AN INVESTIGATION ON MAGNETOHYDRODYNAMICS FLUID FLOW BETWEEN TWO PARALLEL INFINITE PLATES SUBJECTED TO AN INCLINED MAGNETIC FIELD AND VARYING TEMPERATURES

BY

JACOB OTIENO RADING

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS IN THE SCHOOL OF SCIENCE

UNIVERSITY OF ELDORET, KENYA

2015

DECLARATION

Declaration by Candidate

This thesis is my original work and has not been submitted for academic award in any institution; and shall not be reproduced in part or full, or any format without prior written permission from the author and/or University of Eldoret

Name: **RADING JACOB OTIENO**
SC/PGM/010/11
Signature:………………………….. Date……………………………..

Declaration by Supervisors

This thesis has been submitted with our approval as the University Supervisors:

1. **Prof. Jacob Bitok**
   Department of Mathematics and Computer Science
   University of Eldoret
   P. O. Box 1125 – 30100, Eldoret, Kenya
   Signature…………………………… Date………………………………

2. **Prof. Alfred W. Manyonge**
   School of Mathematics, Statistics and Actuarial Science
   Maseno University
   Private Bag, Maseno, Kenya
   Signature…………………………… Date………………………………

DEDICATION

I dedicate this work to my mother, Mary A. Okelo, my wife, Susan Soi, and my two children, Sandra and Frank Rading.

ABSTRACT

Magnetohydrodynamics (MHD) is the study of electrically conducting fluids in the presence of magnetic fields. This study examined the motion of a viscous, electrically conducting and incompressible fluid flowing steadily between two infinite parallel horizontal plates along one dimension. The plates were subjected to an inclined magnetic field and heated at varying temperatures. The objectives of the study were to determine the effects of variation of temperature of the plates, angle of inclination of magnetic field, Reynold's number, Hartmann number Prandtl number and pressure gradient on the velocity of the flow of the fluid. The equations governing the fluid motion were used to develop the model; these are a combination of Maxwell's equation of magnetism and Navier-Strokes equation of fluid dynamics, continuity equation, momentum equation and energy equation. The model governing the flow was developed with appropriate boundary conditions imposed, simplified using the boundary layer approximations, non-dimensionalized and solved analytically. The solutions were simulated and with the use of mathematical software MATLAB; the results for various values of varying temperatures (from 20^0C to 50^0C), the angle of inclination of magnetic field (from $\frac{\pi^c}{6}$ to $\frac{\pi^c}{2}$), Reynolds number (from 0.2 to 0.8), Hartmann number (from 2.4 to 3.0), Pressure gradient (from 2.0 to 8.0) and Prandtl number (from 0.07 to 0.22) were obtained and presented graphically. The objectives of the study were to determine the effect of the following variables on the flow velocity: the angle of inclination of magnetic field, change in temperature, Prandtl number and Reynold's number. The results obtained from the study showed that when the angle of inclination of magnetic field is reduced, the flow velocity increases; increase in velocity of the fluid also results from increase in temperature of the fluid when the plates are heated; increase in Hartmann number retards the velocity of the flow; the velocity will increase when the pressure gradient increases; increase in Reynolds number leads to increase in velocity while increase in Prandtl number reduces the temperature of the fluid and as a result retards the velocity of the flow.

TABLE OF CONTENTS

Page

LIST OF TABLES

Page

LIST OF FIGURES

LIST OF ABBREVIATIONS, ACRONYMS AND SYMBOLS

α	Angle of inclination of magnetic field
ρ	Fluid density
σ	Electrical conductivity of the fluid
ν	Kinematic viscosity
$\vec{E}$	Electric field strength
$\vec{B}$	Magnetic flux density
$\vec{J}$	Current density
$\vec{H}$	Magnetic field intensity
T	Temperature
T_w	Temperature of the plate
T_∞	Temperature at the free stream
$\frac{\partial}{\partial x}$	Partial derivative with respect to x
∇	Divergence operator
P \`	Pressure gradient
$\vec{D}$	Electric flux density
$\vec{u}$	Velocity of the fluid along x-axis
$\vec{F}$	Body force per unit mass
$\vec{v}$	Velocity of the fluid along y-axis

$\vec{w}$	Velocity of the fluid along z-axis
ε_0	Permittivity of free space (electric constant)
μ_0	Permeability of free space (magnetic constant)
μ	Fluid viscosity
θ	Dimensionless temperature
V_o	Suction velocity
β	Thermal expansion coefficient
k	Thermal conductivity
C_P	Specific heat capacity
γ	Charge density
MHD	Magnetohydrodynamics
N-S	Navier-Stokes
MATLAB	Matrix Laboratory computer software

ACKNOWLEDGEMENT

I am very grateful to all people who have made this work a success.

I am greatly indebted to the Catholic university of Eastern Africa for granting me the time and support to undertake my studies. I greatly thank Dr. Kakooza Celestine and Dr. Jacinta Adhiambo for their continual guidance and encouragement throughout the tenure of this work.

I express my profound gratitude to my Supervisors: Prof. Jacob Bitok and Prof. Alfred Manyonge for their dedication, guidance and support during the period of my studies and particularly for being patient and available to ensure the completion of this research.

I also express my heartfelt gratitude to the staff of the Department of Mathematics and Computer Science of the University of Eldoret, my classmates and friends for their suggestions, assistance and support during my studies.

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

According to Munso et al (2002) a fluid is any substance that deforms continuously when acted on by a shearing stress of any magnitude. A fluid as a substance that flows; one which does not flow is termed as a solid. He divides fluids into two categories: liquids which are incompressible and gases which are compressible. Another type of fluid is plasma: this refers to an electrically neutral medium of positive and negative particles. When the charges move they generate electric current and magnetic fields, as a result, they are affected by each other's field, Chalton (2004).

Fluid dynamics is a discipline of science that deals with fluids (plasma, liquids and gases) in motion. The motion of fluids may either be steady or unsteady. In steady flow the flow variables such as velocity, pressure, cross section, density etc. are independent of time whereas for unsteady flow the flow variables are functions of time, i.e. if F (x, y, z, t) represents a flow variable, then

$\frac{dF}{dt} = 0$for steady flow

and

$\frac{dF}{dt} \neq 0$ for unsteady flow

This research does not deal with the motion of an individual particle in a fluid but a continuum of particles in plasma; it is assumed that the distance between fluid molecules (mean free path) is very small.

Fluid dynamics therefore offers a systematic structure that embraces empirical and semi-empirical laws derived from flow measurements and used to solve practical problems. The solution to a fluid dynamics problem typically involves properties of fluids such as velocity, pressure, density and temperature as a function of space and time, Douglas et al (2005).

1.1.1 Boundary Layer

Boundary Layer is the layer of fluid in the immediate vicinity of boundary surface. The boundary layer effects occur at the field region in which all changes occur in the flow pattern. The flow region is split into two regions, namely the thin boundary layer region near the wall and potential flow region outside the boundary layer. The velocity, $\vec{u}$, within the boundary layer increases from $\vec{u} = 0$ at the wall to velocity, $\vec{U}$, at the region away from the wall where the velocity is nearly equal to free stream, referred to as the velocity boundary layer. The Boundary layer thickness is arbitrarily defined as that distance from the wall in which the velocity reaches 99% of the velocity of free stream ($\vec{u} = 0.99\vec{U}$). The thickness of the boundary layer grows along the surface over which the fluid is flowing from the leading edge.

When the plates are heated, there exists a thermal boundary layer. This occurs if the temperatures of the fluid near the plates and that at the free stream differ. Particles in contact with the plates achieve the same temperature as the plate and in turn pass the heat by convection to adjacent layers, creating temperature gradient. There is also velocity boundary layer which occurs simultaneously with the thermal boundary layer.

The concept of boundary layer forms the starting point for simplification of equations for any flow of an incompressible viscous fluid in which the Navier-Stokes equations are solved subject to certain boundary conditions. These equations are non-linear and thus a straight forward solution is not possible. Therefore there is a need to make simplifying assumptions which should be consistent with the given physical solution.

1.1.2 Heat Transfer

Heat is energy in transit due to temperature difference. Whenever there exists a temperature difference in a medium or between media, heat transfer must occur from higher temperature medium to a lower temperature medium. There are three modes of heat transfer, namely conduction, convection and radiation. In this study, conduction and convection may take place when there is variation of the temperature of the two parallel plates.

Conduction refers to the transfer of heat within a body or between two bodies in contact, by atomic and molecular interaction, and by the drift of free electrons. Conduction is governed by Fourier's law which states that the heat flux, $\vec{q}$,resulting from thermal conduction is proportional to the magnitude of temperature gradient and opposite to it in sign. In one dimensional form,

$\vec{q} = -k\dfrac{\partial T}{\partial x}$,   where k is the thermal conductivity.

Convection is the transfer of heat by the motion of a fluid. It arises from temperature and density differences within the fluid or between the fluid and its boundary, or even from an external motive forces. In free convection, the fluid bounding a heat source is heated, becoming less dense and rises. Consequently, the surrounding cooler and denser fluid then moves to replace it and, as a result, becomes heated and the process repeats itself thus forming convection currents. The driving force for free convection is buoyancy, a phenomenon that results from differences in fluid density when gravitation field or any other type of body force is present in the system such as buoyancy, drag, electromagnetic forces, etc.

When a solid body is exposed to a moving fluid having temperature difference from that of the solid, energy is carried from or to the solid body by the fluid, obeying Newton's law of cooling defined as $\vec{q} = -k(T_w - T_\infty)$

where $\vec{q}$ is the heat transfer per unit time $(\vec{q} = \dfrac{\partial Q}{\partial t})$, T_w is the surface temperature, T_∞ is the free stream temperature and k is the constant of proportionality, referred to as the heat transfer coefficient.

1.1.3 Magnetohydrodynamics (MHD)

MHD is the study of the flow of electrically conducting fluids in the presence of magnetic fields. It is the physical mathematical framework that concerns the dynamics of magnetic fields in electrically conducting fluids e.g. in plasma, liquid metals and electrolytes.

According to Witalis (1986) the central point of MHD theory is that magnetic fields can induce currents in a moving conductive fluid, which in turn creates forces on the fluid and also changes the magnetic field itself. The presence of magnetic fields leads to forces that in turn act on the fluid (typically plasma), thereby altering the geometry (topology) and strength of the magnetic fields.

Such phenomenon occurs both in natural and in man-made devices. Naturally, MHD takes place in the sun, the earth's interior, the stars and their atmosphere etc. In the man-made phenomena, many new devices have been invented which utilize the MHD interaction directly. These include power generators and propulsion units.

MHD phenomena result from a magnetic field and a conducting fluid flowing across it. Thus, electromagnetic force is produced in the field flowing across a transverse magnetic field and then magnetic field combines to produce a force that resists the motion of the fluid. The current also generates magnetic field which distorts the original magnetic field. An opposing force on the field can be produced by applying an electric field perpendicularly to the magnetic field or the fluid can propagate in both to produce MHD waves.

1.2 Statement of the problem

Many studies have been carried out in fluid mechanics, particularly on MHD. For instance, Manyonge, Kiema and Iyaya (2011) carried out a study on steady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field. They discovered that high Hartman flow and high magnetic field strength decrease the velocity of the flowing fluid. Ganesh and Krishnambal (2007) studied the unsteady Stoke's flow of an electrically conducting viscous incompressible fluid between two parallel porous plates of a channel in the presence of a transverse magnetic field, when the fluid is being withdrawn through the walls at the same rate. They found that when Hartmann number M was increased from $M = 1$ to $M = 2$ or $M = 5$, the magnitude of the axial velocity profiles decreased. They also discovered that the velocity gradually increases with the increase of magnetic Reynolds number.

Giovanni (2010) carried out a study of asymptotic behaviour of steady motion of a viscous fluid in pipes of arbitrary cross-sections under a transverse magnetic field when the Reynolds number trends to infinity. He found that just as Hartmann (1937), the boundary layer occurs on the entire boundary. He also discovered that the Hartmann boundary layer occurs not directly on V and h but in their principle parts γ and H.

All these related studies have addressed many factors that affect the velocity of electrically conducting fluid flowing between two parallel plates; however the effect of subjecting the plates to temperate changes during the flow has not been addressed. This study, therefore, considers a steady viscous flow of an incompressible electrically conducting fluid, along the x - axis, between two horizontal parallel infinite plates situated at $y = -L$ and $y = L$; both the plates at $y = -L$ and $y = L$ are stationary. An inclined magnetic field at an angle α is applied to the direction of the flow. There is no external applied electric field and it is assumed that steady flow conditions have been attained. When the plates are heated, the temperatures of the fluid near the plates and that at the free stream will differ, thus creating a thermal boundary layer. It is required to analyze the velocity profiles, considering that the plates are heated at varying temperatures. The temperature of the plate is taken to be higher than that at the free stream $(T_w > T_\infty)$. Particles in contact with the plates achieve the same temperature as the plate and pass the heat to adjacent layers by convection, creating a temperature gradient. Figure 1 below is a geometrical presentation of the flow.

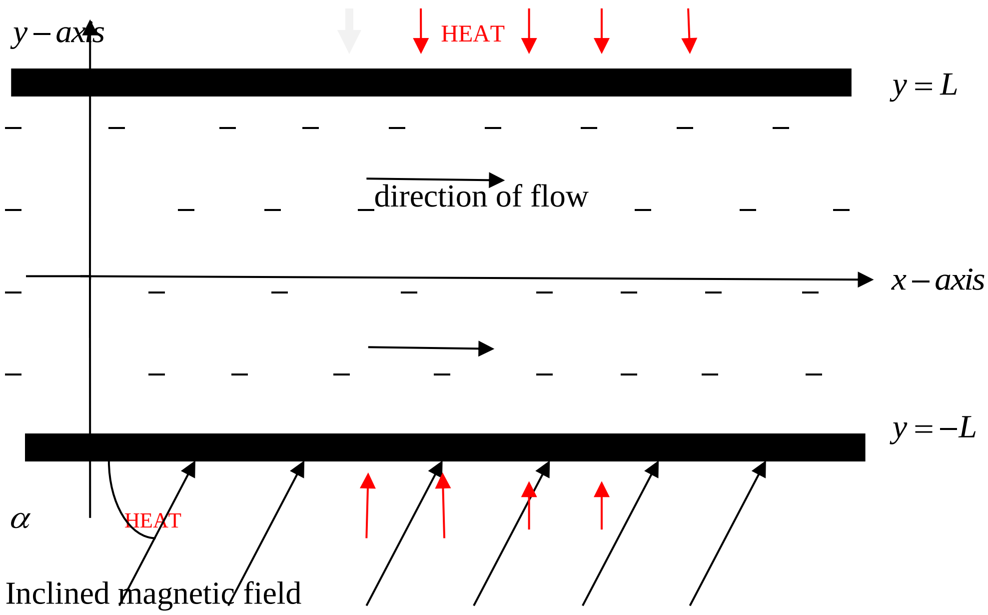

Figure 1: Geometric configuration of the fluid flow along x-axis

1.3 Objectives of the Study

The specific objectives of this study are:

(i) To determine the effect of variation of temperature of the plates on the velocity of electrically conducting fluid flowing between two parallel plates in magnetic field.

(ii) To determine the effect of different angles of inclination of the magnetic field on the velocity of electrically conducting fluid flowing between two parallel plates.

(iii) To determine the effect of Prandtl number on the velocity ofelectrically conducting fluid flowing between two parallel plates.

(iv) To determine the effect of Reynolds number on the velocity ofelectrically conducting fluid flowing between two parallel plates.

(v) To determine the effect of Pressure gradient on the velocity of electrically conducting fluid flowing between two parallel plates.

1.4 Significance of the Study.

This study focuses on the MHD flow between two parallel infinite plates subjected to an inclined magnetic field and heated at varying temperatures. Its findings will be of great significance in Applied Mathematics and Engineering. The findings will assist in further development of plasma physics and geophysics. The study will also assist in improvement of the current devices which employ MHD flow i.e. MHD power generators, cooling systems aerodynamics, heating polymer technology, MHD pumps, electromagnetic flow meter, etc. Thermal radiation effects on flow and heat transfer processes are also of major importance in the space technology and high temperature processes.

1.5 Assumptions of the Study

In order to solve the flow problem of this study of MHD, the following assumptions have been made:

(i) There is no external applied electric field thus electric dissipation is negligible.

(ii) The fluid flows with constant density, ρ, and the velocity $\vec{v}$ of the fluid is much smaller than the velocity of light, c, i.e. $\frac{\vec{v}^2}{c^2} < 1$. This is to avoid relativistic effects.

(iii) The temperature of the plates will be varied significantly where the difference between the temperature of the plates and the free stream is moderately large, causing free convection currents.

(iv) The plates are considered to be infinite.

(v) The flow is restricted to laminar flow.

(vi) The viscosity μ is constant

CHAPTER TWO

LITERATURE REVIEW

2.1 Empirical Review of Literature

The principle of Magnetohydrodynamics was first demonstrated in experiments of Faraday and Richie(Faraday,1832). Faraday experimented with the flow of mercury in a glass tube placed between poles of magnet; he discovered that a voltage was induced across the magnetic field, perpendicular to the direction of the flow and the magnetic field. He also observed that the current generated by this induced voltage interacts with the magnetic field to slow down the motion of the fluid and he was aware that the current produced distorted the field of the magnet. The practical application of Faraday's ideas came with Smith and Stephen's invention, in 1917, of an instrument for measuring ship's speed and was based on the principle that the induced voltage is proportional to the flow rate.

Young et al (1920) studied tidal motions with an induced voltage device; a technique since then widely used in oceanography. Further fundamental work was done by Hartman and Lazarus (1937). They studied the influence of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid between two infinite parallel stationary plates. They also did a similar study but using two infinite parallel stationary and insulating plates. Since then, work in MHD flow has received much attention and has extended in numerous ways.

Rossow (1957) did a study on free convection of MHD flows. He discovered that for incompressible constant property flat plate boundary layer flow, the skin friction and heat transfer were reduced substantially when a transverse magnetic field was applied.

Agarwal et al (1980) studied the combined buoyancy effects of thermal and mass diffusion on MHD natural convection flows. They calculated the skin friction and the rate of heat transfer from the plate to the fluid. They found that the rate of heat transfer from the plate to the fluid decreased as suction velocity increased.

Rapits and Tzirandis (1981) analyzed the free convection flow of an electrically conducting incompressible and viscous fluid past a vertical infinite plate under action of transverse magnetic field in rotating fluid. The plate and fluid were assumed to rotate in a solid body in a steady state. They obtained the exact solutions for the velocity and the temperature and discussed the effects of Hartmann number on the flow characteristics.

Soundalgekar et al (1981) studied the velocity and temperature field in MHD Falkner-Skan flow. They reduced the problem to a system of two differential equations in terms of variables and solved them numerically. They observed that the velocity, skin friction and the rate of heat transfer increased while temperature reduced when the strength of magnetic field was increased.

Campos (1985) carried out an investigation on vertical hydromagnetic waves in a compressible atmosphere under an oblique magnetic field. The results showed that the velocity perturbation transverse to the magnetic field and gravity satisfied a decoupled second order Alfven wave equation and the forth order wave equation. The coupled slow-fast mode was solved exactly using a variable, the velocity perturbation transverse to the magnetic field in the plane of the latter and gravity. It was shown that for oblique waves in an oblique magnetic field there are three cut- off frequencies namely the gravity, acoustic and magnetic cut-offs.

Malashetty and Umavathi (1997) investigated on MHD flow and heat transfer in an inclined channel where one phase is electrically conducting. The transport properties for both fluids were assumed constant. The resulting governing equations were coupled and approximate solution obtained using perturbation method. The results were presented for various values of the viscosities, thermal conductivities, heights of the fluid, Hartmann number, Grashof number and angle of inclination. The results showed that the velocity and temperature can be increased or decreased with suitable values of viscosity, thermal conductivity, the heights and the angle of inclination.

Chaturani and Saxena (2000) investigated a two-layered MHD model for parallel plate haemodialysis, under the influence of uniform transverse magnetic field. Analytical expressions for velocity profiles in the core region and peripheral plasma layer along with flow rate, effective viscosity and effective Reynolds number were obtained. It was discovered that the effect of Peripheral Plasma Layer (PPL) thickness and Hartmann number on physiologically important fluid dynamic quantities (velocity, flow rate, effective viscosity, effective Reynolds number and induced magnetic field) may be used in designing the dialysers.

Agarwal et al (2005) carried out an investigation of MHD unsteady viscous flow through a porous straight channel. The study was carried out with two parallel porous flat walls in the presence of transverse magnetic field under a time varying pressure gradient. Exact solutions for the problem were obtained when the pressure gradient was constant. It was found that the effect of magnetic field in the presence of suction is to accelerate the flow and in the presence of injection is to retard the flow. It was also found that as the value of Hartman number increases the flow becomes more steady rapidly.

Mhone and Makinde (2006) carried out an investigation on unsteady MHD flow with heat transfer in a divergence channel. The non-linear governing equations were obtained and solved analytically using perturbation technique. Results showed that the effect of increasing values of heat on steady flow is to dampen the velocity profile. This is known as Hartmann flow. Moreover for a channel of varying across different sections of the channel, the dampening is more pronounced in the centre of the channel. This creates a stagnation point and consequently fluid is pushed to the walls of the channel, thereby increasing the velocity in the boundary layer.

Guria et al (2006) studied unsteady hydromagnetic flow in a rotating channel in the presence of a uniform magnetic field which is inclined at an angle θ with axis of rotation, under the influence of periodic pressure gradient. An exact solution of the governing equations was obtained. For large values of coriolis parameter and frequency parameter, there arises thin double-decker boundary layers near the walls of the channel. The thickness of these layers were found to increase with increase in the angle of inclination of magnetic field. They found that both primary and secondary velocity decreased with increase in θ (angle of inclination)

Ganesh and Krishnambal (2007) studied the unsteady Stoke's flow of an electrically conducting viscous incompressible fluid between two parallel porous plates of a channel in the presence of a transverse magnetic field, when the fluid is being withdrawn through the walls at the same rate. An exact solution was obtained for all values of Reynolds number (R_e) and Hartmann number (M). Expressions for the velocity components and the pressure were obtained and the graphs for axial and radial velocity profiles were drawn and interpreted. They found that when Hartmann number was increased between $M = 1$ and $M = 5$, the magnitude of the axial velocity profiles decreased. They also discovered that the velocity gradually increased with the increase of magnetic Reynolds number.

Singh (2007) investigated steady flow of viscous incompressible fluid between two parallel infinite plates under the influence of inclined magnetic field. The upper plate was taken to be moving with a constant velocity while the lower plate was held stationary under the influence of inclined magnetic field. The problem was solved by method of solution of linear differential equations with constant coefficients. Analytical expression for fluid velocity was obtained and illustrated graphically. The results showed that velocity profiles decreased as the strength of magnetic field was increased. It was also noted that with increase of inclination of magnetic field, there was a decrease in the velocity profile.

Kwanza et al (2010) studied the unsteady free convection MHD flow past a semi-infinite vertical porous plate in the presence of strong magnetic field. A finite difference method was used to solve the non-linear partial differential equations, after non-dimensionalizing the equations. The effect of Hall and ion- slip currents together with that of viscous dissipation and radiation absorption among other parameters on velocity, temperature and concentration profiles were presented graphically. It was found that in the presence of heating of the plate by free convection current, the velocity boundary layer thickness decreased. The results also showed that increase in mass diffusion parameter increased the primary velocity profiles and decreased the secondary velocity profiles.

Giovanni (2010) carried out a study of asymptotic behaviour of steady motion of a viscous fluid in pipes of arbitrary cross-sections under a transverse magnetic field when the Reynolds number tends to infinity. He examined the mathematical aspects of the formation of the Hartmann's boundary layer. He compared his model with that of Hartmann (1937) who modeled the flow between two parallel walls, $x=-L$ and $x=L$, and z-axis parallel to the fluid velocity which is a function of a sole variable x. His study was a 2-dimensional case which used singular perturbation problem that presented a transition from a second order elliptic system to a first-order system. He found that just as Hartmann, the boundary layer occurs on the entire boundary. He characterized the first order boundary value problem that is solved by the limit solution and he proved that temperature tends to the temperature distribution that would exist in the absence of the flow. He also discovered that the Hartmann boundary layer occurs not directly on V and h but in their principle parts γ and H.

Ziya and Manoj (2011) studied MHD heat and mass transfer free convection flow near the lower stagnation point of an isothermal cylinder imbedded in porous domain with the presence of radiation. The equations of conservation of mass, momentum, energy and concentration which governed the study were obtained and transformed into a system of non-linear ordinary differential equations. They were solved by Runge-Kutta and shooting methods. Velocity profiles, temperature distribution and concentration distribution for the flow were presented for various values of radiation parameter, viscosity variation parameter, thermal conductivity variation, Prandtl number and Schmidt number. The skin friction, local Nusselt number and Sherwood number were also calculated for all parameters involved in the problem. The findings showed thatincrease in Schmidt number lead to decrease in skin friction and Nusselt number but it lead to increase in Sherwood number.

Manyonge et al (2012) carried out a study on steady MHD poiseuille flow between two infinite parallel porous plates in an inclined magnetic field. They examined the motion of a two-dimensional steady flow of a viscous, electrically conducting, incompressible flowing between two infinite parallel porous plates under the influence of transverse magnetic field and constant pressure gradient. The lower plate was assumed porous while the upper plate was not. The resulting coupled governing equation of motion was solved by analytical method. Analytical expression for the fluid velocity obtained was expressed in terms of Hartman number. They discovered that high Hartman flow and high magnetic field strength decreases the velocity of the flowing fluid.

2.2 Knowledge gap

All the related studies have addressed many factors that affect the velocity of electrically conducting fluid flowing between two parallel plates, either porous or non-porous plates. The factors widely dealt with in the reviewed studies included transverse magnetic field, pressure gradient, Reynolds Number and Hartmann Number, among others, however the effect of heating the plates at different temperatures during the flow has not been addressed. This study, therefore, focuses on the effect of temperature changes of the plates on the velocity of the fluid flowing between two parallel infinite plates. An inclined magnetic field at an angle α is applied to the direction of the flow.

CHAPTER THREE

METHODOLOGY

This study examined the flow along the x-axis between the horizontal parallel plates heated at varying temperatures with the help of equations governing the fluid motion.

To describe MHD, a combination of Navier-Stokes (N-S) equations of fluid dynamics and Maxwell's equations of electromagnetism are used. The foundation axioms of fluid dynamics are the conservation laws, specifically conservation of mass, conservation of linear momentum and conservation of energy.The models governing the flow are then developed with appropriate boundary conditions imposed, simplified using the boundary layer approximations and then solved simultaneously analytically.

3.1 Continuity Equation

The continuity equation is the mathematical expression of the law of conservation of mass. According to Raisinghania (2013), the law of conservation of mass states that the increase in the mass of the fluid within any closed surface drawn in the fluid at any one time must be equal to the excess of the fluid that flows out.

The differential form of continuity equation is:

$$\frac{\partial \rho}{\partial t} + \nabla.\left(\rho \vec{u}\right) = 0 \quad \text{, where } \rho \text{ is the mass density.} \tag{3.1}$$

If ρ is constant, as in the case of incompressible flow the mass equation simplifies to a volume continuity equation $\nabla.\vec{u} = 0$

3.2 Momentum equation

The law of conservation of momentum states that if a closed system is not affected by external forces, its total linear momentum is constant. The law also requires that the time rate of change of the linear momentum of the system must be equal to the sum of external forces acting on the system.

In the model described in Figure 1, the moving fluid element experiences external forces in the x-direction. The external forces include:

(i) Body forces which are proportional to the volume and which are on the fluid particle from an external force field such as gravitational, electrical, magnetic or centrifugal fields.

(ii) Surface forces which are proportional to the surface area of the fluid element. The surface forces are due to pressure distribution acting on the surface, imposed by the outside fluid surrounding the fluid element, or due to viscous shear stress distribution imposed by the outside fluid 'pushing' on the surface by means of friction.

An appropriate mathematical statement for conservation of momentum is:

$$\rho\left(\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x}\right) = -\frac{\partial P}{\partial x} + \mu\,\frac{\partial^2 u}{\partial x^2} + \overrightarrow{F_x} \tag{3.2}$$

where $\vec{F}_x = \vec{J}\,X\,\vec{B}$

This implies that the sum of local acceleration and convective acceleration equals the total sum of body forces, pressure forces and boundary forces.

3.3 Navier-Stokes Equations

Together with supplementary equations (e.g. conservation of mass) and well defined boundary conditions, the Navier-Strokes equation model fluid motion accurately. Taking the incompressible flow assumption into account and assuming constant viscosity, the N-S equations in vector form read:

$$\rho\,\frac{\partial \vec{u}}{\partial t} + \rho\vec{u}.\nabla\vec{u} = \mu\nabla^2\vec{u} - \nabla p + \vec{F} \tag{3.3}$$

3.4 Energy Equation

The energy equation is derived from the conservation of energy on the basis of the first law of thermodynamics. According to this law the total energy added to the system (both by heat and by work done on the fluid) increases the energy per unit mass of the fluid. For this study, the energy equation of an incompressible, viscous steady laminar fluid flow is:

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} = \frac{1}{\rho C_p}\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) \tag{3.4}$$

3.5 Maxwell's Equations

Maxwell's equations describe how electric and magnetic fields are generated and altered by each other and by charges and currents. When the equations are put together, their significance is

recognized in predicting the existence of electromagnetic waves. In this study, the following differential forms of equations will be used:

$$\left.\begin{aligned}
&\nabla.\vec{D} = \gamma \\
&\vec{D} = \varepsilon_0 \vec{E} && \text{for free space} \\
&\nabla.\vec{B} = 0 \\
&\vec{B} = \mu_0 \vec{H} && \text{for free space} \\
&\nabla X \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\
&\nabla X \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}
\end{aligned}\right\} \tag{3.5}$$

3.6 Non-dimensional Parameters

It is desirable to find the non-dimensional parameters which characterize the flow in order to bring out essential features for the flow. The flow motion is characterized by a combination of forces such as inertial, viscous force, magnetic force, gravitational force, pressure etc. The dynamic similarity can always be obtained by considering the ratio of two forces which result in a dimensionless parameter responsible for the fluid motion. Section 3.6.1 to 3.6.3presents some of the non-dimensional parameters commonly used in MHD flows.

3.6.1 The Hartmann Number

This is obtained from the ratio of magnetic force to the viscous force. It gives a measure of relative importance of drag forces resulting from magnetic induction and viscous forces. It is defined as:

$$M = \mu_e \vec{H} L \sqrt{\frac{\sigma}{\mu}} \qquad \text{or} \qquad M = \vec{B} L \sqrt{\frac{\sigma}{\mu}} \tag{3.6}$$

where $\vec{B}$ is the magnetic flux, μ_e is the magnetic permeability, L is the characteristic length scale, μ is the fluid viscosity, σ is the electrical conductivity of the fluid.

3.6.2 Prandtl Number

This is the ratio of kinetic viscosity to the thermal diffusivity; it depends on the properties of the fluid. It plays a very important role in problems of convection in which one must consider the simultaneous exchange of momentum through viscosity and heat through conduction.

$$P_r = \frac{\mu C_p}{k} \tag{3.7}$$

where K is the thermal conductivity of the fluid, μ is the dynamic viscosity and C_p is the specific heat capacity of the fluid at constant pressure.

3.6.3 Reynolds Number

This is the ratio of the inertial forces to viscous forces. It is defined as

$$R_e = \frac{DU\rho}{\mu} \tag{3.8}$$

Where D is the characteristic dimension/characteristic travelled length; U is magnitude of velocity; ρ is fluid density and μ is fluid dynamic viscosity.

For laminar flow, the range of Reynolds number is $0 \le R_e \le 2100$ while for transcient flow the range is $2100 \le R_e \le 4000$ and for turbulent flow R_e >4000. Reynolds number is used to predict similar flow patterns in different fluid flow situations.

3.7 Set of Governing Equations

The continuity equation (3.1) can be written in Cartesian coordinate as

$$\frac{\partial \rho}{\partial t} + \rho(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}) = 0 \qquad (3.9)$$

For a steady incompressible flow, ρ is constant and $\frac{\partial \rho}{\partial t} = 0$, therefore equation 3.9 becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (3.10)$$

Since the flow is only along x-axis, the velocity along y-axis $v = 0$ and along z-axis $w = 0$. This means that $\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$. Equation (3.11) reduces to

$$\frac{\partial u}{\partial x} = 0 \qquad (3.11)$$

This makes the non linear term in the Navier-Stokes equations zero. We neglect the body forces which are mainly due to gravity in N-S equations and consider the Lorentz force.

For steady flow $\frac{\partial \vec{u}}{\partial t} = 0$, the momentum equation (3.2) reduces to

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 \vec{u}}{\partial y^2} + \frac{F_x}{\rho} \qquad (3.12)$$

where F_x refer to the force on a point charge due to electromagnetic fields (the Lorentz force).

We consider $\vec{F} = \vec{F}(F_x, F_y, F_z)$, yet $F_y = F_z = 0$ and $\vec{F} = \vec{J} X \vec{B}$. Thus $F_x = \vec{J} X \vec{B}$. Equation (3.12) therefore reduces to

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 \vec{u}}{\partial y^2} + \frac{\vec{J} X \vec{B}}{\rho} \qquad (3.13)$$

Now $\vec{J} = \sigma \vec{E}$, and because of the interaction of the two fields, namely, velocity and magnetic fields, an electric field vector $\vec{E}$ is induced at right angles to both $\vec{V}$ and $\vec{B}$. This electric field is given by $\vec{E} = \vec{V} \text{ x } \vec{B}$.

This is because we are considering the flow in x-direction and the flow will be affected by the magnetic flux which is inclined at an angle to the flow. Since this study focuses on the effect of different angles of inclination α of the magnetic field, the velocity and magnetic flux profiles will be $\vec{V} = \vec{V}(\vec{u}, 0, 0)$ for velocity profile

$\vec{B} = \vec{B}(0, \vec{B} Sin\alpha, 0)$ for magnetic profile

Now $\vec{J} = \sigma \vec{E} = \sigma(\vec{V} X \vec{B})$

$$\vec{J} = \sigma \begin{vmatrix} i & j & k \\ \vec{u} & 0 & 0 \\ 0 & \vec{B} Sin\alpha & 0 \end{vmatrix}$$

$$\vec{J} = i(0) - j(0) + k(\sigma \vec{u} \vec{B} Sin\alpha)$$

$$\vec{J} = \sigma \vec{u} \vec{B} Sin\alpha \; k \qquad (3.14)$$

Hence $\vec{F}_x = \vec{J} X \vec{B} = \begin{vmatrix} i & j & k \\ 0 & 0 & \sigma \vec{u}\vec{B} Sin\alpha \\ 0 & \vec{B} Sin\alpha & 0 \end{vmatrix}$

$$\vec{J} X \vec{B} = -\sigma \vec{u}\vec{B}^2 Sin^2\alpha \; i \tag{3.15}$$

Substituting in equation (3.13) and taking kinematic viscosity $v = \dfrac{\mu}{\rho}$

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\frac{\partial^2 \vec{u}}{\partial y^2} - \frac{\sigma u \vec{B}^2 Sin^2\alpha}{\rho} \tag{3.16}$$

The flow along y-axis is zero, therefore

$$\frac{\partial v}{\partial y} = 0 \tag{3.17}$$

Integrating equation (3.17) with respect to y, we get $v =$ constant, say $-V_0$ so that $-V_0 = \dfrac{1}{u}$

Therefore

$$v = -V_0 \tag{3.18}$$

Using (3.18) and considering that $\dfrac{\partial T}{\partial t} = 0$, from the given assumptions of compressible flow, the energy equation (3.3) reduces to

$$-V_0 \frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{3.19}$$

Hence the final set of governing equations to this problem is the coupled system of differential equations obtained by combining equations (3.16) and (3.19).

3.8 Non-demensionalization of the governing equations for the problem

Non-dimensionalization is the partial or full removal of units from an equation involving physical quantities by a suitable substitution of variables. The process is very important because the results obtained for a surface experiencing one set of conditions can be applied to a geometrically similar surface experiencing entirely different conditions.

In order to bring out the essential features of the flow problem under investigation, some non-dimensional numbers which may be helpful in this study include the Hartmann Number, Prandtl Number and Reynolds Number.

We therefore use the following non-dimensional parameters to non-dimensionalise the final set of governing equations (* denotes non-dimensional quantity)

$$X^* = \frac{X}{L}, \qquad y^* = \frac{y}{L}, \qquad u^* = \frac{u}{U}$$

$$V_0 = \frac{V_0^*}{U} \qquad \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*} \qquad p^* = \frac{p}{\rho U^2} \tag{3.20}$$

From equations (3.20)

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y}, \qquad \frac{\partial u}{\partial u^*} = U, \qquad \frac{\partial y^*}{\partial y} = \frac{1}{L},$$

$$\frac{\partial u}{\partial y} = \frac{U}{L}\frac{\partial u^*}{\partial y^*} \tag{3.21}$$

Similarly

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y^*}\left(\frac{U}{L}\frac{\partial u^*}{\partial y^*}\right).\frac{\partial y^*}{\partial y}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{U}{L^2}\frac{\partial^2 u^*}{\partial y^{*2}} \tag{3.22}$$

From $p^* = \frac{p}{\rho U^2}$, we get

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial p^*}.\frac{\partial p^*}{\partial x^*}.\frac{\partial x^*}{\partial x}$$

$$\frac{\partial p}{\partial p^*} = \rho U^2, \qquad \frac{\partial x^*}{\partial x} = \frac{1}{L}$$

$$\frac{\partial p}{\partial x} = \frac{\rho U^2}{L}\frac{\partial p^*}{\partial x^*} \tag{3.23}$$

Using equations (3.21), (3.22) and (3.23) in equation (3.16), we get

$$0 = -\frac{1}{\rho}\frac{\rho U^2}{L}\frac{\partial p^*}{\partial x^*} + \frac{vU}{L^2}\frac{\partial^2 u^*}{\partial y^{2*}} - \frac{\sigma u^* U \vec{B}^2 Sin^2\alpha}{\rho} \tag{3.24}$$

Simplifying equation (3.24) by multiplying both sides by $\frac{L^2}{\vec{v}U}$ and dropping the (*)

$$0 = \frac{L^2}{vU}\left(-\frac{U^2}{L}\frac{\partial p^*}{\partial x^*} + \frac{vU}{L^2}\frac{\partial^2 u^*}{\partial y^{2*}} - \frac{\sigma u^* U \vec{B}^2 Sin^2\alpha}{\rho}\right) \tag{3.25}$$

We get

$$0 = -\frac{UL}{v}\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \frac{\sigma u L^2 B^2 Sin^2\alpha}{\vec{v}\rho} \tag{3.26}$$

But kinematic viscosity, $v = \frac{\mu}{\rho}$, therefore

$$0 = -\frac{UL}{v}\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \frac{\sigma L^2 B^2 (Sin^2\alpha) u}{\mu} \tag{3.27}$$

We use the Hartmann number, $M = BL\sqrt{\frac{\sigma}{\mu}}$ to simplify equation (3.27) so that

$$M^2 = \frac{B^2L^2\sigma}{\mu}$$

We can therefore write equation (3.27) as

$$0 = -\frac{UL}{v}\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (M^2 Sin^2\alpha)u \tag{3.28}$$

Given that Reynolds number, $R_e = \frac{UL}{v}$, equation (3.28) can be simplified to:

$$0 = \frac{\partial^2 u}{\partial y^2} - R_e\frac{\partial p}{\partial x} - (M^2 Sin^2\alpha)u \tag{3.29}$$

Suppose we let pressure gradient $\frac{\partial p}{\partial x}$ to be a variable, say $-P(x)$,equation (3.29) becomes

$$0 = \frac{\partial^2 u}{\partial y^2} + PR_e - (M^2 Sin^2\alpha)u \tag{3.30}$$

$$\frac{\partial^2 u}{\partial y^2} - (M^2 Sin^2\alpha)u = -PR_e \tag{3.31}$$

From equation (3.19) we have

$$-V_0^*\frac{\partial T^*}{\partial y^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} \tag{3.32}$$

From equations (3.20)

$$-V_0^* = -UV_0 \tag{3.33}$$

$$\frac{\partial T^*}{\partial y^*} = \frac{\partial T^*}{\partial \theta}\frac{\partial \theta}{\partial y}\frac{\partial y}{\partial y^*}$$

But $\frac{\partial T^*}{\partial \theta} = (T_w^* - T_\infty^*)$ and $\frac{\partial y}{\partial y^*} = L$

$$\frac{\partial T^*}{\partial y^*} = (T_w^* - T_\infty^*)\frac{\partial \theta}{\partial y}L = L(T_w^* - T_\infty^*)\frac{\partial \theta}{\partial y} \quad (3.34)$$

$$\frac{\partial^2 T^*}{\partial y^{*2}} = \frac{\partial}{\partial y^*}\left(\frac{\partial T^*}{\partial y^*}\right) = \frac{\partial}{\partial y}\left(L(T_w^* - T_\infty^*)\frac{\partial \theta}{\partial y}\right)\frac{\partial y}{\partial y^*}$$

$$\frac{\partial^2 T^*}{\partial y^{*2}} = L^2(T_w^* - T_\infty^*)\frac{\partial^2 \theta}{\partial y^2} \quad (3.35)$$

Substituting equations (3.33), (3.34) and (3.35) in (3.32) we get

$$-UV_0L(T_w^* - T_\infty^*)\frac{\partial \theta}{\partial y} = \frac{k}{\rho C_p}L^2(T_w^* - T_\infty^*)\frac{\partial^2 \theta}{\partial y^2} \quad (3.36)$$

Dividing through by $LU(T_w^* - T_\infty^*)$ we get

$$-V_0\frac{\partial \theta}{\partial y} = \frac{kL}{\rho C_p U}\frac{\partial^2 \theta}{\partial y^2} \quad (3.37)$$

But $\frac{U}{L} = \frac{\mu}{\rho}$ thus $\frac{1}{\mu} = \frac{L}{\rho U}$

Equation (3.37) becomes

$$-V_0\frac{\partial \theta}{\partial y} = \frac{k}{\mu C_p}\frac{\partial^2 \theta}{\partial y^2} \quad (3.38)$$

Given that Prandtl number, $P_r = \frac{\mu C_p}{k}$

$$\frac{1}{P_r}\frac{\partial^2\theta}{\partial y^2}+V_0\frac{\partial\theta}{\partial y}=0$$

or $$\frac{\partial^2\theta}{\partial y^2}+P_rV_0\frac{\partial\theta}{\partial y}=0 \qquad (3.39)$$

Equations (3.31) and (3.39) give the set of differential equations for the model

3.9 Solution of the Problem

The ordinary differential equation (3.31) is non-homogeneous.

By use of differential operator, D, the auxiliary equation is written and solved as:

$$D^2u-(M^2Sin^2\alpha)u=0$$

$$(D^2-M^2Sin^2\alpha)u=0$$

$$D=\pm\sqrt{M^2}Sin^2\alpha$$

$$D=\pm MSin\alpha$$

The solution of the complementary equation is given by

$$u_c=Ae^{MSin\alpha y}+Be^{-MSin\alpha y} \qquad (3.40)$$

Where A and B are constants to be determined using boundary conditions.

The particular integral of equation (3.31) is given by

$$u_p=\frac{1}{(D^2-X)}PR_e \qquad \text{where } X=M^2Sin^2\alpha$$

$$u_p=-\frac{PR_e}{0-X}e^{0y}$$

$$u_p=\frac{PR_e}{M^2Sin^2\alpha} \qquad (3.41)$$

The general solution, u, is obtained by adding (3.40) and (3.41), i.e. $u = u_c + u_p$

$$u = Ae^{MSin\alpha y} + Be^{-MSin\alpha y} + \frac{PR_e}{M^2 Sin^2\alpha} \tag{3.42}$$

Where A and B are arbitrary constants.

From equation (3.39), the auxiliary equation becomes

$D^2 + D\delta = 0$ where $\delta = P_r V_0$

The solutions are $D = 0$ and $-\delta$.

The general solution of equation (3.39) is

$$\theta = C_1 + C_2 e^{-\delta y} \tag{3.43}$$

Where C_1 and C_2 are arbitrary constants.

3.10 Boundary Conditions

The ordinary differential equation (3.42) is solved using the following boundary equations:

at $y = -L$ $\quad \vec{u} = 0$

at $y = L$ $\quad \vec{u} = \vec{U}$

$y^* = \frac{y}{L}$ $\quad \vec{u}^* = \frac{\vec{u}}{\vec{U}}$

When $y = -L$ $\quad y^* = \frac{-L}{L} = -1$

When $y = L$ $\quad y^* = \frac{L}{L} = 1$

When $u = 0$ $\quad u^* = \frac{0}{L} = 0$

When $u = \vec{U}$ $\qquad u^* = \frac{\vec{U}}{\vec{U}} = 1$

Let $MSin\alpha = a$ and $\frac{PR_e}{M^2 Sin^2\alpha} = Q$ such that equation (3.42) becomes

$$\vec{u} = Ae^{ay} + Be^{-ay} + Q \tag{3.44}$$

Using the boundary conditions $y = -1,\ u = 0$ and $y = 1,\ u = 1$ to solve equation (3.44) we get

$$0 = Ae^{-a} + Be^{a} + Q \tag{3.45}$$

$$1 = Ae^{a} + Be^{-a} + Q \tag{3.46}$$

Multiplying equation (3.45) by e^{-a} and (3.46) by e^{a} and then subtracting, we get

$$e^{a} = Ae^{2a} - Ae^{-2a} + Qe^{a} - Qe^{-a}$$

$$e^{a} = A(e^{2a} - e^{-2a}) + Q(e^{a} - e^{a})$$

$$A = \frac{e^{a} - Q(e^{a} - e^{-a})}{e^{2a} - e^{-2a}} \tag{3.47}$$

Similarly multiplying equation (3.45) by e^{a} and (3.46) by e^{-a} and subtracting, we get

$$e^{-a} = Be^{-2a} - Be^{2a} + Qe^{-a} - Qe^{a}$$

$$e^{-a} = B(e^{-2a} - e^{2a}) + Q(e^{-a} - e^{a})$$

$$B = \frac{e^{-a} - Q(e^{-a} - e^{a})}{e^{-2a} - e^{2a}} \tag{3.48}$$

Substituting the values of A and B in (3.44) we get

$$u = \frac{e^{a} - Q(e^{a} - e^{-a})}{e^{2a} - e^{-2a}} e^{ay} + \frac{e^{-a} - Q(e^{-a} - e^{a})}{e^{-2a} - e^{2a}} e^{-ay} + Q$$

$$u = \frac{e^{a+ay} - Q\sinh ae^{ay} - Q\sinh ae^{-ay} + e^{-a-ay}}{e^{2a} - e^{-2a}} + Q$$

$$u = \frac{(e^{a+ay} + e^{-a-ay}) - Q\sinh a(e^{ay} + e^{-ay})}{e^{2a} - e^{-2a}} + Q$$

$$u = \frac{(e^{a(1+y)} + e^{-a(1+y)}) - Q\sinh a(e^{ay} + e^{-ay})}{\sinh 2a} + Q$$

$$u = \frac{Cosha(1+y) - Q\sinh a(\sinh ay)}{\sinh 2a} + Q \qquad (3.49)$$

Substituting the values of a and Q in equation (3.49)

$$u = \frac{Cosh(M\sin\alpha(1+y)) - \frac{PR_e}{M^2Sin^2\alpha}Sinh(MSin\alpha(\sinh(M\sin\alpha y)))}{Sinh(2MSin\alpha)} + \frac{PR_e}{M^2Sin^2\alpha} \quad (3.50)$$

To solve equation (3.43) the following boundary conditions are applied:

at $\quad y = -1$, $\quad \theta = \dfrac{T_\infty^* - \infty}{T_w^* - T_\infty^*} = 0$, $\quad u = 0$

at $\quad y = 0$, $\quad \theta = \dfrac{T_w^* - T_\infty^*}{T_w^* - T_\infty^*} = 1$, $\quad u = 1$

Equation (3.43) therefore becomes

$$0 = C_1 + C_2e^{\delta} \qquad (3.51)$$

$$1 = C_1 + C_2 \qquad (3.52)$$

Subtracting equation (3.51) from (3.52) we get

$$1 = C_2(1 - e^{\delta})$$

$$C_2 = \frac{1}{1 - e^{\delta}}$$

Substituting C_2 in equation (3.51) we get

$$0 = C_1 + \frac{e^{\delta}}{1-e^{\delta}}$$

$$C_1 = \frac{-e^{\delta}}{1-e^{\delta}}$$

Substituting C_1 and C_2 in equation (3.43) we get

$$\theta = -\frac{e^{\delta}}{1-e^{\delta}} + \frac{e^{-\delta y}}{1-e^{\delta}}$$

$$\theta = -\frac{e^{\delta}}{1-e^{\delta}} + \frac{e^{-\delta}.e^{\delta}.e^{-\delta y}}{1-e^{\delta}}$$

Let $\eta = \dfrac{e^{\delta}}{1-e^{\delta}}$

Therefore $\theta = -\eta + \eta e^{-\delta(1+y)}$

$$\theta = \eta[e^{-\delta(1+y)} - 1] \tag{3.53}$$

Substituting the values of η and δ in equation (3.53) and noting that V_0 is a constant equal to $\dfrac{1}{\vec{u}}$

$$\theta = \frac{e^{\frac{\Pr}{u}}}{1-e^{\frac{\Pr}{u}}}[e^{-\frac{\Pr}{u}(1+y)} - 1] \tag{3.54}$$

The two equations (3.50) and (3.54) give us the model for this study. They are used to find the effect of different parameters on the velocity, $\vec{u}$, of the fluid i.e. the Prandtl number, Reynolds number, pressure gradient, Hartmann number, different angles of inclination of magnetic field and the temperature changes.

CHAPTER FOUR

RESULTS AND DISCUSSION

In order to analyze the flow, the models (or the governing equations) have been developed with appropriate boundary conditions imposed, solved analytically and plotted by use of MATLAB version 7.0.0.19920(R14). Each of the parameters (Hartmann number, angle of inclination for magnetic field, temperature changes, pressure gradient, Reynolds number and Prandtl number) was varied at one time while keeping the others constant. Figure 2 to Figure 8 demonstrate graphically the effects of variation of these parameters on velocity profile and others on temperature profile.

Table 1: Variation of Reynolds number (R_e)

α	M	P	R_e
$\pi/3$	2.5	2.0	400
$\pi/3$	2.5	2.0	300
$\pi/3$	2.5	2.0	200
$\pi/3$	2.5	2.0	100

Using the conditions in the Table 1, the governing equation (3.50) can be represented in the graph as show in Figure 2.

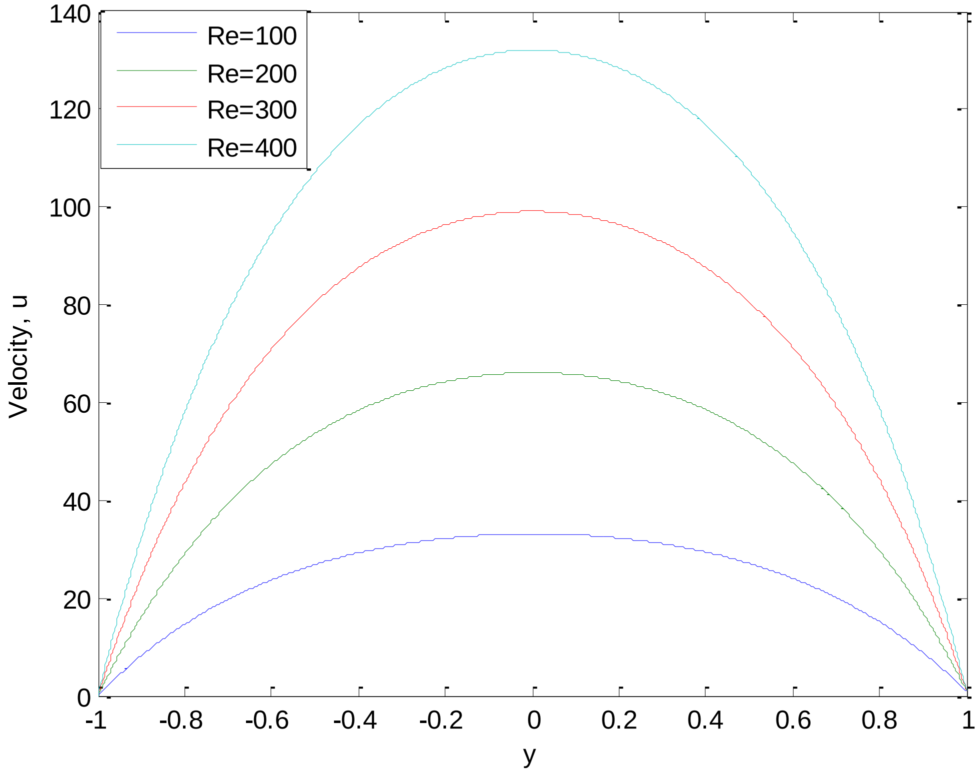

Figure 2: Velocity profile for different values of Reynolds number (Re)

In Figure 2, y-axis represents the changes in velocity when Reynolds number varies while in x-axis, y represents the space between the two plates along which the fluid flows. Figure 2 shows that increase in Reynolds number leads to increase in velocity. This is becausefor low Reynolds number viscous forces are dominant and the flow is characterized by smooth constant fluid motion, while for higher Reynolds number inertial forces increase in strength.

The range of variation of Reynolds number was chosen to be between 100 and 400 because the study focused on laminar flow. For laminar flow, the range of Reynoldsnumber is $0 \leq R_e \leq 2100$ while for transcient flow the range is $2100 \leq R_e \leq 4000$ and for turbulent flow R_e >4000. Reynolds number is used to predict similar flow patterns in different fluid flow situations.

Dmitry et al (2012) explain that there is a broad range of Hartmann number in which the flow is neither laminar nor fully turbulent, but has laminar core, Hartmann boundary layers and turbulent zones near the wall parallel to the magnetic field. This happens in the range of Hartmann number from 0 to 400 only when Reynolds number is 10^5. For lower Reynolds number the Hartmann number tends to zero for steady flow. This supported the choice of Hartmann number as 2.5 for this study.

Table 2: Change in the angle of inclination of magnetic field (α)

α	M	P	R_e
$\pi/2$	2.5	2.0	0.2
$\pi/3$	2.5	2.0	0.2
$\pi/4$	2.5	2.0	0.2
$\pi/6$	2.5	2.0	0.2

Using the conditions in the Table 2, the governing equation (3.50) can be represented in the graph as in Figure 3.

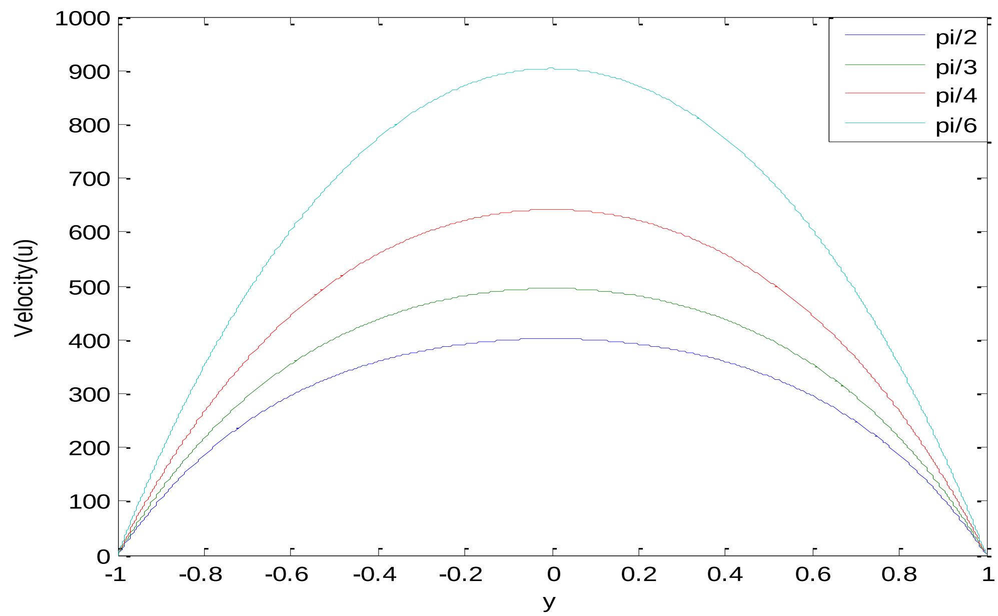

Figure 3: Velocity profile for different angles of inclination of magnetic field (α)

From Figure 3, there is evidence that when the angle of inclination of magnetic field to the plate is decreased the velocity of the fluid increases. At 90^0 it indicates a special case where the effect of fluid flow is observed at transverse magnetic field, which is the normal MHD flow.

Table 3: Variation of Hartmann number (M)

α	M	P	R_e
$\pi/3$	2.4	2.0	0.2
$\pi/3$	2.6	2.0	0.2
$\pi/3$	2.8	2.0	0.2
$\pi/3$	3.0	2.0	0.2

Using the conditions in Table 3, the governing equation (3.50) can be represented in the graph as in Figure 4 below.

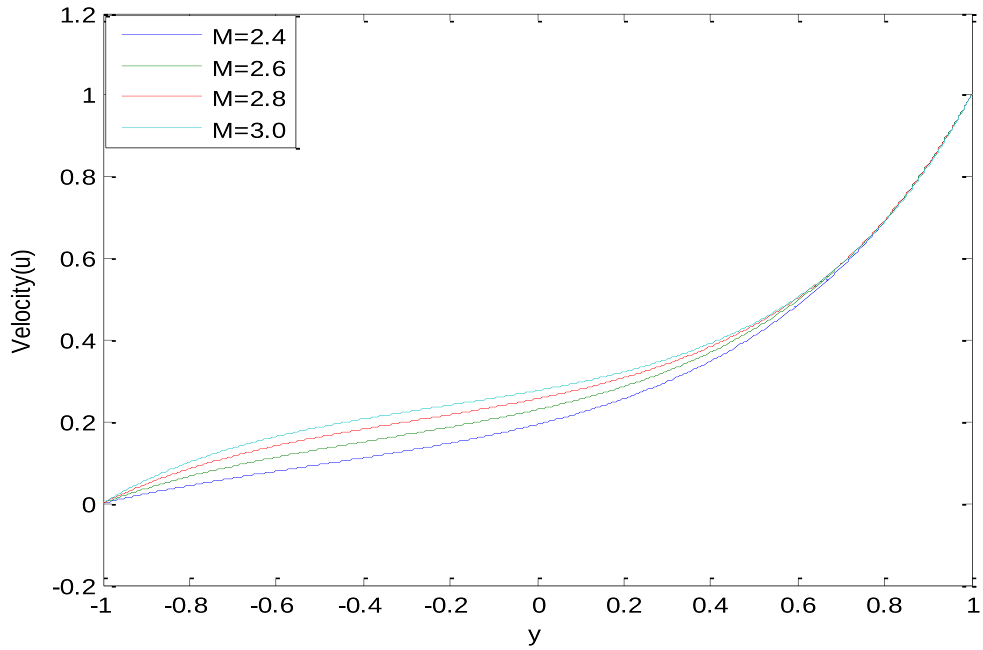

Figure 4: Velocity distribution for different values of Hartmann number (M)

Figure 4 illustrates that increase in Hartmann number retards the velocity of the flow. Hartmann number is a ratio of magnetic force to viscous force; this slows down the motion of the fluid due to the action of Lorentz force.

Table 4: Changes in Pressure gradient (P)

α	M	P	R_e
$\pi/3$	2.0	2.0	0.2
$\pi/3$	2.0	4.0	0.2
$\pi/3$	2.0	6.0	0.2
$\pi/3$	2.0	8.0	0.2

Using the conditions in Table 4, the governing equation (3.50) can be represented in the graph as in Figure 5 below.

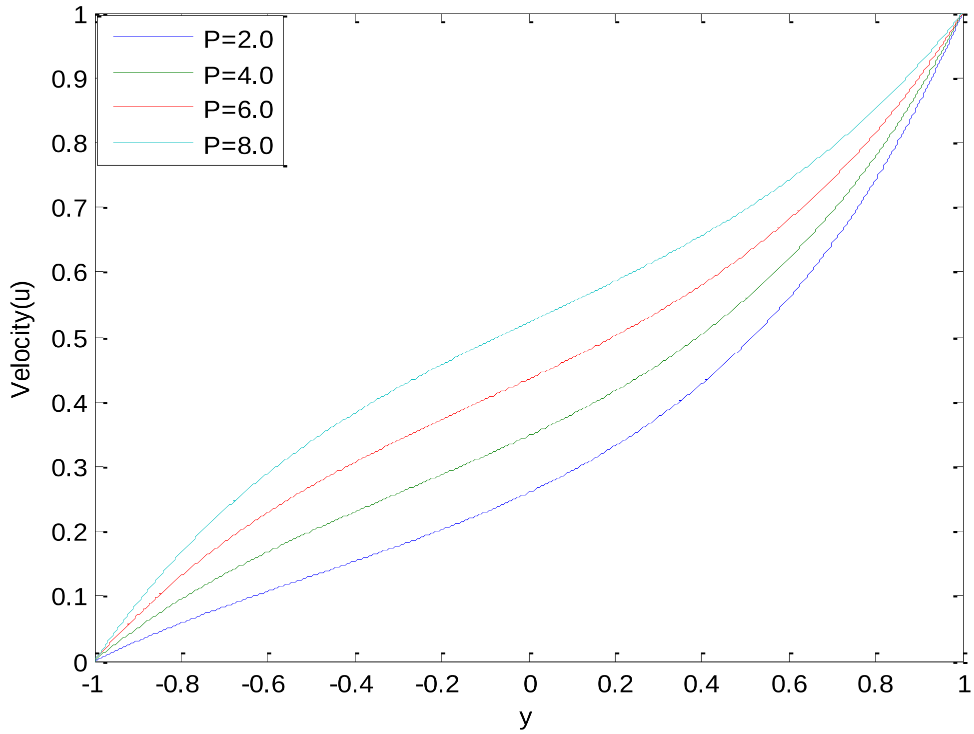

Figure 5: Velocity distribution for different values of Pressure gradient (P)

Figure 5 illustrates the effect of pressure gradient on the flow velocity. Because of the force the pressure will have on the fluid, the velocity will increase when the pressure gradient increases.

Table 5: Effect of temperature variation on velocity(u)

Temp.(θ)	Pr
20^0	0.07
30^0	0.07
40^0	0.07
50^0	0.07

Using the conditions in Table 5, the governing equation (3.54) can be represented in the graph as in Figure 6 below to show the effect of temperature on velocity profile.

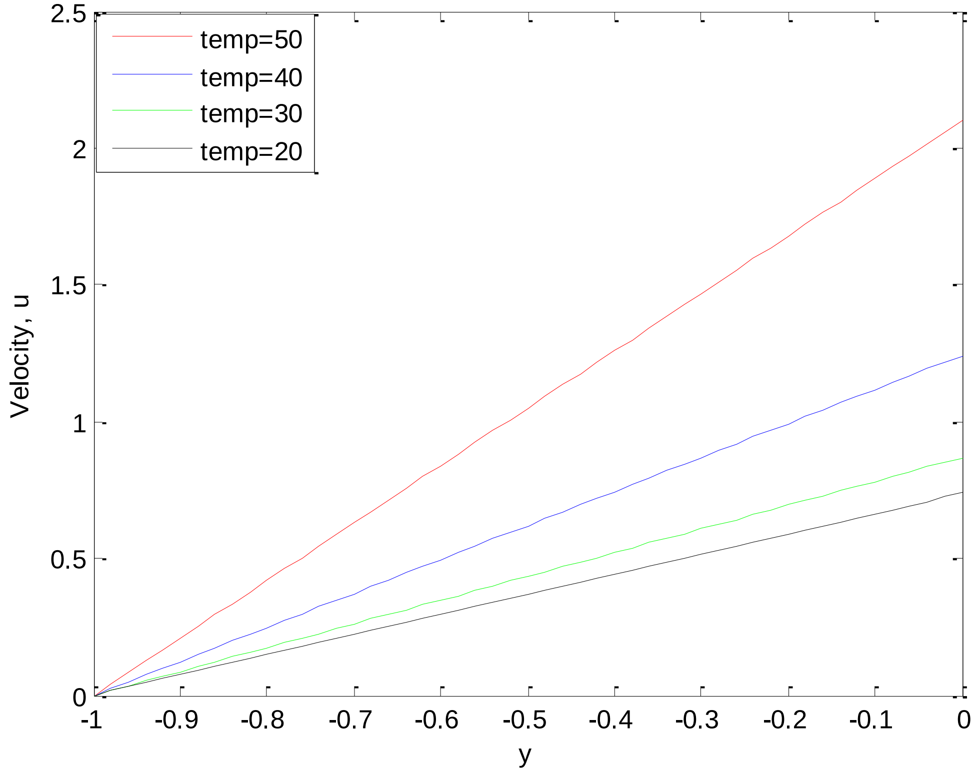

Figure 6: Effect of temperature variation on velocity (u)

Figure 6 shows that the velocity of the fluid increases with increase in temperature of the fluid. This is due to reduced density of the fluid particles.

Welty et al (2014) explain that Prandtl number varies with differences in fluids; for instance, Prandtl number for gases range between 0.7 and 1, for water ranges from 4 to 7 while that of liquid metals is of the order of 10^{-2}. This facilitated the choice of low values of Prandtl number in this study since the study dealt with electrically conducting fluids which include some liquid metals.

Table 6: Variation of Prandtl number (Pr)

U	Pr
0.5	0.07
0.5	0.12
0.5	0.17
0.5	0.22

Using the conditions in Table 6, the governing equation (3.54) can be represented in the graph as in Figure 7 below to show the effect of velocity on temperature profile.

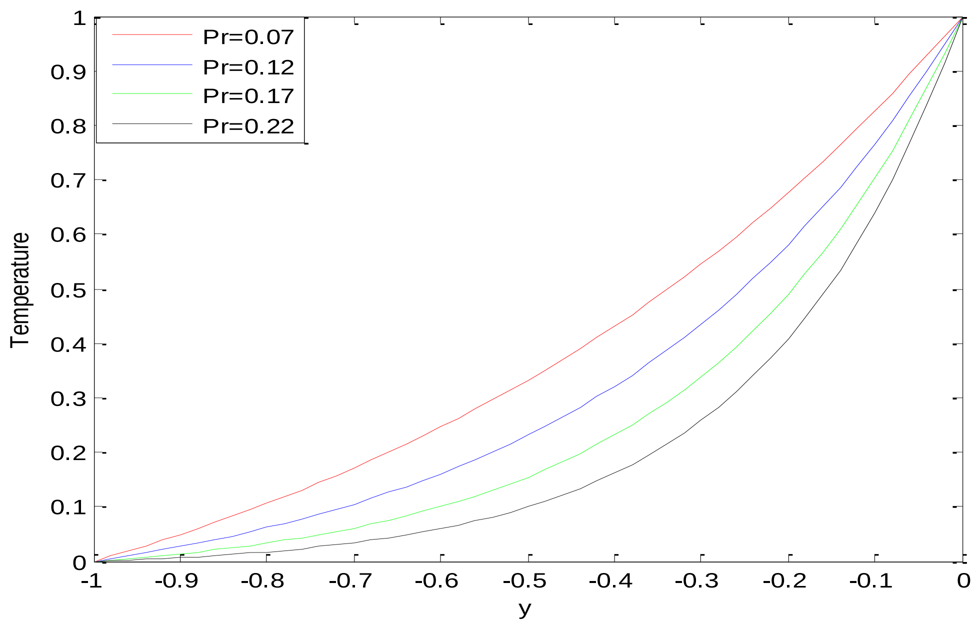

Figure 7: Temperature distribution for different values of Prandtl number (Pr)

Figure 7 illustrates that increase in Prandtl number leads to reduction of temperature of the fluid. Prandtl number is the ratio of viscous force to the thermal force which depends on other factors such as thermal conductivity of the fluid, specific heat capacity and dynamic viscosity. It therefore implies that when Prandtl number is increased the velocity of the fluid reduces.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

This study was focused on the steady flow of electrically conducting fluid, along x-axis between the horizontal plates heated at varying temperatures and subjected to an inclined magnetic field. The two plates were stationary, one at $y=-L$ and the other at $y=L$; the governing equations were non-dimensionalized and solved analytically, whereas the velocity and temperature profiles were computed using MATLAB and graphically illustrated at selected values of the emerging flow parameters. The results obtained from this study showed that when the angle of inclination of magnetic field is reduced, the velocity of the fluid increases. As the angle of inclination tends to zero, the flow velocity increases. This implies that when there isno angle of inclination of the magnetic field, the field has no effect on the flow while the highest effect is experienced at 90^0 (or transverse magnetic field). Velocity of the fluid and its temperature has positive correlation. Increase in the velocity of the fluid results from increase in temperature of the fluid when the plates are heated. This is due to reduced density of the fluid particles.

The results also showed that increase in Reynolds number leads to increase in velocity. This is because for low Reynolds number viscous forces are dominant and the flow is characterized by smooth constant fluid motion, while for higher Reynolds number inertial forces increase in strength.

Increase in Prandtl number reduces the temperature of the fluid and as a result retards the velocity of the fluid.

For the effect of pressure gradient on the flow velocity: because of the force the pressure will have on the fluid, the velocity will increase when the pressure gradient increases.

Increase in Hartmann number retards the velocity of the flow. Hartmann number is a ratio of magnetic force to viscous force; this slows down the motion of the fluid due to the action of Lorentz force.

5.3 Recommendations

Further research could be done on the same problem when considering that induced magnetic field and Hall effects currents are not negligible. This problem can also be investigated when the flow is unsteady and while considering other methodologies like application of the Finite Element Method or Finite Difference Method.

REFERENCES

Agarwal, H.L., Ram, P.C., and Sing, S. S. (1980). Combined buoyancy effects of Thermal and mass diffusion on MHD natural convection flows. *Journal of Chemical Engineering.Vol.58(1),* 131-133.

Agarwal, H.L., Ram, P.C., and Yadav, N. (2005). Hydrometric unsteady viscous Flow through a porous channel. *The Journal for scientific research* –221005.

Campos, L.M.B.C.(1985).Vertical hydromagnetic waves in a compressible Atmosphere under an oblique magnetic field. *Journal of Geophysical and Astrophysical Fluid dynamics. Vol. 32,* 217-272.

Charlton, F. (2004).*Fluid Dynamics*. CBS publishers & Distributors. Darya Ganj, New Delhi, India.

Chaturani, P.,and Saxena, S.(2000).Two layered MHD flow through parallel plates with applications. *Indian journal of Pure and Applied Mathematics, Vol.32(1),* 55-68.

Dmitry, K., Oleg, Z., and Boeck, T.(2012). Numerical Study of magnetohydrodynamics duct flow at high Reynolds and Hartmann Numbers. *Journal of Fluid Mechanics, Vol. 704*, 421-446.

Douglas, J.F.,Gasiorek, J.M., Swaffield, J.A., and Jack, B.L.(2005):*Fluid Mechanics*, (5^{th}Edition).Pearson Education Ltd. New Delhi, India.

Faraday, M. (1832). Experimental Researches in electricity. *Philosophical Transactions of the Royal Society of London (1776-1886).* 1832-01-01. 122:125–162

Gannesh, S., and Krishnambal, S. (2007).Magnetohydrodynamics Stoke's flow of Viscous fluid between two parallel porous plates. *Journal of Applied science, Vol.17(1)*, 374-379.

Giovanni, C. (2010). A single perturbation problem in magnetohydrodynamics. *IMA Journal of Applied Mathematics, Vol.75(2),* 240-245.

Guria, M., Das, B.K., and Jana, R.N.(2006). Unsteady hydromagnetic flow in a Rotating channel in the presence of inclined magnetic field. *International journal of Pure and Applied Mathematics. Vol.32(3)*, 379-392.

Hartman, J., and Lazarus, F. (1937). Influence of transverse uniform magnetic field on the flow of a viscious incompressible electrically conducting fluid between two infinite parallel, stationary and insulating plates. *Kgl, Danske Videnskab, Selskab, mat. Fys. Medd, Vol 6(1)*, 125-132.

Kwanza, J.K.,Marigi, E.M., and Kinyanjui, M. (2010).Hydromagnetic free Convection current effects on boundary layer thickness. *Journal of Energy conservation and management, Vol.51(6),*1326-1332.

Malashetty, M. S., and Umavathi, J. C. (1997). Two-phase magnetohydrodynamic flow and heat transfer in inclined channel. *International Journal on Multiphase flow*, *Vol 23*, No.3, 545-560.

Manyonge, W.A., Kiema, D.W., and Iyaya, C.C.W. (2012). Steady MHD Poiseulle Flow between two infinite parallel porous plates in an inclined magnetic field. *International Journal of Pure and Applied Mathematics. Vol. 76(5),* 661-668.

Mhone, P.Y., and Makinde, O.D. (2006). Unsteady MHD flow with heat transfer in a divergence channel. *Journal of Applied Mathematics and Computation. Vol.181(2),*966-972.

Munson, B.R., Young, D.F., Okiishi, T.H. (2009). *Fundamentals of Fluid Mechanics,* (5th Edition).Wiley India Pvt. Ltd, New Delhi, India.

Raisinghania, M. D., (2013). *Fluid Dynamics*. S Chand Publications, New Delhi, India.

Rapits, A.A., and Tzivandis, G.J. (1981). Free convection flow of an electrically conducting incompressible and viscous fluid past a vertical infinite plate under action of transverse magnetic field in a rotating fluid. *Journal of Physics, Vol. 14,* Page 12.

Rossow, V. J. (1957). Flow of electrically conducting fluid over a flat plate in the presence of transverse magnetic field. *N.A.C.A.T.N, 3971*.

Singh, C.B. (2007). Hydromagnetic steady flow of Viscous incompressible fluid between two parallel infinite plates under the influence of inclined magnetic field. *Kenya Journal of Sciences Series A, Vol.12(1),* 2007.

Soundalgekar, V. M., Takhar, H. S., and Singh , M. (1981). Velocity and temperature field in MHD Falkner-Skan flow. *Journal for Physical Society of Japan. Vol. 50*, 3139-3143.

Welty, J. R., Rorrer, G. L., and Forster, D. G. (2014).*Fundamentals of Momentum, Heat and Mass Transfer*.6th Ed. New Jersey: Wiley Publishers.

Witalis, E. A. (1986).Hall Magnetohydrodynamics and Its Applications to Laboratory and Cosmic Plasma", *IEEE Transactions on Plasma Science(ISSN 0093-3813), vol. PS-14*, PP. 842-848.

Young, F. B., Gerrard, H., and Jevons, W. (1920). Electrical disturbances due to tides and waves. *Philos. Mag., Ser.6*, 40: 149-159.

Ziya, U., and Manoj, K. (2011). MHD heat and mass transfer free convection flow near the lower stagnation point of an isothermal cylinder imbedded in porous domain with the presence of radiation. *Journal of Mechanical and Industrial Engineering.Uttakarhand, India, Vol. 5(2)*, 133-138.