nu} \tag{1.26}$$

where D is the pipe diameter, R_c is the radius of curvature, $\overline{V}$ is the area-averaged or bulk velocity, and ν is the fluid kinematic viscosity. In the present work, De is equal to the Reynolds number.

$$R_{eD} = \frac{\overline{V}}{D\nu} \tag{1.27}$$

The flow at small Dean numbers has been studied widely because of its appearance in, for example, arterial flows Boiron et al (1957)[7], but at high Dean numbers corresponding to turbulent flow where most industrial applications are found there is only scattered evidence available. Studies of the secondary flow in pipes with a 900 bend were conducted using hot wire anemometry by Sudo et al(1998)[8] for Reynolds numbers of up to 610^4, and their measurements revealed secondary flows that persisted 10 diameters downstream of the bend. However, in some cases, a high degree of unsteadiness is observed, as in the study by Tunstall and Harvey (1968)[9] at a high Reynolds number of about 1.810^5, which may indicate the presence of a single large streamwise vortex that is switching sign. Our purpose is to provide high resolution velocity maps of the primary and secondary flow upstream and downstream of a 90^0 bend, with Rc/R (where Rc is the radius of the bend and R is the radius of the pipe). The measurements were obtained using stereoscopic particle image velocimetry (spiv) at Reynolds numbers up to 105 . One of the particular areas of interest is the behaviour of the turbulent structure, specifically the very large scale motions (VLSM), where we build on the recent work of Helstrom et al (2011)[10] who investigated the three dimensional character of the VLSM in fully-developed pipe flow (that is in the flow upstream of the bend).

1.5 Statement of the Problem

Most pipes flow systems have straight pipes which incur frictional losses. Components like valve, bends, elbows and tees contribute to the overall head loss of the flow system. The energy losses arising from the flows need to be considered in computing friction losses. The lose due to valve and fittings, the loss due to bends and the loss due to the fluid leaving the pipe system is to be analyzed. The determination and analysis of these loses require considerations.

1.6 Objectives of the Study

The objectives of this study are

- To identify and discuss analytical methods that can give exact and reliable solutions in the analysis of flow in pipe system components particularly at contractions and expansions along horizontal pipes.
- model and discuss minor and major losses in straight pipe flows that lead to head loss

1.7 Significance of the Study

- This study and results will create awareness to researchers and research organizations to invest more as this will lead to improved agricultural setups using irrigation
- The study has an academic benefit to universities, research institutions, and industrial processes.

Chapter 2

Literature Review

In the past, differential calculus studies were restricted to laminar flow and interpretation of the complex functions such as Navier-Stoke's equations that define the general concept of flow motion Morison et al (1982) and the suitable boundary conditions that simplifies the situation. In his classic paper Taylor (1953)[11], pointed out that in the longitudinal dispersion of soluble matter in a moving fluid, the solute is more slowly dispersed by molecular or turbulent diffusion alone than the dispersion due to the shear effect caused by the combined effects of convection and lateral diffusion. Aris (1953)[12] subsequently proposed an idea of moment method in solving the model removing restrictions imposed by Taylor (1953)[11]. Gill and Sankarasubrarian (1970)[13] used elegant α u-time approaches to study the dispersion of passive solutes in Newtonian fluid flows. Regarding the non-Newtonian fluid flows, Fan and Hwang (1965)[14], calculated the time asymptotic longitudinal dispersion coefficient in the steady laminar flow of the Ostwald-cle Waele fluid in laminar flow in a tube. Since axial dispersion is enhanced by larger velocity gradients across the tube flatter profiles for pseudoplastic fluids which are shear-thinning power law fluids with power law index N exceeding unity result in a decrease in the longitudinal dispersion coefficient. Taylor's initiative approach was also used by Fan and Wang (1965)[14], to study the dispersion of solute in flows of Birmingham plastic fluids. The exact method of analysis of convective diffusion developed in Helstrom (2011)[10] was extended by Sankarasubrarian and Gill (1970)[13] to include the characteristic of non-Newtonian flows. Results were given for specific case of dispersion of solute in steady laminar flow of a non-Newtonian power law fluid which shows that the constant coefficient Taylor dispersion model is inadequate for describing the average concentration distributions for small values of time or for axial locations close to the inlet. The friction dependence of the friction factor on Reynold's number and relative roughness were done in experiments by Nikuradse (1933)[3], and has since been amplified by many others. One difficulty lies in the determination of the roughness of the pipe. Nikuradse (1933)[3] used artificially roughened pipes produced by gluing sand grains of known size onto pipe walls to produce pipe with sand paper-type surface. Mei (2011)[15] correlated the original data for Nikuradse in terms of the relative roughness of commercially available pipe materials consequently a number of authors have attempted to model a number of inviscid flow and the assumption of smooth channel sides improvement in analytic techniques based on sound initial condition extension to three dimensional calculation additional work on laminar flow in various types of channels for example cracks analysis of pipe boundary functions as well as experimental validation of these techniques is considered as a landmark in analysis of flow of this type. Simplification of geometry and boundary condition by employing suitable assumption are usually considered in any model. However, in experiments and application next to laminar flow (with low Reynold's numbers) velocity changes causes considerable forces on the parts of the pipe Ern and Lelievre (2011)[16] who gives a method of procedure for calculating the magnitude and direction of this force. Properties of flow at the pipe system components have adopted the streamline coordinate system in analysis of flow in the pipe. In the application of fluid analysis theory of work and energy relationship Ern (2011)[16], considered fluid flow as stream of particles that can be considered for an analysis independently hence generalizing entire flow. Turbulent flow in pipe bends is of great practical interest due to the associated pressure losses and the distortion of the velocity profile. Bends tend to introduce secondary flows that lead to scour, and non-uniform heat transfer Berger et al (1983)[17]. Much research has been done in the area of secondary flows as it is a phenomena related to many fluid problems, including flow through heat exchangers, industrial management piping systems, scouring and river meandering. Early studies showed that secondary vortices in the stream wise direction downstream of the bend had major losses Harlock (1955)[5], the so-called Dean motions, Monty and Stewert (2007)[18].

Chapter 3

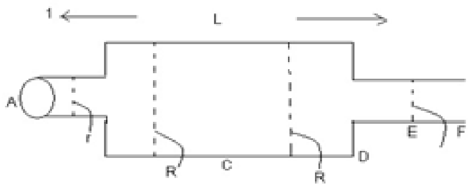
Methodology

3.1 Mathematical Models for Pipe System Components

In this chapter we consider a number of mathematical models that describe fluid flow in various pipe configurations and the associated pressure drops and the figures are produced using MATLAB software.

3.2 Expansion, Contraction and diverging pipe system

We consider the horizontal straight pipe with varying cross-sectional area as shown in the figure below. The model diagram is as shown in figure 1 below. The letters A-F represent flow areas of the fluid. r and R are the radii at respective sections and L is the length of the pipe. The pipe is insulated against heat exchange and consequence heat changes. The model used in this study analyses head loss or pressure drop in abrupt expansion and contraction for a diverging pipe system. Fluid enters into the pipe system of length L at point A with pipe radius r. It then diverges into a bigger pipe of radius R at point B to D through C and eventually converges again into same pipe of radius r at point E. The centre of axis is along the horizontal axis. The fluid here flows into two distinct regions. A



11.png

Figure 3.1: **A model of a horizontal straight pipe with a varying cross sectional area**

region of abrupt expansion from A to C and a region of abrupt contraction from C to E.

3.2.1 Abrupt expansion, the diverging case

When the fluid suddenly leaves the smaller pipe and enters the wider part of the pipe there is a sudden deceleration of the fluid and the fluid being unable to move in sharp corners, it tears the boundary at the enlargement so that eddy currents are developed and these eddies dissipate a large amount of fluid energy and expansion losses occur. Hence from the law of conservation of mass

$$\sum F = \rho n Q(V_2 - V_1) \tag{3.1}$$

where V_1 and V_2 are fluid velocities at points 1 and 2 in the pipe. If A_1 is the area between A and B and A_3 is the area between D and E, then for incompressible flow

$$A_1 V_1 = A_3 V_3 = Q \tag{3.2}$$

If P_1 and P_2 are the pressures at A_1 and A_2, then

$$P_1 A_3 - P_3 V_3 = \rho Q(V_3 - V_1) \tag{3.3}$$

where ρ, A, and V refers to the fluid density, cross-sectional area of the pipe and speed of fluid respectively. From the above equations we find

$$A_3 \frac{P_1 - P_3}{\rho g} = \frac{\rho A_3 V_3}{\rho g}(V_3 - V_1) \tag{3.4}$$

$$\frac{P_1 - P_3}{\rho g} = \frac{V_3}{g}(V_3 - V_1) \tag{3.5}$$

Applying Bernoulli's equation to a horizontal streamline along the centreline of the pipe, and ignoring friction losses between the fluid and the pipe walls, but including the head loss at the expansion h_L, we find

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + h_L \tag{3.6}$$

Thus

$$h_L = \frac{P_1 - P_3}{\rho g} + \frac{V_1^2 - V_3^2}{2g} \tag{3.7}$$

and using equations (3.5) and (3.7) we have

$$h_L = \frac{(V_1 - V_3)^2}{2g} \tag{3.8}$$

The loss is a fraction of the square velocity as in turbulent flow. Exit flow from a pipe into a large reservoir is similar to a sudden expansion with $V_3 = 0$. So the expression becomes

$$h_L = \frac{V_1^2}{2g} \tag{3.9}$$

2. Abrupt contraction At the point F in the pipe, the pressure P is uniform, the equations of continuity, momentum, energy and stagnation pressure becomes

$$A_1 V_1 = A_3 V_3 \tag{3.10}$$

$$P_1 A_1 = P_2 A_3 - (P_0(A_1 - A_3) = A_1 V_1 (V_3 - V_1) \tag{3.11}$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + h_L \tag{3.12}$$

and

$$\frac{V_1^2}{2g} + \frac{P_1}{\rho} = \frac{P_0}{\rho} \tag{3.13}$$

If D_1 and D_3 denote diameters at A_1 and A_3 respectively, then simplifying

$$h_L = V_1 \frac{A_1 V_1 / A_3}{2g} = (1 - \frac{A_1}{A_3})^2 (1 - \frac{A_1}{A_3}) = 1 - \frac{D_1^2}{D_3^2})^2 \frac{V_1^2}{2g} \tag{3.14}$$

Thus the equation for the theoretical head loss is modified to the form

$$h_L = k \frac{V_1^2}{2g} \tag{3.15}$$

Where k is an experimental value equal to or less than and D is the diameter of the pipe. From equation (3.9), the head loss between the points $D$and F is

$$h_L = \frac{V_0}{2g} - \frac{V_0}{C_c} \tag{3.16}$$

where $C_c = \frac{A_c}{A_3}$ and A_c is cross-sectional area of vena contract coefficient of contraction. For flow from a large reservoir through a pipe with square entry

$$\frac{A_3}{A_1} = 0.k = \frac{1}{2} \tag{3.17}$$

Hence the final equation for calculating head loss arising from abrupt contraction without any re-entry is given by

$$h_L = \frac{1}{2}(1 - \frac{A_3}{A_1}) \frac{V_3^2}{2g} \tag{3.18}$$

where $k = 1/2$. As $A_1 \to \infty$ the value of k tends to 0.5 and this corresponds to flow from a large reservoir into a sharp edged pipe provided that the end of the pipe does not protrude into the reservoir. The Table below depicts the loss coefficient for sudden contraction.

Table 1: Loss coefficient for sudden contractions

$\frac{A_2}{A_1}$	0	0.1	0.2	0.3	0.4	0.6	0.8	1.1
k	0.5	0.46	0.43	0.36	0.30	0.18	0.06	0

3.2.2 Forms of Flow Resistance (Head loss due to friction)

One form of resistance to flow is due to the viscosity of the liquid. Viscosity is the amount of work needed to move one 'box' of liquid against another 'box' of liquid. Every liquid has its own value for this resistance to flow. SAE 30 motor oil has a lower viscosity and flows much easier than SAE 50 motor oil. The values for water are lower than for the motor oil. Another characteristic for any liquid is its attraction to a surface. It attaches itself to a surface and cannot be moved. The liquid in the 'box' on the very surface of a pipe does not flow or move. It always remains stationary. The liquid in the 'box' above it has to slide against it and that requires an amount of energy to overcome friction between the two 'boxes'. The higher the viscosity of the liquid is; the higher the resistance to flow; therefore the higher the friction loss. A layer is formed by this non-moving liquid and reduces the inside diameter of the pipe. This increases the velocity of the liquid passing through it. The head loss from friction is related to the velocity energy $v^2/2g$ of the liquid squared.

The liquid is not moving at the pipe wall but has a much higher velocity at the centre of the pipe.

$$h_L = \frac{v^2}{2g} \tag{3.19}$$

The condition of the inside of a pipe also has a great effect on the head loss of the flow of liquid. The rougher it is; the thicker the layer of non-moving or slow moving liquid near the pipe wall. This reduces the inside diameter of the pipe, increasing the velocity of the liquid. With the increase in velocity comes an increase in friction losses.

3.2.3 Pipe Fittings

Any time a liquid flow changes direction, there is resistance. Since all liquids have weight, they also have momentum. This means the liquid will always try to continue moving in the same direction. When the liquid encounters a change in direction (such as an elbow), its momentum carries the flow to the outer edge of the fitting because the liquid is trying to flow around the outer edge of the fitting, the effective area of the fitting is reduced. The effect is similar to attaching a smaller diameter pipe in the in the system. The velocity of the liquid increases and the head loss due to friction increases. Pipe fittings and valves disturb the normal flow of liquid, causing head loss due to friction. We have two methods currently to predict the head loss in pipe fittings and valves. They are the K factor and the Equivalent length of the pipe in linear feet methods. The fittings such as elbows, tees, strainers, valves, have all been tested and assigned "K " factors based on the head loss measured through them.

These are normally found in pump handbooks including the hydraulic Institute Data books. The pipe fittings and valves were tested and values assigned for the head loss measured through them. Instead of assigning a factor as in the "K " factor method, an "equivalent length of pipe in inner feet" valves was assigned. This means that a particular fitting will have a head loss equal to a given length of straight pipe of the same size.

3.2.4 Turbulent Pipe flow through a 90^0 bend

We provide high resolution velocity maps of the primary and secondary flow upstream and downstream of a 90^0 bend, with $R_c/R = 1$ (where R_c is the radius of bend and R is the radius of the pipe). The measurements were obtained using stereo- scopic particle image velocimetry (SP I V) at Reynolds numbers up to 105 . One of the particular areas of interest is the behaviour of the turbulent structure, specifically the very large scale motions (V LSM), where we build on the recent work of Helstrom et al (2011) who investigated the three dimensional character of the V LSM in fully-developed pipe flow (that is, in the flow upstream of the bend).

3.2.5 Energy Loss

Any time a liquid is compelled to change direction or to change velocity, there is a change in energy. The energy lost by the liquid is converted to heat created by friction. Since the amount of liquid exiting a pipe has to equal the amount entering the pipe, the velocity must be equal. If the velocity is equal, then the velocity energy (head) must be equal. This only leaves one place for the energy to come from; pressure energy. The measured pressure entering the pipe will be higher than the measured pressure exiting the pipe.

3.2.6 Friction Loss Tables

In an effort to easily predict the head loss in pipes and fittings, there were a number of studies made many years ago. These have been published,as formulas and tables for different size pipes; fittings; and flow ratings. the most commonly used are "Darcy, Weisbach" and "Williams and Hazen". They are good predictors of head loss but have some basic differences. The "Darcy, Weisbach" tables are based on the head loss in clean pipe.

$$H_L = F \times (\frac{L}{D}) \times (\frac{V^2}{2g}) \tag{3.20}$$

where H_L is the total head loss, F is friction factor related to the roughness inside the pipe, L is the length of the pipe, D is diameter of the pipe, V is average liquid velocity in the pipe, $2g$ is two times the universal gravitation constant. The "Williams and Hazen" tables takes a different approach. They are based on the head loss in ten-year old pipe. Their values are adjusted for different pipe age and materials.

$$\frac{H_L}{100} = 0.2083 \times (\frac{100}{C})^1.85) \times (\frac{Q^1.85}{D^4.8665}) \tag{3.21}$$

where H_L is head loss per 100 feet of pipe, C is correction factor to account for pipe roughness, Q is liquid flow rate in GPM, D is inside pipe diameter. The tables are for ten-year old steel pipes. Variations of this, such as new pipe, plastic pipe, cast iron pipe or other types are addressed through the use of correction factors. Ten-year old steel pipe has a 'C' value of 100 or a multiplier of 1.0 because that is what the tables are based on. Clean new steel pipe has a 'C' value above 100 or a multiplier below 1.0, which translates to lower head loss. Based on testing for ten-year old, steel pipe, the tables are 'divided by' the different pipe sizes. The following Table shows head losses at various rates of flow.

Table 2: Total head losses at various rates of flow

Qkg/s	V_1m/s	V_2m/s	$V_1^2/2g$mm	$V_2^2/2g$mm	Expansion (1-2)	Contraction (3-4)	Bend (5-6)
0.554	1.394	0.806	99.0	33.0	25	30	31
0.524	1.318	0.762	88.5	29.5	21	29	30
0.514	1.293	0.747	85.2	28.4	20	21	28
0.462	1.161	0.671	68.8	23.0	18	16	24
0.427	1.074	0.621	58.8	19.6	19	16	21
0.392	0.986	0.570	49.5	16.5	12	12	12
0.329	0.827	0.478	34.9	11.6	9	8	11

The friction head loss is estimated by choosing a suitable value of friction factor f for fully developed flow along a smooth pipe. The Prandtl equation

$$rewritethisequation \qquad (3.22)$$

is used. Typical values derived from this equation are presented in the table below.

Table 3: Friction factor f for smooth walled pipe

$10^4 \times R_e$	1	1.5	2	2.5	3	3.5
$10^3 \times f$	7.73	6.96	6.48	6.14	5.88	5.67

3.2.7 Factors that affect Head Loss

1. Flow rate When the flow rate (GPM) increases the velocity of the liquid increases at the same rate. The friction or resistance to flow (due to viscosity) also increases. The head loss is related to the square of the velocity so the increase in loss is very quick.

2. Inside diameter of the pipe When the inside diameter is made larger, the flow area increases and the velocity of the liquid at a given flow rate is reduced. When the velocity is reduced there is lower head loss due to friction in the pipe. on the other hand, if the inside diameter of the pipe is reduced, the flow area reduces the velocity of the liquid increases and the head loss due to friction increases.

3. Roughness of the pipe wall As the roughness of the inside pipe wall increases so does the thickness of the slow or non-moving boundary layer of the liquid. The resulting reduction in flow area increases the velocity of the liquid and increases the head loss due to friction.

4. Corrosion and Scale deposits Scale deposits and corrosion both increase the roughness of the inside pipe wall. Scale build up has the added disadvantage of reducing the inside diameter of the pipe. All these added up to a reduction in flow area, an increase of the velocity of the liquid and an increase in head loss due to friction.

5. Viscosity of the liquid The higher the viscosity of the liquid the higher the friction is from moving the liquid. More energy is required to move a high viscosity liquid than for a lower viscosity liquid.

6. Length of the pipe head loss due to friction occurs all along a pipe. It will be constant for each foot of pipe at a given flow rate.

7. Fittings: Elbows, tees, valves and other fittings are necessary to a piping system for a pump. It must be remembered that fittings disrupt the smooth flow of the liquid being pumped. When the disruption occurs, head loss due to friction occurs. At a given flow rate the losses for the fittings will be calculated using a factor that must be multiplied by a velocity head figure, or as the head loss equivalent to a straight length of the pipe.

8. Straightness of the pipe: Because of momentum, liquid wants to travel in a straight line. If it is disturbed due to crooked pipe, the liquid will bounce off the pipe walls and the head loss due to friction will increase. There is no accurate way to predict the effects since "crooked" can mean a lot of things.

3.2.8 Energy Equation and Concept of Heads

Assuming there is no shaft work or heat-transfer effects in a pipe-system, the steady flow energy equation is

$$z_1 + \frac{P_1}{\gamma} + \alpha_1 \times \frac{V_1^2}{2g} - h_f - \Sigma h_L = z_2 + \frac{P_2}{\gamma} + \alpha_2 \times \frac{V_2^2}{2g} \tag{3.23}$$

where α_1, α_2 are kinetic energy correction factors which are dimensionless V_1, V_2 are the cross-sectional average fluid velocities at points 1 an 2 respectively. g is acceleration due to gravity P_1 and P_2 are pressures at points 1 and 2 respectively z_1 and z_2 are elevations above the datum at points 1 and 2 respectively h_f is head loss due to boundary shear and h_L are minor losses. The equation applies only for control volumes which are single stream tubes i.e. Control volumes with only one inflow and one one outflow and with the inflow rates equal to each other. The head is defined as the rate at which kinetic and potential energies are transported by the flow plus the rate at which the fluid does work against the internal pressure, all divided by the rate at which the weight of the fluid is being transported by the flow. That is

$$rewritethisequation \tag{3.24}$$

Thus P or P = gh where h can be the frictional head loss, the pressure head, the head from the pump, and P is the rate of doing work (against shear stresses or pressure) or the power (from the pump) or the rate of transport of energy (the rate at which kinetic energy is transported by the fluid when the head is the velocity head). Since h is defined only for single stream tubes and since h is obtained by dividing P by g, head is like an intensity in that it is associated with each unit of fluid.

3.2.9 Frictional Head Losses

Frictional head losses are losses due to shear stress on the pipe walls. The general equation for head loss due to friction is the Darcy-Weisbach equation,

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{3.25}$$

, where f is Darcy-Weisbach friction factor L is length of the pipe, D is pipe diameter V is cross-sectional average flow velocity. The equation is valid for pipes of any diameter and for both laminar and turbulent flows. For laminar flow,

$$f_{\text{laminar}} = \frac{64}{R_e} \tag{3.26}$$

where R_e is the Reynold's number defined for pipe flow as

$$R_e = \frac{\rho V D}{\mu} = \frac{V D}{\nu} \tag{3.27}$$

Substituting f_{laminar} and Reynold's number into the Darcy-Weisbach equation yields

$$re - writethisequation \tag{3.28}$$

for turbulent flow, the friction factor is obtained from a moody diagram or from the Colebrook-White equation given by

$$rewritethisequation \tag{3.29}$$

where K_s is the equivalent sand roughness and Ks is the relative roughness. The values of f in the moody diagram and in the Colebrook-White equation are empirical.

3.2.10 Falling Head Viscometer

Substituting

$$rewriteequations 3.31 - 3.39 \tag{3.30}$$

into V = Q and solving for Q, the result for a laminar flow can be written as for a tank and tube shown in figure below, $h_{f_{\text{laminar}}=h}$ if the entrance loss and the velocity head at the downstream end of the tube are negligibly small. Taking the tank as a control volume, the continuity equation gives Since the depth of liquid in the tank is h minus a constant (Z_b) and since there is only one outflow, and separating variables, the result can be written as Integrating with h = h0 at t = 0 gives where measuring h as a function of time as a liquid drains from the tank and plotting h 0 on a semi logarithmic graph produces a straight line when there is a laminar flow in the tube. The slope of the line is f . Determination of f allows m to be evaluated.

3.2.11 Losses in Pipe Bends

Bends are provided in pipes to change the direction of flow through it. An additional loss of head, apart from that of due to fluid friction, takes place in the course of flow through pipe bend. The fluid takes a curved path while flowing through a pipe bend as shown in figure Whenever a fluid flows in a curved path, there must be a force acting radially inwards on the fluid to provide the inward acceleration, known as centripetal acceleration. This results in an increase in pressure near the outer wall of the bend, starting at some point A and rising to a maximum at some point B. There is also a reduction of pressure near the inner wall giving a minimum pressure at C and a subsequent rise from C to D. Therefore between A and B and between C and D the fluid experiences an adverse pressure gradient (the pressure increases in the direction of flow). Fluid particles in this region, because of their close proximity to the wall have low velocities and cannot overcome the adverse pressure gradient and this leads to a separation of flow from the boundary and consequent losses of energy in generating local eddies. Losses also take place due to a secondary flow in the radial plane of the pipe because of a change in pressure in the radial depth of the pipe. This flow in conjunction with the main flow, produces a typical spiral motion of the fluid which persists even for a down stream distance of fifty times the pipe diameter from the central plane of the bend. This spiral motion of the fluid increase the local flow velocity and the velocity gradient at the pipe wall, and therefore results in a greater frictional loss of the head than that which occurs for the same rate of flow in a straight pipe of the same length and diameter. The additional loss of head (apart from that due to usual friction) in flow through pipe bends is known as bend loss and is usually expressed as a fraction of the velocity head as $KV^2/2g$. where V is the average velocity of flow through the pipe. The value of K depends on the total length of the bend and the ratio of radius of curvature of the bend and pipe diameter R/D. The radius of curvature R is usually taken as the radius of curvature of the centre line of the bend. The factor K varies slightly with Reynold's number Re in the typical range of Re encountered, in practice, but increases with surface roughness.

Chapter 4

Numerical Computation

4.1 Numerical Computations

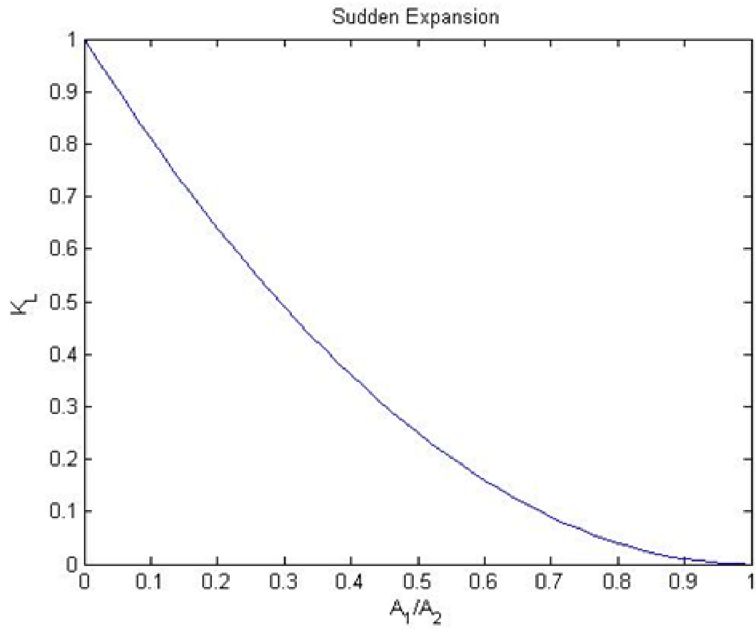
In this chapter we analyze and present graphical solutions of the models discussed in chapter three. The figures are produced by MATLAB software.

4.1.1 Sudden expansion

The graph for sudden expansion is given in the figure below from equation (2.10) where k_L is the head loss.

4.1.2 Sudden Contraction

The graph for sudden contraction is given in the figure below from equation (2.19)



41.png

Figure 4.1: **sudden expansion**

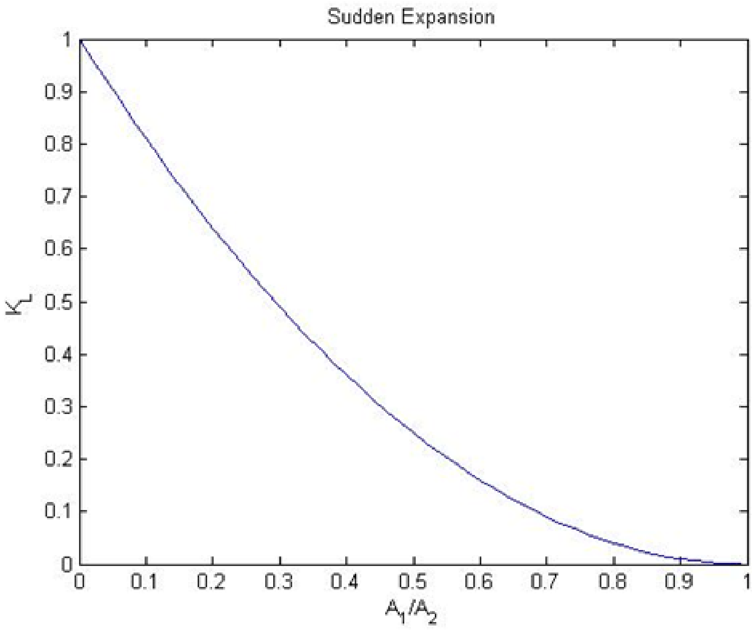
4.1.3 Scaled Energy Content

The figure below shows the scaled energy content using velocity vectors.

$$k_L = (1 - \frac{A_2}{A_1})^2 \tag{4.1}$$

4.1.4 Auto-correlation of Tangential Velocity

The figure below shows the auto-correlation of the tangential velocity, taken in the center of one of the cells separated by the symmetry line. It shows a highly oscillating trend and by choosing a velocity pair with a temporal shift corresponding to either a negative or positive correlation,



42.png

Figure 4.2: **sudden contraction**

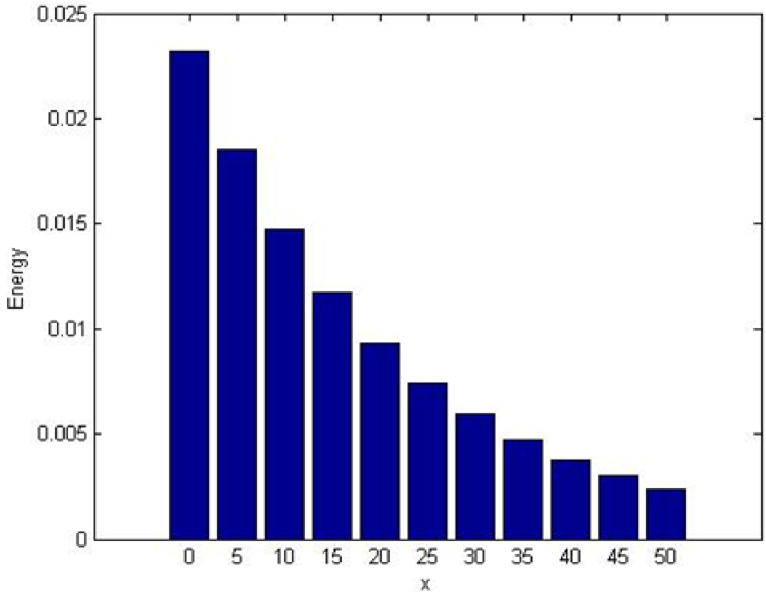
we can visualize the corresponding flow structure.

4.1.5 Head losses in pipes

4.1.6 Head loss against pipe length from equation (2.28)

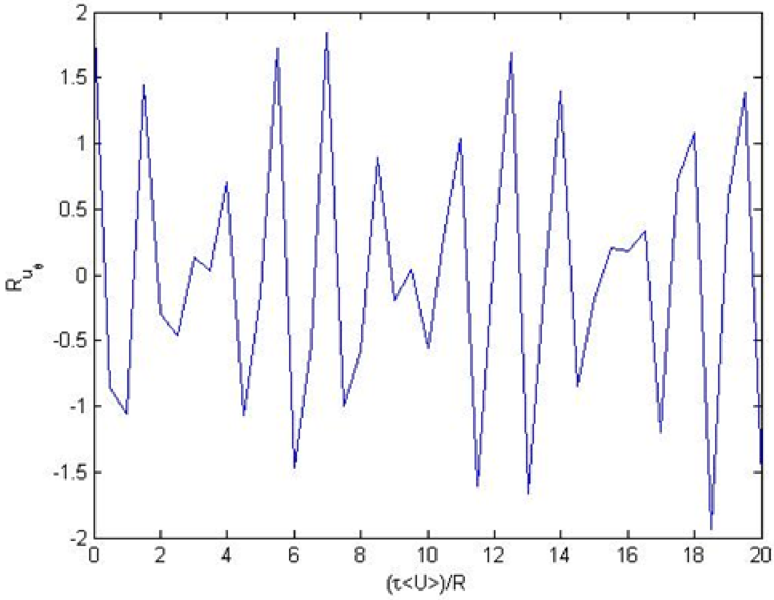
4.1.7 Head loss against velocity from equation (2.28)

4.1.8 Head loss due to bend in pipe



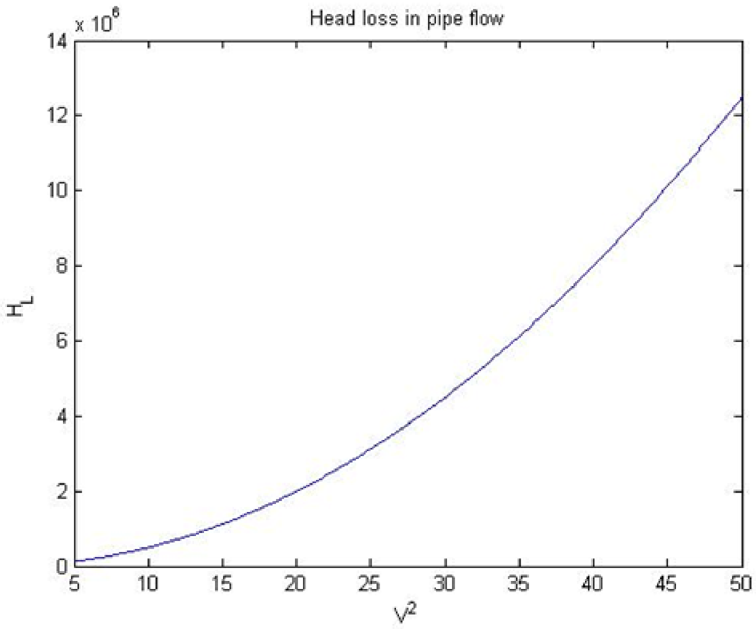
43.png

Figure 4.3: **Energy content**



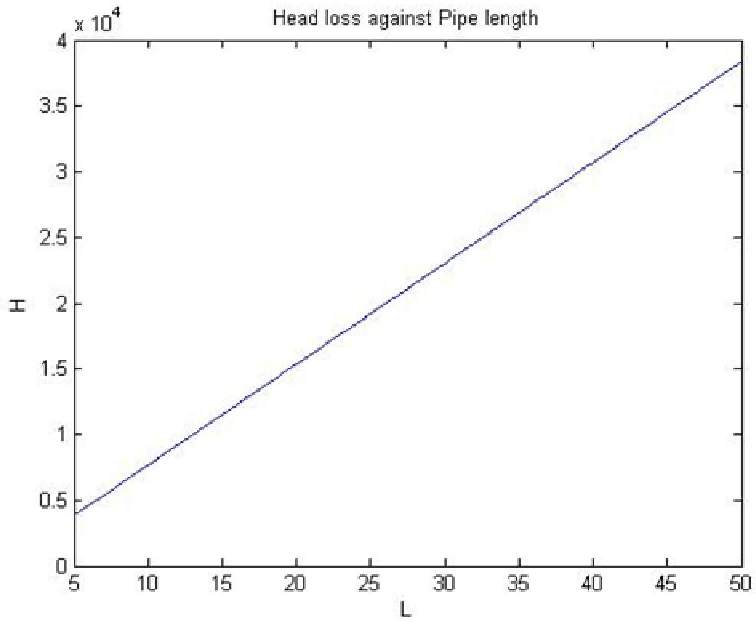
44.png

Figure 4.4: **Auto-correlation of tangential velocity**



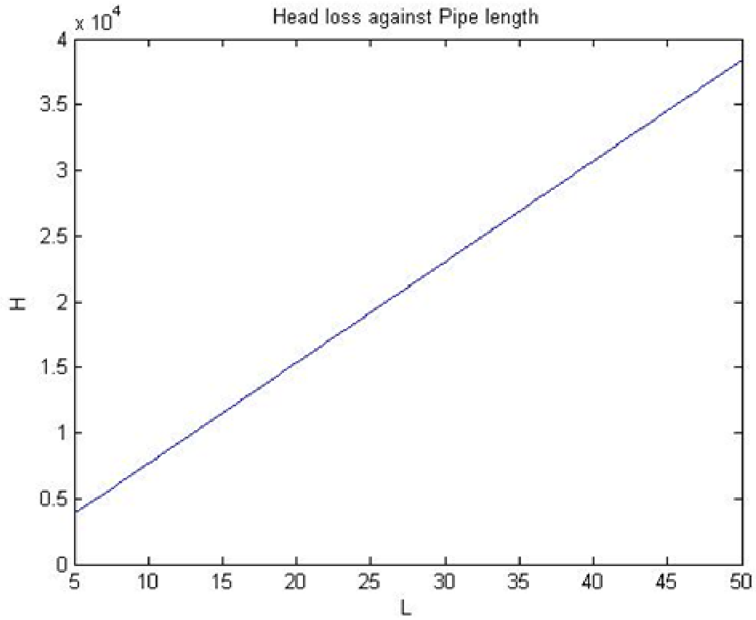
45.png

Figure 4.5: **Head Loss in pipes**



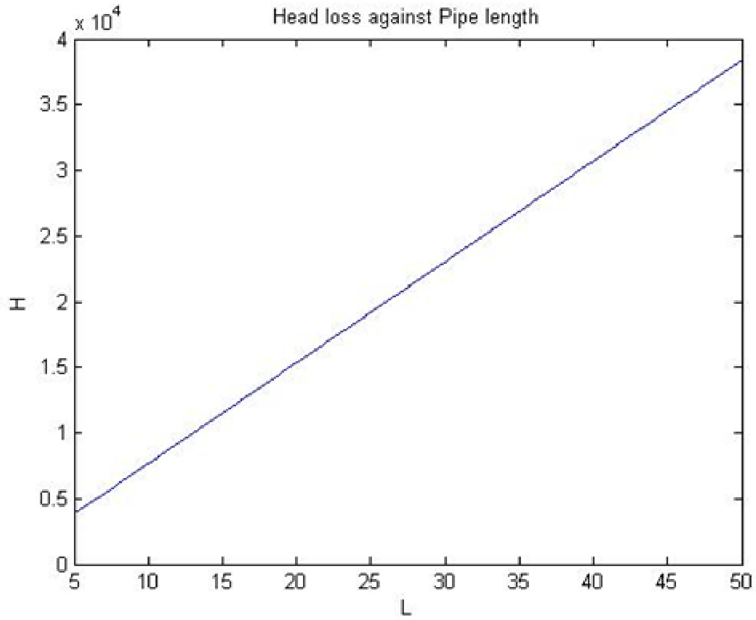
46.png

Figure 4.6: **Head Loss against pipe length**



47.png

Figure 4.7: **Head Loss against velocity**



48.png

Figure 4.8: **Head Loss in pipe bends against velocity**

Chapter 5

Conclusion and Recommendations

5.1 Conclusion

In pipe flow, the head loss term k_L gradually decreases with the ratio of minimum to maximum areas of the pipe components used as seen in figure 2 and figure 3 for sudden expansion and sudden contraction. We have a slight ratio difference between maximum and minimum areas of the pipe systems used. The energy flow structure of the fluid in the pipe also decreases with distance. The head loss in pipe systems increases with the speed of the flow as shown if figure 4. the higher the head loss, the higher the speed. Figure 5 also shows that the head loss h_L increases with increasing length of the pipe. The head loss to bends in pipes have gradual increasing speed when they are compared as seen in figure 6. In this work, we developed and analyzed models for head loss in abrupt expansion, contraction, friction, bends and pipe fittings. We modified and simplified fluid flow models using general fluid mechanics concepts.

5.2 Recommendations

1. We recommend comprehensive research in fluid flow so as to make the flow in pipe system more efficient

2. The government and research institutions should find more researchers in this field of study.

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