

**A MODIFIED NUCLEAR MODEL FOR BINDING ENERGY OF NUCLEI AND
THE ISLAND OF STABILITY**

BY

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DECLARATION

Declaration by the Candidate

I declare that this is my original and personal work and has not been presented for a degree in any other university. This thesis is not to be reproduced without the prior written permission of the author and/or University of Eldoret.

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DEDICATION

To my beloved mum Grace for her unconditional love, advice and support.

To our children Gael and Abby you are blessings. My wife Jacinta, for her love and care.

ABSTRACT

A new nuclear model of quantifying binding energy of nuclei is proposed. The nucleus is assumed to be composed of two regions; the inner core region and surface region. The inner core is assumed to be composed of Z proton-neutron pairs ($Z=N$) and the surface region is composed of the unpaired neutrons for a nucleus in which $N>Z$. The interaction between the core and neutrons in the surface region is assumed to be such that it leads to an average potential V_o in which each neutron in the surface region can move. Knowing the experimental values for the binding energy of nuclei, this average interaction potential V_o has been calculated for light, medium, heavy nuclei and Super heavy elements. V_o varies for isotopes, Isobars and Isotones. For isotopes the value of V_o decreases as the nuclei surface region neutron number (N) increases. A decrease in V_o is quite large when the neutron number increases by unity in light nuclei compared to heavy nuclei. For isotones, the value of V_o increases with an increase in proton number (Z). This is evident for both light nuclei and heavy nuclei. For Isobars, all elements ranging from light to SHE, V_o increases with decreasing $A-2Z$. The value of V_o is calculated for 254 super heavy nuclei or elements (SHE or SHN) starting from ${}^{234}_{92}\text{U}$ to ${}^{295}_{118}\text{Ei}$. The calculated values of the interaction potential V_o range from 671.688 MeV for ${}^{234}_{92}\text{U}$ to the value 938.961 MeV for ${}^{295}_{118}\text{Ei}$. However, a very high value of 1022.206 MeV is noted for ${}^{267}_{110}\text{Ds}$. Definite variations in V_o are obtained for Isotones, Isotopes and Isobars. For Isotones, the value of V_o increases while there is an increase in proton number (Z increases). For isotopes, V_o value decreases while number of neutrons in neutron skin region increases. For isobars, V_o value increases while proton number (Z) increases. This trend in variation of V_o is evident across all the nuclides, the obtained values of V_o are within the expected theoretical values, also the variation of V_o is in agreement with “Woods-Saxon potential”.

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LIST OF SYMBOLS

SEMF	Semi-empirical mass formula
nn	Nucleon-Nucleon
NN	Neutron-Neutron
pp	Proton-Proton
np	Neutron-proton
BCS	Bardeen-copper-Schrieffer
NSCL	National Superconducting Cyclotron Laboratory
BW	Bethe Weizsacker
S_n	Neutron separation Energy
S_p	Proton separation Energy
h	Planck's Constant (6.626×10^{-34} JS)
k_B	Boltzmann's Constant (1.3807×10^{-23} J/K)
M	Mass of nucleus
R	Radius of the Nucleus
S	Slow-neutron capture
P	Photo-dissociation

Rp	Rapid proton capture process
Z	Proton Number
N	Neutron Number
A	Mass number
SHE	Super Heavy Elements (SHE)
SHN	Super Heavy Nuclei (SHN)
MeV	Million Electron Volts
T_z	Isospin
EXOSAT	European X-ray Observatory Satellite
ROSAT	Rontgen satellite
T_c	Transition Temperature
IBM	Interacting Boson Model

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CHAPTER ONE

INTRODUCTION

1.1 Composition of Nuclei and Nuclear forces

After Chadwick discovered neutron, Heisenberg (Heisenberg, 1932; Heisenberg, 1933) proposed the hypothesis that nuclei are composed of neutrons and protons. This hypothesis is now confirmed experimentally such that A of the nucleus is the summation of neutrons (N) and protons (Z), it is also experimentally confirmed that the nuclear forces inside the nuclei are charge independent. However, Coulomb forces in the modified form do exist between protons inside the nucleus. In addition, there are pairing forces between the nucleons inside the nucleus.

1.1.1 Phenomena of pairing in finite and infinite nuclear system

The study of Physics is a vast area ranging from microscopic systems to macroscopic systems. The study of pairing in nuclear systems is an important area in nuclear Physics. In neutron star and nuclear matter which are examples of infinitely nuclear system, the study of pairing and superfluidity has been studied over years (Cooper *et al.*, 1959; Khanna, 1962), even before pulsars were discovered (Hewish *et al.*, 1968) of which later became known as to be magnetic neutron stars that rotated rapidly. Nucleons pairing got a lot of interest because experiments were done on two different frontiers that is the field of astrophysics and terrestrial. In astrophysics, X-ray satellites gave birth to data on emission from neutron stars. Example of these satellites were ROSAT, ASCA and EXOSAT. On terrestrial front, a concerted nuclei far from stability has been stimulated by increasing quality of radioactive beam and heavy-ion facilities, with special focus on neutron-rich species (Riisager, 1994). During modeling of the chemical and physical

properties of the new formed nuclei, pairing plays prominent role. Over fifty years ago, Mayer (1950) pointed out that $J = 0$ ground states would yield as a result of short-ranged, attractive nucleon-nucleon interaction. Pairing in infinite nuclear matter and nuclei is as a result of n-n potential containing close-range attractive parts. Both for infinite matter, and finite nuclei information on pairing correlations is abundantly available from spectroscopic data. Even in the presence of random interactions, signatures of pairing still remain in finite many-body systems.

The concept of charge being equal for the nuclear forces is yielded through relatively weak Coulomb type electric force and the effect of a pair of protons resembles that of a pair of neutrons. Furthermore, there is a similarity in proton-neutron interaction. This led to the concept of isotopic invariance of the n-n effect. Either a Neutron or a Proton with $\tau = 1/2$ may be in one of the two states, $\tau_z = -\frac{1}{2}$ (Z) or $\tau_z = +\frac{1}{2}$ (N). Of course, the equality is unequal, but is widely employed when discussing nuclei. It leads to a quantum number T_z called isospin, and its projection $T_z = (N-Z)/2$, where in the nucleus the number of neutrons (protons) is N (Z). We can define with this isospin symmetry two distinct states within the two-nucleon system. A $T = 1$ nucleon-nucleon system can have spin-projection $T_z = 1, 0, -1$. $T_z = 1$, and this corresponds to a neutron-neutron system, $T_z = 0$ to a proton-neutron system, and $T_z = -1$ to a proton-proton system. The nucleons being fermions, in this case have total spin $J = 0$ in order for the full wave function to maintain antisymmetry of the sum of n-n wave function. For $T = 0$, n-p systems will only have $T_z = 0$ and $J = 1$. Therefore, in the nucleus there exists two different types of elementary particles and they are dependent on spin and isospin of the pair-particle model. Pairing discussions of nuclei originates from discussing briefly quantum numbers of a system

composed of two nucleon. A ground-state with total angular momentum quantum number and parity, $\pi, J_{\pi} = 0+$ is what all even-even nuclei have. Pairing interaction that couples particles in time-reversed states can be postulated. Using this type of simple pairing interaction, one can also understand the fact that the ground state from excited states is well separated in even-even nuclei, although in the even-odd neighbour nucleus, several states exist near the ground state. The isovector ($T = 1$) of a duo-body system is associated with the behaviour of the even-even ground states. A prediction of a pair condensate in these systems is made from a simple model of interacting n-n (Tamagaki, 1970).

The evidence of isoscalar ($T = 0$) pairing in nuclei, is an open question of concern. A nuclei with $N=Z$ has a unique aspect that same shell-model orbitals is occupied by neutrons and protons. Consequently, it is expected that np connection to a great extent the np-pair is enhanced by the large spatial extend of neutron and proton one functions. Nuclei with a sizable neutron excess where the isospin $T = 1$ neutron-neutron (nn) and proton-proton (pp) pairing dominate is where our knowledge about nuclear pairing majorly comes from. Experiments for explorations of nuclear systems in the corridors of $N=Z$ line having a lot of valence np pairs, investigating the connection between n-p pairs and like particles channels. Weigner energy, the extra binding that occurs in $N = Z$ nuclei is one of the evidence related to $T = 0$ pairing. A Statement that if BCS pairs of neutrons, are made of ground-state of a nucleus, then a pair of neutron transfer is favoured unlike one-neutron transfer (Yoshida, 1962). If nuclei with unclosed shells come into contact (Peter *et al.*, 1999) a collective enhancement pair transfer is expected. In rapidly rotating nuclei, pairing fluctuations are also expected (Shimizu *et al.*, 1989).

To study the wave function of the ground state, light model i.e ${}^6_2\text{He}$, Neutron pair transfer have been used (Shimizu *et al.*, 1989). A clear description of nuclear ground state, nuclear collective motion and its low-lying excitations are depicted in terms of bosons. Interacting Boson Model (IBM), $L = 0$ (S) and $L = 2$ (D) is one such model where bosons are selected with pairs of nucleon having the equal quantum numbers and the ground state can be seen as a condensate of such pairs. Studies of Shell-model for duo structure of ground state and its difference in valence number, both protons and neutrons can give an indication of validity and microscopic principles on boson model. Thus it is important to understand that the neutron-proton pairing plays an important role in understanding the properties of nuclei in the region of light to super heavy nuclei. It is this concept that has been used in this thesis to build a new structure for the atomic nuclei.

1.1.2 Limits of stability

What is the amount of neutrons and protons that can be added to a nucleus that is stable for it to be unstable? This is not yet known? Where experiments cannot reach an answer to this question provides critical information for theoretical predictions and for relevant future energy technologies, which will yield experimental constraints for theories of fission. Today, only an answer for the lightest elements is known. The ideologies about this question of how many neutrons will a nucleus hold is still valid to date. New features about the strong interaction is learnt as more precise measurements of many neutron-rich isotopes becomes evident, in which this restricted level referred to as the neutron drip line exist naturally is ultimately dictated. Neutron dripline is defined like this keeping the proton number Z of a nucleus constant, go on adding neutrons till the separation energy

of the neutron is zero, or the last neutron drips, and the nucleus does not exist. Similarly proton dripline is defined as keeping the neutron number N constant, keep adding protons till the separation energy of the proton is zero, and or the proton drips. NSCL recently did measurements precisely on neutron-rich isotopes of magnesium and aluminium and the drip line indication was that it was far from line of stability than what was earlier perceived. Information on how strong interaction saturates will be provided by mapping of the neutron drip line. An understanding of the origin of these elements is also vital to us. It is thought that in the era of stellar explosions super heavy elements naturally formed in an environment where there was rapid neutron capture on nuclei till a neutron drip line was approached. Heaviest elements such as uranium can be made through a series of neutron captures and nuclear beta decays. One of the key astrophysical processes that we believe must occur is the r-process nucleosynthesis.

1.2 Statement of the problem

Protons, neutrons, and electrons belong to a class of particles called gas of fermions which can be found in contexts as different as neutron stars and block of metal. When fermions interaction is on the brink of forming fermion pairs (like in the formation of Cooper pairs by the phonon exchange just before the transition temperature T_c), the thermodynamics of the gas become dependent only on the gas density and temperature. In the last few years, a renewed interest in studying the effect of pairing correlations of excited nuclei's thermodynamic properties has been there. Level density, new accurate measurements at low excitation energy have been there. Thus the influence on behaviour of the specific heat in the isotopes $^{161, 162}\text{Dy}$ and $^{171, 172}\text{Yb}$ pairing on low temperature was extracted from the measurement of level density measurements. Thermodynamic

properties of even-odd, even-even and odd-odd nuclei and their isotopes have been studied using many-body theories and by using different pairing forces. However, it is the binding energy of nuclei that determines the stability of nuclei and it is this problem that has been studied in this thesis. Special attention has been given to the calculation of the average interaction potential energy.

1.3 Research Objectives

1.3.1 General Objective

To develop an interaction potential that can be used to calculate the interaction between the unpaired neutrons on the surface region and the paired nucleons (np) in the core region.

1.3.2 Specific Objectives

1. To derive the interaction potential V_0 from the experimental values of the binding energy B .
2. To compute V_0 for isotopes, isotones and isobars, for light ($0 < A < 50$), medium ($49 < A < 100$), heavy nuclei ($99 < A < 234$) and super heavy nuclei ($233 < A < 295$).
3. To compare the variation of V_0 for isotopes, isotones, and isobars for light, medium, heavy and super heavy elements.

1.4 Justification of study

However, there have been different theoretical approaches on this study but due to the dynamics of nuclear properties and their applications; new approaches may yield more precise results that impact on further developments on its applications in nuclear technology, especially nuclear fission for nuclear reactors and fusion reactors.

It is important to introduce new theoretical derivations of binding energy because when the binding energy formula was proposed in the beginning, concepts like shell structure of nuclei, pairing types, laboratory production of nuclei with abnormally large neutron or proton numbers called designer nuclei, and the concept of drip-line were not known. Hence binding energies formula may have to be modified as new properties of nuclei are discovered experimentally from time to time. Study of neutron – rich nuclei gives us distinct data on the binding energy surface in this region. Since very heavy and super heavy nuclei can undergo spontaneous fission (SF), for creation of new and stable super heavy nuclei, it is necessary to know the binding energy of nuclei that can lead to stability of SHN.

1.5 Significance

The astrophysical processes of neutron rich nuclei make the importance of this study more relevant. These include production of heavy nuclei which is as a result of explosive burning processes in light nuclei including rapid proton capture and the slow and rapid neutron capture processes. The study of super heavy elements is important because it may yield high stable super heavy nuclei. The properties of rare isotopes emerge as crucial ingredient for the development of nuclear models that may explain the march towards neutron drip line and proton drip line, and recently even deuteron drip line and alpha-particle dripline.

CHAPTER TWO

LITERATURE REVIEW

2.1 Background Information

Pairing transitions have been studied in isotopic chains of *Ca*, *Ni*, *Sn* and *Pb* (Niu *et al.*, 2013). Energy gaps have been calculated at zero temperature and at finite temperature using separate pairing force. Considering even-even effects in these isotopes, it is found that a general rule is followed by the critical temperature for a pairing transition; $T_c = 0.6 \Delta_n(0)$, where $\Delta_n(0)$ is the neutron pairing gap at zero temperature. In infinite systems it is well known that they can undergo phase transitions, examples being superconductors and superfluids. However, finite many-body systems can also undergo phase-transitions or exhibit phase-transitions although surface effects and statistical fluctuations can inhibit the transition. In warm nucleus for example, superfluidity vanishes when temperature increases (Egido *et al.*, 2000).

In terms of the shell model this can be understood, whereby protons and neutrons are energized from levels below the Fermi surface to levels above the Fermi surface when temperatures are increased, leading to level blocking and hence pairing correlations disappear. An experimental evidence in the form of *S*-shaped curve of heat capacity or specific heat as a function of temperature exists (Melby *et al.*, 2001). It is also found that $T_c \cong 0.5$ MeV for $^{161,162}\text{Dy}$, and $^{166,167}\text{Er}$ is the critical temperature for quenching of pair correlations. The vanishing of pair correlations in finite temperature mean- field theory occurs when the temperature is increased as a phase transition at the critical temperature. Calculation of the critical temperature is found to be $T_c=0.57 \Delta(0)$ in the finite temperature BCS theory (Sano and Yamasaki, 1963) with a constant pairing force G , and

$T_c=0.5 \Delta(0)$ using a degenerate model that has been simplified, where $\Delta(0)$ is the pairing energy gap at zero temperature. In the Shell Model, critical temperature calculated is found to be in better agreement with the forecast for the critical temperature obtained using BCS method (Levit and Alhassid, 1984). The BCS theory was used in studying the superfluidity disappearance with rise in temperature in nuclei (Goodman, 1884) and the pairing transition was predicted at the critical temperature $T_c=0.57 \Delta_n(0)$ for the case of pairing force. Using the BCS theory, one can study the superconductivity of metals (Bardeen *et al.*, 1957) and Superfluidity of atomic nuclei (Khanna, 1962).

A linear relation can be obtained between the critical temperature T_c and the pairing gap $\Delta(0)$ at zero temperature with the assumptions of a constant pairing force G in some energy interval around the Fermi surface, as well as a constant single-particle level density of such that $G \gg 1$. The resulting critical temperature is $T_c=0.57 \Delta(0)$, determined by setting the finite temperature energy gap equal to zero at $T=T_c$. In the BCS theory Cooper pairs are formed by electrons (or particles) only in the time-reversed orbitals whereas the more general Bogoliubov theory incorporates additional correlations and thus two particles from different single-particle orbitals can also form a pair. This could lead to a higher critical temperature at which all the correlated pair states are broken. In nuclei, because of the shell structure of single-particle states the level density is not a constant, and thus there could be variations from a linear relation between the critical temperature and the zero-temperature pairing gap. The Bogoliubov theory is used to study such deviations between the critical temperature T_c and the zero-temperature pairing gap $\Delta(0)$.

The most important quantity that determines the physical properties and stability of nuclei is the binding energy, B , of the nuclei as we go from light, $Z=1$ to heavy nuclei and super heavy nuclei. A number of nuclear models have been proposed from time to time (Weizsacker, 1935; Bohr and Wheeler, 1939; Bethe and Backer, 1936; Gharamany *et al.*, 2012; Gangopadhyay, 2016; Bao *et al.*, 2013; Meyerhof, 1967; Strutinsky, 1967; Dufflo and Zuker, 1999), but none of the models could explain all the properties of nuclei from light to heavy and SHN. Recent experimental studies on Super Heavy Nuclei (SHN) and the synthesis of heaviest nuclei (Yuri and Utyonkov, 2015; Yuri, 2012) emphasized the need to correctly predict the binding energy of the nuclei so as to determine their stability since T_{SF} (half-life for spontaneous fission) for the very heavy nuclei varied from 10^{-19} s to 10^{14} years. Thus it was felt that a new model is needed to calculate the binding energy of nuclei for light, heavy and super heavy nuclei. To calculate the binding energy of nuclei, a new model for the structure of the atomic nucleus is proposed. It is assumed that the nucleus is composed of two regions. The inner core region is composed of equal number of neutrons and protons, and the surface region is composed of unpaired neutrons when $N>Z$. The interaction between the core and the surface region is such that an average potential V_o is created in which each neutron of the surface region moves. For such an interaction, the contribution to the binding energy B of the nucleus will be $V_o(N-Z)$.

2.2 Binding energy of the nucleus

When the neutrons and protons come together to form a nucleus, there is an evolution of energy, called the binding energy B of the nucleus. This is due to the disappearance of a fraction of the total mass of N neutrons and Z protons. It is this energy that leads to the

formation of the nucleus as a bound system. If the mass that disappears is denoted by Δm , also called the mass defect of a nucleus, then,

$$B = \Delta m.c^2$$

Alternatively, this will be the energy required to fragment the nucleus into its constituents. Hence, the total mass M of a nucleus will be less than the sum of the masses of the neutrons and protons that constitute nucleus, i.e.,

$$M < (ZM_p + NM_N)$$

Where M_p is the mass of the free proton and M_N is the mass of the free neutron.

The binding energy per nucleon is called binding fraction, $f = \frac{B}{A}$. The value of f varies characteristically with A , and in the region of super heavy nuclei, it is not easy to determine its exact value. The variation of binding fraction of nucleus and its characteristics can be summarized as,

- a) For most nuclei the binding energy per nucleon is about 8 MeV.
- b) Binding fraction is less for light nuclei, of the order 1.1 MeV for deuteron, and then it abruptly increases to 7.8 MeV for ${}^4_2\text{He}$, and stabilizes around 7.8 MeV for heavy nuclei.
- c) The most stable nuclei are nickel and iron.

2.3 Semi-Empirical Mass formula (SEMF)

As a function of A , Z and N the phenomenological understanding of nuclear binding energies was presented by semi-empirical mass formulae (SEMF) which has always been

at the center of our understanding of several properties of the atomic nuclei (Sree-Harsha, 2018). Often referred to as Bethe-Weizsäcker (*BW*) formula, SEMF is the oldest and simplest form of binding energy (Bethe *et al.*, 1936). With good accuracy, the *BW* formula describes the various properties of nuclides such as fission, fusion, and alpha-decay barrier potential energies (Royer and Remaud, 1984; Royer, 2000). However, various suggestions have been made to add additional terms to *BW* formula, such that the theoretical results can agree with experimental results. The Wigner term, pairing term, Coulomb exchange term, surface asymmetry term among the many (Michael, 2008) are the additional terms added, such that,

$$B(A, Z) = a_v A - a_s A^{\frac{2}{3}} - a_c Z(Z-1)A^{-\frac{1}{3}} - a_a (N-Z)^2 A^{-1} + a_p \delta A^{-\frac{1}{2}}$$

Where the coefficients a_v , a_s , a_c , a_p and a_a are volume, surface, Coulomb, asymmetry, pairing coefficient, respectively. There are many choices for these coefficients (Mahdi and Reza, 2018; Dong and Ren, 2005).

If the summation of the above energies is considered, an observed variation of binding energy for a nucleus with A varying is roughly accounted for by the impression of a nucleus as a drop of incompressible liquid (Mirzaei *et al.*, 2017; Krane, 1988).

2.3.1 Volume Energy

$a_v A$ is referred as volume energy of the nucleus and is proportionate to A . The coefficient a_v is smaller than the binding energy possessed by the nucleons. For a collection of nucleons with roughly same size are brought closely together into a smaller volume, each proton or neutron have a distinct number of different nucleon in contact with it.

Subsequently, this nuclear energy is proportional to the volume (Mirzaei *et al.*, 2017) and is the largest contribution to the binding energy.

2.3.2 Surface Energy

The term $a_s A^{\frac{2}{3}}$ is known as the surface energy term. The radius should be proportional to $A^{\frac{1}{3}}$ and the surface area to $A^{\frac{2}{3}}$ if the volume of the nucleus is proportional to A . This explains why the term is proportional to $A^{\frac{2}{3}}$. The term constitutes the surface energy, in which it is the main characteristic of finite elements depicting the truth that particles in the surface always have fewer neighbours than at normal density. However, in comparison to classical liquids and crystals, the kinetic energy affects structures of the nuclear surface and the associated energy. It is seen in the lighter nuclei that the surface energy term is responsible for an increase in binding energy per particle with increasing A (Mirzaei *et al.*, 2017). Nucleons in the core in the vicinity of the surface experience a mean weaker binding.

2.3.3 Coulomb Energy

$a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}}$ is the Coulomb energy term. Electrostatic repulsion between each pair of protons is the basis for this term. In heavy nuclei it is observed that the Coulomb repulsion is responsible for the gradual decrease in the binding energy per particle as Z increases and A increases leading to heavy nuclei (Mirzaei *et al.*, 2017)

2.3.4 Asymmetry Energy

$a_a \frac{(N-Z)^2}{A}$ is the asymmetry term and is due to the fact that N increases faster than Z as A increases. For a given number of nucleons an imbalance in the number of neutrons and protons causes the energy to be higher than it needs to be. The Pauli principle is due to higher asymmetry energy term (Mirzaei *et al.*, 2017).

2.3.5 Pairing Energy

The term $a_p \delta(A, Z)$ is called the pairing term. The effect of spin-coupling is captured by this term (Ishkhanov, 2014) and is given as

$$\delta = \begin{cases} A, & Z \text{ even}, N \text{ even} \\ 0, & A \text{ odd} \\ -A, & Z \text{ odd}, N \text{ odd} \end{cases}$$

A very important role in low-energy nuclear phenomena is played by the pairing effect. It can be treated in terms of a simple generalization of the independent-particle description. The correlation effect is brought about by the predominantly attractive character of the nucleonic force; thus, no similar pairing effect is observed in atoms with the repulsive Coulomb interactions of the electron (Krane, 1988).

2.4 Separation Energy

Nuclei, accompanied by the emission of a neutron, proton, alpha, to name a few. Will disintegrate into lighter nuclei when excited to energies beyond a certain minimum. The nucleus excitation energy at which these emissions become energetically possible is called the separation energy. Separation energies are the analog to work functions of atoms or molecules. While a single parameter “work function” specifies the ionization of

an atom or molecule, nuclear transformations can occur by the emission of a neutron, proton, among the many. For each mode, corresponding separation energy is to be specified. For each species, there is a corresponding energy, such as neutron separation energy, proton separation energy, alpha separation energy, among others. for the emission of a neutron, proton, and alpha, respectively. Knowledge of these energies is very useful since we could know at what energies a specific nucleus will be unstable against the emission of neutrons, protons, to name a few. (Chary, 2014). Separation energy of a neutron is defined as the energy required to remove a neutron from a nucleus in the ground state leaving the residual nucleus also in the ground state. We can similarly define the separation energy of proton, deuteron just to mention a few. Separation energy $S_n(A)$ is related to the binding fraction f by a formula, (Blatt and Weisskopf, 1952),

$$S_n(A) \cong f + (A-1) \frac{\partial f}{\partial A}$$

Thus, separation energy and binding fraction in a nucleus are closely related to each other. The one and two nucleon separation energies of a nucleus (A, Z) are given by,

$$S_n(A, Z) = BE(A, Z) - BE(A-1, Z)$$

$$S_p(A, Z) = BE(A, Z) - BE(A-1, Z-1)$$

$$S_{2n}(A, Z) = BE(A, Z) - BE(A-2, Z)$$

$$S_{2p}(A, Z) = BE(A, Z) - BE(A-2, Z-2)$$

When S_n (neutron separation energy) or S_p (proton separation energy) is varied against Z or N , a big drop is evident after the N or Z values correspond to magic numbers (Basu, 2004).

2.5 Super Heavy Elements (SHE) and Island of stability

The mass of a nucleus is not determined by its charge rather, there exist nuclei of the same charge Z (proton number) but of different masses was first discovered by J. Thomson (1913). Such nuclei are called isotopes. It was assumed that in each of the isotopes, the mass was assumed to be roughly equal to the mass of an integral number of protons. The assumption that a nucleus is composed of protons was contradicted by the fact that in most of the nuclei the mass number A is generally twice or more than twice the proton number Z . Then appeared neutron discovery by Chadwick (1932) and Curie and Joliot (1931) and this led Heisenberg (1932) to propose the hypothesis that nuclei are composed of neutrons and protons. Hence, as we know it, nuclear Physics to-day began its exposition from the year 1932. Now the assumption that a nucleus is composed of neutrons (N), protons (Z), and its mass number $A=N+Z$, is now confirmed experimentally. Protons being positively charged compared to the mass of the neutron which has no charge have slightly less mass. Inside the nucleus, both are referred to as nucleons and this nomenclature was accepted due to the fact that the nuclear forces between neutrons and protons inside the nucleus are charge independent.

In fact, different types of nuclei have been given different names depending upon whether the proton number is constant (Z constant), and such nuclei are called isotopes (A and N varies). When the neutron number N is constant (A and Z varies), the nuclei are called Isotones. There is another set of nuclei in which Z and N are the same in two or more nuclei, and such nuclei are called isomers or mirror nuclei. For instance in ${}^3_1\text{H}$ there are two neutrons, whereas in ${}^3_2\text{He}$ there are two protons (Leo *et al.*, 2019).

Now the properties of nuclei change drastically as the atomic number Z changes between 2 and 120 or so. A number of nuclear models have been proposed from time to time to explain the properties of nuclei in different regions of mass number A , but no nuclear model can explain all the properties of nuclei.

It is not exactly known as to how many neutrons and protons can get together to form a bound atomic nucleus especially in that region of periodic table when Z is more than 92 to $Z=120$. By now there are some 3200 isotopes in the nuclear regime (Theonnessan and Barmann, 2016; Leo *et al.*, 2019). Out of these only 286 primordial nuclei existed in their present form since the creation of the Earth, and these are the stable isotopes. They constitute the valley of stability on the nuclear table of elements. As one adds protons and or neutrons to nuclei, one may move away from the region of stable isotopes, and may enter the region of short-lived radioactive nuclei, and such nuclei may be beta-unstable. At some point when a last nucleon (proton or neutron) is added to the nucleus, the binding fraction f may become zero, and hence the nucleon simply drips off. This stage is called dripline for neutron or proton, and at this stage nuclear existence ends. Even the strong nuclear force can not keep the last nucleon attached to the nucleus. As per some recent theoretical calculations, the number of bound nuclei with atomic number Z between 2 and 120 is of the order of 7000 (Erber and Birge, 2012; Agbemava *et al.*, 2014). By now it is an established fact one of the most important properties of nuclei is its stability that is directly related to its average binding energy, and binding energy per nucleon. An important role of nuclear binding energy is played in the study of nuclear mass, decay half-life, nuclear fission, and a very significant nuclear property such as nuclear stability.

The limits of nuclear stability are determined by interaction of nucleons in the nucleus. The nature of interactions between the nucleons and the limits of nuclear stability are still not known especially in super heavy nuclei in the so called “Island of Stability”. However, what is certain is that the super heavy nuclei (SHN) are at the limits of coulomb stability (Yuri, 2012). We must also understand that the term “Island of Stability” is from nuclear Physics describing the possibility of elements with particularly stable “magic numbers” of protons and neutrons. Some isotopes of transuranic elements are far more stable than others. In the 1960s, the idea of the island of stability was first proposed through developments by Strutinsky (1967) and others. The understanding of the lighter nuclei nuclear structure led to predictions by Sobiczewski *et al.*, (1966) on new nuclear shell closures. Myers and Swiatecki (1966) and Viola and Seaborg (1966) independently predicted the existence of heavy nuclei that would occupy a so-called island of stability. Since that time, the concept of an island of stability has dominated the Physics of Super Heavy Nuclei (SHN) (Yuri and Krzysztof, 2015).

The atomic nucleus is built up in “shells” in a manner similar to the electron shells in atoms. A large energy gap will separate energy levels from quantum states in two different shells by a relatively large energy gap. So when the energy levels of a given shell in the nucleus are completely filled by the number of neutrons and protons, the binding energy per nucleon will reach a local minimum such that a particular configuration will have a longer life time than the nearby isotopes that do not have filled shells (Yuri and Krzysztof, 2015; Dean, 2007). In the shell-model representation, those nuclei with closed shell Z or N are called “magic” and also when both Z and N numbers are magic, the nucleus is called “double magic” (Mahdi and Reza, 2018). Several models

have been presented to study of binding energy of SHE (Mahdi and Reza, 2018; Sobiczewski and Pomorski, 2007; Nilsson *et al.*, 1968; Myers and Swiatecki, 1966; Dong and Ren, 2005). For the study of SHN mostly macro-micro approach is used (Moller *et al.*, 1995; Moller and Nix, 1994; Yuri *et al.*, 2006; Brack *et al.*, 1972) and some mass formulas were proposed that combine the liquid-drop ideology with the shell-model corrections of Strutinsky (1967) and Myers and Swiatecki (1966).

To improve the agreement with experimental results, different corrections were introduced in the mass formula (Cwiok *et al.*, 1999; Hofman and Munzenberg, 2000). Three different ways for describing the binding energy of super heavy nuclei have been in use. First, SHN is considered as a part of a whole system of nuclei for which a global mass formula is found (Kolesnikov and Vymiatnin, 1980). Another way is the detailed local description of energy of SHN taking into account the effects of shells and subshells. The third way of description, applied for nuclei in the region limited by principal magic numbers, is attached to the beta-stability line (Kolesnikov, 2016). Although the SHN are at the limits of Coulomb stability, for SHE to exist it is enabled by shell stabilization lowering the ground-state energy creating a fission barrier (Yuri, 2012). What are the limits of existence of nuclei? What are the highest proton Z at which the nuclear landscape and periodic table of chemical elements cease to exist (Afanasjev *et al.*, 2018). Super heavy elements are also referred to as transactinides and are elements with more than 100 protons in its nucleus (Royer and Gautier, 2006), the “island of stability” is a term from nuclear Physics that describes the possibility of elements with particularly stable “magic numbers” of protons and neutrons (Yuri *et al.*, 2017). SHE are basically made by taking two lighter nuclei that contain the number of protons in the

element you want to make and then add them together, for instance to make Moscovium ($Z=115$), scientists combine calcium ($Z=20$), with Americium ($Z=95$). This practice is not simple because nuclear fission only happens under extreme conditions. Most SHE are very unstable (Royal society of chemistry, 2020) and it's unlikely that scientists will ever make one that stick around for more than even a few minutes. However super heavy isotopes with half-lives of several orders of magnitude longer than others have been predicted and this is always referred to as "Island of Stability" (Yuri and Utyonkov, 2015; Janecke and Odennell, 2007).

2.6 Neutron Dripline

Isotopes have a constant number of protons but varying number of neutrons. The limit at which any additional neutron for neutron rich isotope of each element will not be bound is called neutron dripline. The one and two neutron separation energy becomes negative as the neutron dripline nucleus gains a more stable configuration by emitting one or two neutrons directly. The neutron dripline location coincides with the limit of existence of the nucleus, meaning that nuclei beyond the neutron dripline decay with time scale of the order 10^{-22} s (Thoennesen and Barmann, 2016).

2.7 Coulomb Energy

In 1785 Charles Coulomb discovered the Coulomb energy law (Lowrie, 2007). A nucleus is composed of A nucleons. These nucleons reside in the core of the nucleus and are held by strong nuclear force. The neutron has no charge whereas the proton is positively charged. The presence of protons which are positively charged creates an electrostatic repulsion between each pair of protons. This electrostatic force between charged particles

is called Coulomb force and it is written as, $F = K_e \frac{Q_1 Q_2}{r^2}$ the quantity F is treated to be attractive if the charged particles have opposite signs and repulsive if the charged particles have like signs. From the famous SEMF, the Coulomb energy term is written as,

$a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}}$, a_c has a value close to 0.7 MeV. The Coulomb term is weak in lighter

nuclei with $N=Z$, but this trend changes drastically as $N>Z$, also the term Z^2 replaces $Z(Z-1)$. For uniformly charged sphere, Coulomb energy can be expressed as (Janecke, 1972),

$E_c = \frac{3}{5} \frac{Z(Z-1)}{R} C_i$, where C_i is the correction term for the diffuseness of nuclear shape.

2.8 Isotopes

One of the properties of atoms is radioactivity. An unstable nucleus is what every radioactive atom will have. Thus it has a tendency to release subatomic particles to obtain stability thereby releasing radiation or energy in the process. In both radioactive and non-radioactive variety of elements, neutrons are present. The number of neutrons will be different in both the varieties. These different varieties of the same element are termed isotopes. A Scottish doctor and writer Margaret Todd in 1913 first proposed the term isotope to a chemist Frederick Soddy. Isotopes exist in two distinct forms: one is the stable isotopes, these are the isotopes which do not break down in its free state. There are isotopes that can not sustain themselves meaning they spontaneously break down to form two different nuclei, this elements are called radioactive isotopes.

There is emission of alpha, beta and gamma rays. Radioactivity and the nuclear energy rely on the instability of isotopes of heavy elements, tapping into the explosive power of

the nucleus. Generally, the chemical properties of isotopes of any element are almost identical owing to their similar arrangement and number of electrons. Isotopes of hydrogen are an exception in this case because size of the nucleus of hydrogen atoms is affected majorly by the numbers of neutrons. In a particular element physical properties vary from each other. This is because the physical properties of any isotope are dependent on the mass. The mass of each isotope of a single element varies from one another. Fractional distillation and diffusion are examples of processes used to separate isotopes from one another. The use of isotopes such as uranium (^{235}U) in nuclear reactors as fuel makes it popular. Radioactive isotopes also find their use in medicinal purposes such as detection of tumors and blood clots.

2.9 Isotones

In nuclear Physics and nuclear chemistry, nuclides having the same amount of neutrons but the atomic number Z and the mass number A are different are called isotones. The name isotone has been derived from the name isotope by changing the “p” in “isotope” from “p” for “proton” to “n” for “neutron”. The term was formed by the German physicist Guggenheimer (Brucer, 1978). An insight into the stability of a particular neutron configuration is provided by the numbers of naturally occurring isotones. The fact that odd neutron configurations are relatively more stable is established by most atoms having an odd number of neutrons and are isotonic in nature.

2.10 Isobars

Isobars are elements that have the same amount of nucleons (sum of protons and neutrons), A is constant ($N+Z$), but contain varying numbers of protons and neutrons.

Examples of isobars include ${}^{14}_6\text{C}$ and ${}^{14}_7\text{N}$, ${}^{40}_{20}\text{Ca}$ and ${}^{40}_{19}\text{K}$. The name was given by Alfred Walter Stewart in 1918 (Brucer, 1978). It was originally taken from the combination of Greek words- *isos* meaning equal and *bar* meaning weight. Since their number of electrons is different, their chemical properties are different. That is isobars are elements, which are chemically different but physically same.

CHAPTER THREE

METHODOLOGY

3.1 Theory

Looking at the actual composition of a nucleus, we have proposed a slightly different and unconventional type of a nuclear model to calculate nucleus binding energy. Some low mass nuclei have $N=Z$, but as A increases, $N>Z$. Nowadays in some of the low mass designer nuclei $N>Z$, such as ${}^6_3\text{Li}$ (Sherill and Bradley, 2008). In the proposed nuclear model, it is assumed that in any nucleus, the core of the nucleus is made up of equal number of neutrons and protons while the unpaired Neutrons reside in the surface region of the nucleus. Thus for a nucleus of mass number $A=Z+N$, the core of the nucleus will be composed of Z proton-neutron pairs and the unpaired neutrons equals $N-Z$, will constitute the surface region of the nucleus. The core is thus composed of Z neutron-proton pairs. The interaction energy between the pairs is equal to $(2Z-1)Z\epsilon_p$ where ϵ_p is the pairing energy between a pair of nucleons. The core will contain $2Z$ nucleons, and thus the number of pairs will be,

$${}^{2Z}C_2 = \frac{2Z!}{2!(2Z-2)!} = Z(2Z - 1) \quad (3.1)$$

3.2 Calculation of ϵ_p and B_{tot}

If ϵ_p is the energy associated with each pair, then the energy E_p of the nucleus due to pairing interaction will be say

$$E_p = \epsilon_p Z(2Z - 1) \quad (3.2)$$

As in the liquid drop model, the core will be assumed to be an in-compressible system and thus the volume energy will be assumed to be E_v and

$$E_v = 2a_v Z \quad (3.3)$$

Similarly the Coulomb energy due to the presence of Z protons in the core of the nucleus was written as (for large nuclei)

$$E_C = -0.72 \frac{Z^2}{(2Z)^{1/3}} \quad (3.4)$$

Since in this model, the surface region was treated separately, there was no such thing as the surface energy. Also $N=Z$ in the core, thus there was no asymmetry energy of the core.

Now we had to decide on the type of interaction that the core of the nucleus could have with the unpaired neutrons in the surface region.

To keep matters simple, the interaction between the core and the neutrons in the surface region was assumed such that it lead to an average potential in which each neutron in the surface region could move. This was close to what the Hartree-Fock theory demands. Thus if the average potential is denoted by V_0 per nucleon then total potential energy would be $-V_0(N-Z)$; denoting this by E_H , we got

$$E_H = -V_0(N-Z) = -V_0(A-2Z) \quad (3.5)$$

Combining equations (3.2), (3.3), (3.4) and (3.5), we got the total binding energy, B_{tot} (A,Z) as

$$B_{\text{tot}}(A,Z) = a_v A - \varepsilon_p Z(2Z-1) - 0.72 \frac{Z^2}{(2Z)^{1/3}} - V_0(A-2Z) \quad (3.6)$$

Equation (3.6) was used to calculate the value of V_o for finite nuclei from low A to medium A and then to heavy nuclei. Equation (3.6) gave the new formula for binding energy according to our model of the nucleus. In Eq. (3.6) all the parameters are known, except the value of V_o , and this became an adjustable parameter since this depended on the number of nucleons in the core, and the number of neutrons in the surface region. Exact calculation of B_{tot} and its comparison with the experimental values of B would exactly determine how V_o may vary as A changes. It is important to understand that the value of V_o changed as the neutron number (N) changed in isotopes of a given nucleus with Z fixed. Calculations indicated how V_o changes as the number of neutrons in the surface region changes. The value of V_o is calculated by equating $B_{tot}=B$ (the experimental value of binding energy). Variation of V_o with change of the number of neutrons is calculated using the formula in Eq (3.6) where B_{tot} will be the experimental value for different nuclei.

3.3 Derivation for V_o

For $A \leq 50$, there are nuclei in which the amount of protons (Z) is equal to that of neutrons (N) number. As there is an increase in mass number, $A=N+Z$, of a nuclide the number of neutron N , increases faster than that of proton, Z . Nuclei for which $N=Z$ are found to be strongly bound and have large binding energy, B , and also large binding fraction $f = \frac{B}{A}$. Nuclei in which $Z \neq N$ ($N > Z$) and have large value of A have their B and

f values reduced due to Coulomb repulsion between the protons. There are nuclei with equal and even numbers of neutrons and protons (even-even expressed as e-e). Then there are nuclei with equal and odd numbers of neutrons and protons (odd-odd expressed as o-

o). There are also nuclei with even number of neutrons and odd number of protons (e-o), and odd number of neutrons and even number of protons (o-e). It is found that no two nuclei have the same binding fraction f , and the value of f rises abruptly for nuclei with $A > 20$ or so, rising to a maximum value of around 8.4 MeV and then starts decreasing as A increases, to some value 7.4 MeV. The value of B and f depends on how we model the nucleus and what kind of interactions between the neutrons and protons are considered.

Due to saturation of density of the nucleus (no other system has a density more than the nuclear density), the nuclear interaction energy is also assumed to be the largest. Assuming that nuclides are made of incompressible matter, and that nuclear force for each nucleon is the same and there is saturation of the nuclear force Bohr and Wheeler (1939) and Weissacker (1935) proposed a model called liquid drop that led to the famed semi-empirical formula for the mass of the nucleus. The nuclear mass depends on the values of A and Z (Bethe, 1936), Coulomb force, the Pauli exclusion principle, the nucleon asymmetry, the pairing force to name a few and the final mass formula and binding energy of the nucleus depends on N , Z , $N+Z$, $Z-N$ or $N-Z$, (Cottingham and Greenwood, 2001). Then there are quite a few other nuclear models that have been proposed to explain the properties of nuclei. The most successful of them, being the nuclear shell model (Goepfert-Mayer, 1949).

Eq (3.6), is re-arranged to get the value of V_o as,

$$V_o = \frac{-B_{tot}(A, Z) + 2a_v Z - \varepsilon_p Z(2Z - 1) - 0.72 \left[\frac{Z^2}{(2Z)^{1/3}} \right]}{(A - 2Z)} \quad (3.7)$$

Using the following parameters

$$a_v = 15.8 \text{ MeV (Chen, 2011, Mirzaei et al., 2017)}$$

$$\epsilon_\rho = 2 \text{ MeV}$$

Eq. (3.7), changes to

$$V_o = \frac{-B_{tot}(A, Z) + 31.6Z - 2Z(2Z - 1) - 0.72\left[\frac{Z^2}{(2Z)^{1/3}}\right]}{(A - 2Z)} \text{ MeV} \quad (3.8)$$

Using the respective binding energy and mass number, we calculate the values of V_0 for some light, medium and heavy nuclei.

3.4 Super Heavy Nuclei (SHN)

From Bethe and Weizsäcker (1935), Bethe and Bacher (1936) and Bohr and Mottelson (1975), assuming similarities between atomic nucleus and liquid drop, a formula for binding energy B of nucleus was first proposed whereby:

$$B(A, Z) = a_v A - a_s A^{2/3} - a_c Z(Z - 1)A^{-1/3} - a_a \left(\frac{A}{2} - Z\right)^2 A^{-1} + a_p \delta A^{-1/2}$$

Where a_v , a_s , a_c , a_a and a_p , respectively are; volume, surface, coulomb, asymmetry and pairing coefficients. Over the years this formula has been modified to include different parameters, There are many choices for these coefficients (Mahdi and Reza, 2018; Dong and Ren, 2005). The formula proposed recently looks like,

$$B_{tot}(A, Z) = 2a_v Z - \epsilon_\rho 2Z(2Z - 1) - 0.72 \frac{Z^2}{2Z^{1/3}} - V_0(A - 2Z) \quad (3.9)$$

Where the coefficient a_v and ϵ_ρ are the volume term and the pairing energy term. The nucleus binding energy formula is given by the Eq. (3.9). In this nuclear model it is

assumed that when $N > Z$, the core of the nucleus is composed of equal number of neutrons and protons that form neutron-proton pairs, and the unpaired neutrons reside in the surface region of the nucleus which is also called neutron skin. Thus for a nucleus of mass number $A = Z + N$, the core of the nucleus will be composed of Z proton-neutron pairs and the unpaired neutrons equal to $N - Z$, will constitute the surface region of the nucleus. If the pairing energy between neutron-proton pair is ϵ_p then the total energy of the core of the nucleons will be $\epsilon_p \binom{2Z}{2} = \epsilon_p Z(2Z - 1)$ since there are $2Z$ nucleons in the core. The interaction between the core and the neutrons in the surface region will be assumed to lead to an average potential V_0 in which each neutron in the surface region can move. Coulomb energy is one of the major parameters in determining the binding energy as shown in different binding energy formula (Mahdi and Reza, 2018; Michael, 2008; Dai *et al.*, 2017; Chemogos *et al.*, 2019; Mackie and Bayn, 1977). Coulomb energy has continuously been corrected in order to understand nuclear correlations, charge-dependence, nuclear force, Coulomb perturbation and nuclear rearrangement energy (Cherop *et al.*, 2019; Janecke, 1972; Nolen and Schiffer, 1969). The semi-empirical mass formula (SEMF) gives the Coulomb energy term as $a_c Z(Z - 1)A^{-\frac{1}{3}}$, where $a_c = 0.71 \text{ MeV}$. When calculating light and medium nuclei binding energy, this term is applied. When it is calculated for heavy and super heavy nuclei, the Coulomb energy term changes drastically. Where the term Z^2 replaces the term $Z(Z - 1)$ (Ghoshal, 2008). Using a modified Coulomb energy equation from (Cherop *et al.*, 2019), we can write an expression for E_c as,

$$E_c = \frac{3}{5} \frac{Z_1 Z_2 e^2}{4\pi \epsilon_0 R_o} e^{\frac{R_o^n}{nR^n}} \quad (3.10)$$

Where the terms Z_1, Z_2 are the proton nuclear charges, e is the electron charge, the permittivity of free space being ϵ_o , the core radius is R_o , nuclear radius being R and n being an integer. The correction terms are the exponential terms of the ratio of the powers of the nuclear core radius (R_o) to the powers of the effective nuclear radius (R). As the power of their radii increases from $n=1$ to $n>21$, the ratio of $\frac{R_o^n}{nR^n}$ tends to the value zero

and the exponential correction term goes to one.

Eq. (3.10) is inserted to Eq. (3.9) to replace the Coulomb energy term. The terms are rearranged to get value of V_o such that,

$$V_o = \frac{2a_v Z - B_{tot}(A, Z) - \epsilon_p Z(2Z - 1) - \frac{3}{5} \frac{Z_1 Z_2 e^2}{4\pi \epsilon_o R_o} e^{-\frac{R_o^n}{R^n}}}{A - 2Z} \quad (3.11)$$

Using the respective binding energy values and mass number we calculate the values of V_o for 254 SHN from Eq. (3.11).

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1 Results

The following values for different physical quantities have been used to calculate V_o , I.e

$$a_v = 15.8 \text{ MeV (Mirzaei } et al., 2017; \text{ Chen, 2011)}$$

$$\epsilon_\rho = 2 \text{ MeV}$$

$B_{tot}(A, Z)$ is obtained from the experimental binding energy values given in (Gharamany *et al.*, 2012; Wang *et al.*, 2012; Wang *et al.*, 2017).

$$h = 6.626 \times 10^{-34} \text{ JS}$$

$$\omega = 6 \times 10^9 \text{ s}^{-1}$$

$$\kappa = 1.3807 \times 10^{-23} \text{ J/K}$$

$$\mu = 8.369 \times 10^{-28} \text{ Kg}$$

$$a_o = 1.3 \times 10^{-15} \text{ A}^{1/3} \text{ M}$$

Calculations gotten after analysis using excel and tabulated in form of tables, graphs and figures. Results for average potential interaction for light, medium and heavy are tabulated on Table 1 and for super heavy nuclei are tabulated in Table 2. These tables are provided in the appendices as appendix I and appendix II.

4.2 Discussions

4.2.1 Light Nuclei

4.2.1.1 Isotopes

Table 3: Isotopes of Sulphur with Proton number $Z=16$ from Table 1.

Nuclei	A	N	Z	A-2Z	$V_0(\text{MeV})$
S	33	17	16	1	824.879
S	34	18	16	2	418.148
S	36	20	16	4	213.292

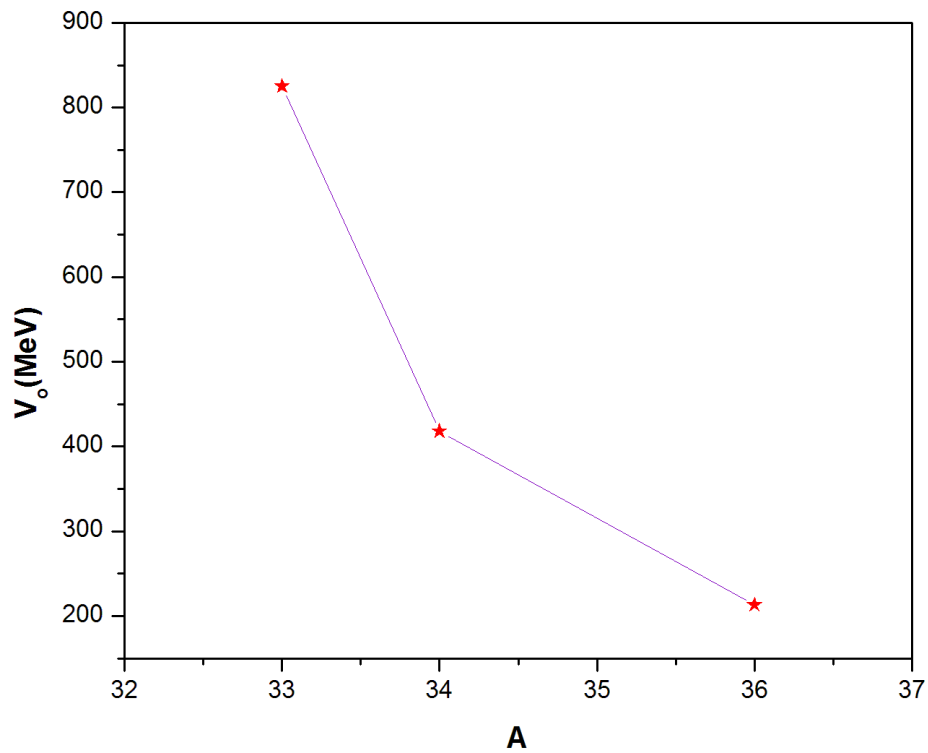


Figure 1: Graph showing variation of V_0 (MeV) against A for isotopes of Sulphur from results of Table 3.

The interaction potential V_0 decreased with an increase in A for sulphur isotopes. For $Z=N$ region the interaction potential energy E_H goes to zero because the surface region has no neutrons. The ratio N/Z increased with an increase in A . For low mass nuclei, the value of V_0 changed abnormally (drastically) or by a very large amount when compared to the change in V_0 for transuranic elements or super-heavy-nuclei (SHN) or (SHE), but the trend in the variation was the same. V_0 decreased as N increased keeping Z constant (for isotopes). This trend is consistent with neutron dripline theory presented by (Kazuhiro *et al.*, 2010).

4.2.1.2 Isotones

Table 4: Isotones with Neutron number $N=20$ from table 1.

Nuclei	A	N	Z	A-2Z	$V_0(\text{MeV})$
S	36	20	16	4	213.292
Cl	37	20	17	3	322.043
Ar	38	20	18	2	544.596
K	39	20	19	1	1216.635

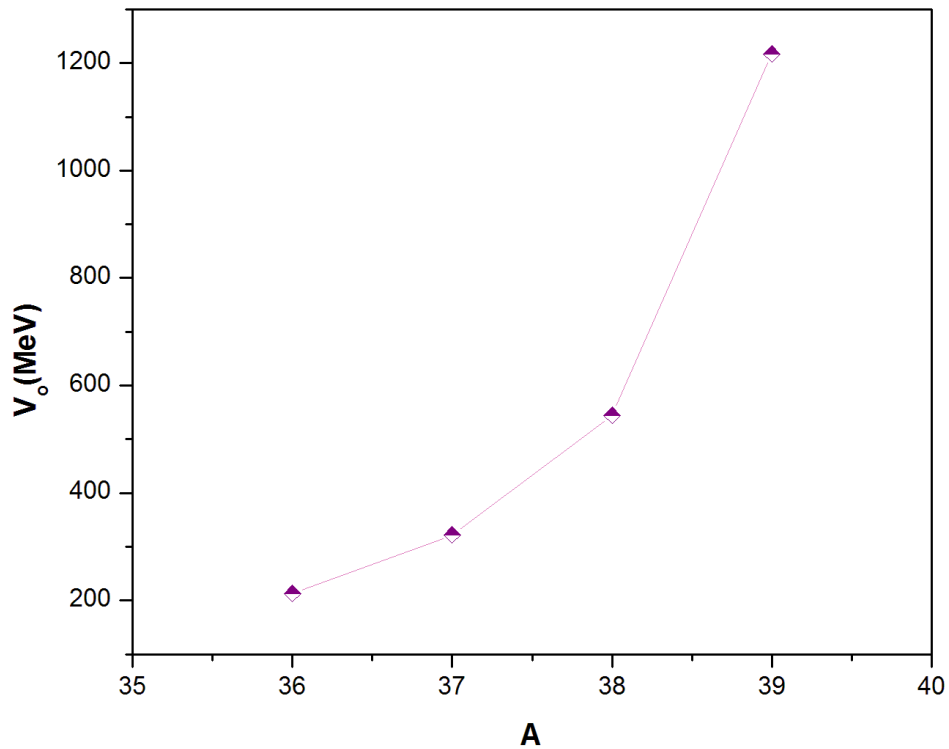


Figure 2: Graph showing variation of V_0 (MeV) against A for Isotones with Neutron number N=20 from results of Table 4.

For isotones with N=20, the value of V_0 increased with an increase in A. The ratio N/Z decreased with an increase in Z. In these light nuclei also, V_0 increased as Z increased, but the increase in the value of V_0 is very large. These obtained results conform to theory of stability by Hansen (1992).

4.2.1.3 Isobars

Table 5: Isobars with A=28 from Table 1.

Nuclei	A	N	Z	A-2Z	$V_0(\text{MeV})$
Ne	28	18	10	8	39.290
Na	28	17	11	6	63.872
Mg	28	16	12	4	115.081
Al	28	15	13	2	269.825

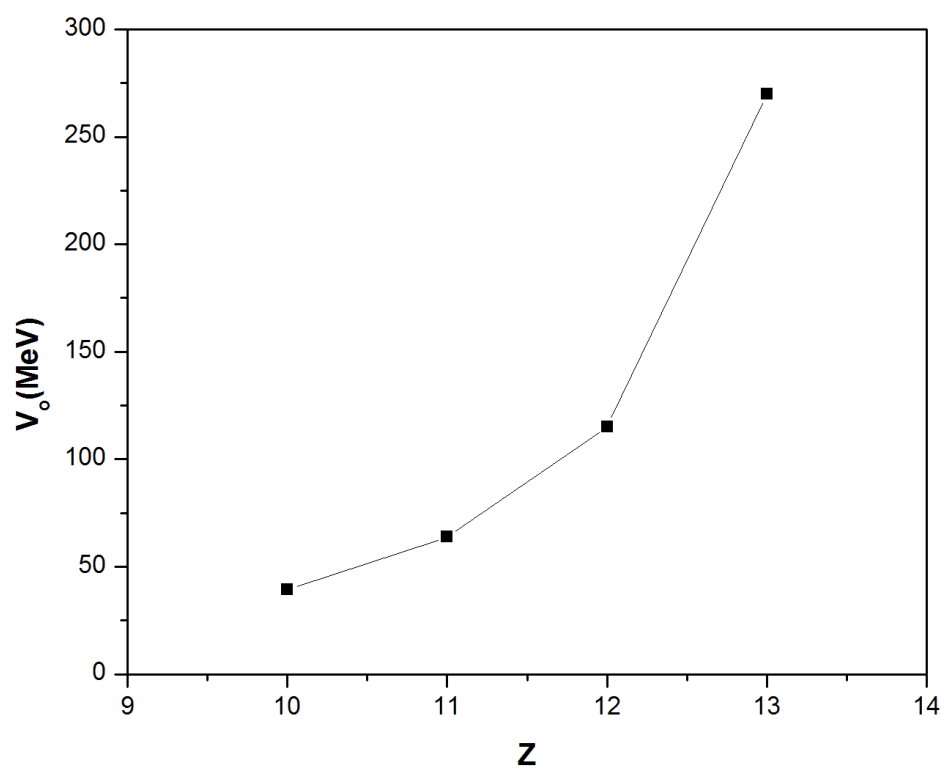


Figure 3: Graph showing variation of V_0 (MeV) against Z for Isobars with Mass number A=28 from results of Table 5.

From Table 5, as the proton number (Z) increases the value of V_0 increases this increases is due to decrease of unpaired Neutrons on the outer region of nucleus. This means the system becomes more bound and approaches line of stability (Hansen, 1992). This is also confirmed by the ration of $N:Z$ for the isobars.

4.2.2. Medium Nuclei

4.2.2.1 Isotopes

Table 6: Isotopes of Selenium (Se) with $Z=34$ from table 1.

Nuclei	A	N	Z	A-2Z	V_0(MeV)
Se	74	40	34	6	721.401
Se	76	42	34	8	543.448
Se	77	43	34	9	483.889
Se	78	44	34	10	436.550
Se	80	46	34	12	365.198
Se	82	48	34	14	314.165

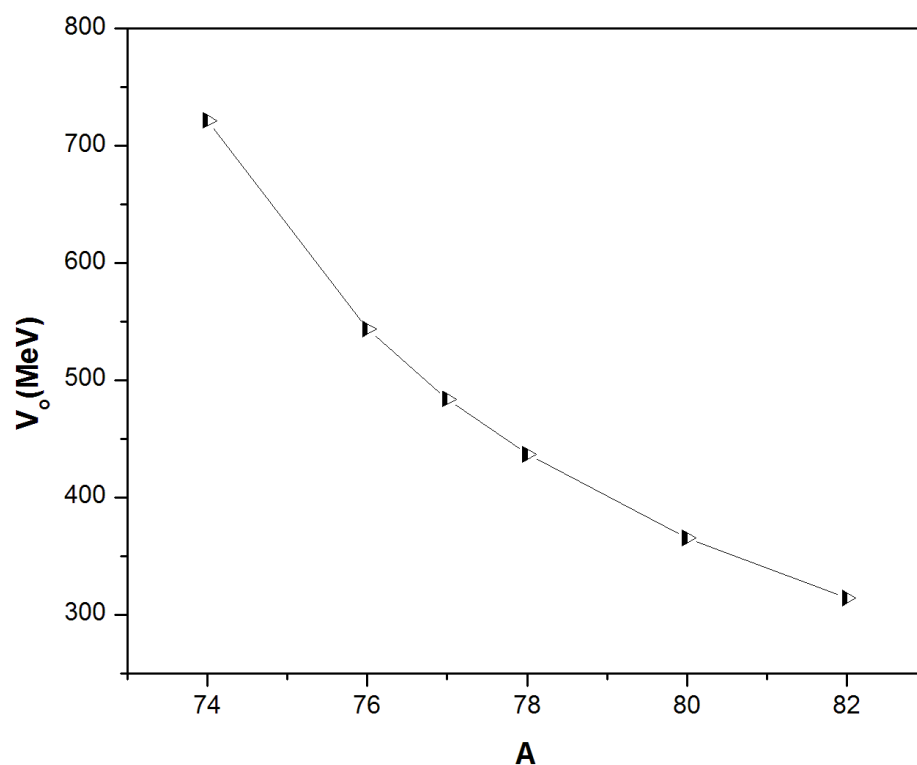


Figure 4: Graph showing variation of V_0 (MeV) against A for Isotopes with Proton number $Z=34$ from results of Table 6.

From Table 6, with constant proton number (Z) and increasingly neutron number (N) the value of V_0 decreased and this was attributed to the increase of unpaired neutrons on the outer region of the nucleus. From the results, the ratio of $N:Z$ increases and this affects the stability of any nuclei, in that the nuclei become unbound and approaches neutron dripline (Hansen, 1992).

4.2.2.2 Isotones

Table 7: Isotones with N=50 from table 1.

Nuclei	Symbol	A	N	Z	A-2Z	$V_0(\text{MeV})$
Krypton	Kr	86	50	36	14	353.423
Rubidium	Rb	87	50	37	13	473.351
Strontium	Sr	88	50	38	12	459.426
Yttrium	Y	89	50	39	11	527.767
Zirconium	Zr	90	50	40	10	610.724
Molybdenum	Mo	92	50	42	8	841.414

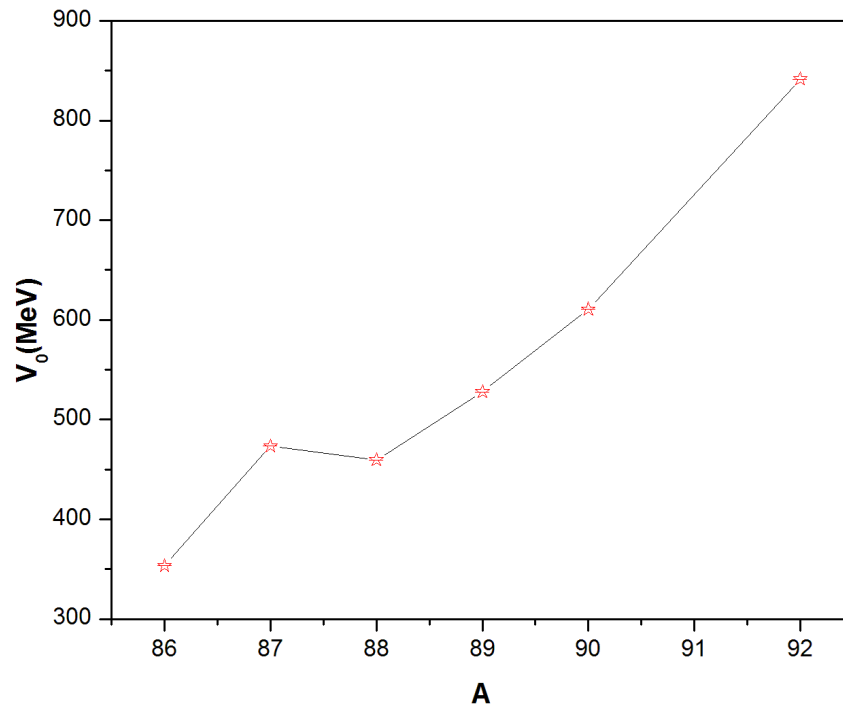


Figure 5: Graph showing variation of V_0 (MeV) against A for Isotones with Neutron number Z=50 from results of Table 7.

From Table 7, As the proton number (Z) increased with constant Neutron number (N), the value of V_o increased. This means that the excess neutrons on the surface region reduced by moving to the core region and forming more $N-Z$ pairs, this increased the interaction between the core region and surface region also the nuclei become more bound (Mahdi and Reza, 2018).

4.2.2.3 Isobars

Table 8: Isobars with $A=75$ from table 1.

Nuclei	Symbol	A	N	Z	A-2Z	$V_0(\text{MeV})$
Gallium	Ga	75	44	31	13	264.457
Arsenic	As	75	42	33	9	432.264
Krypton	Kr	75	39	36	3	1539.397

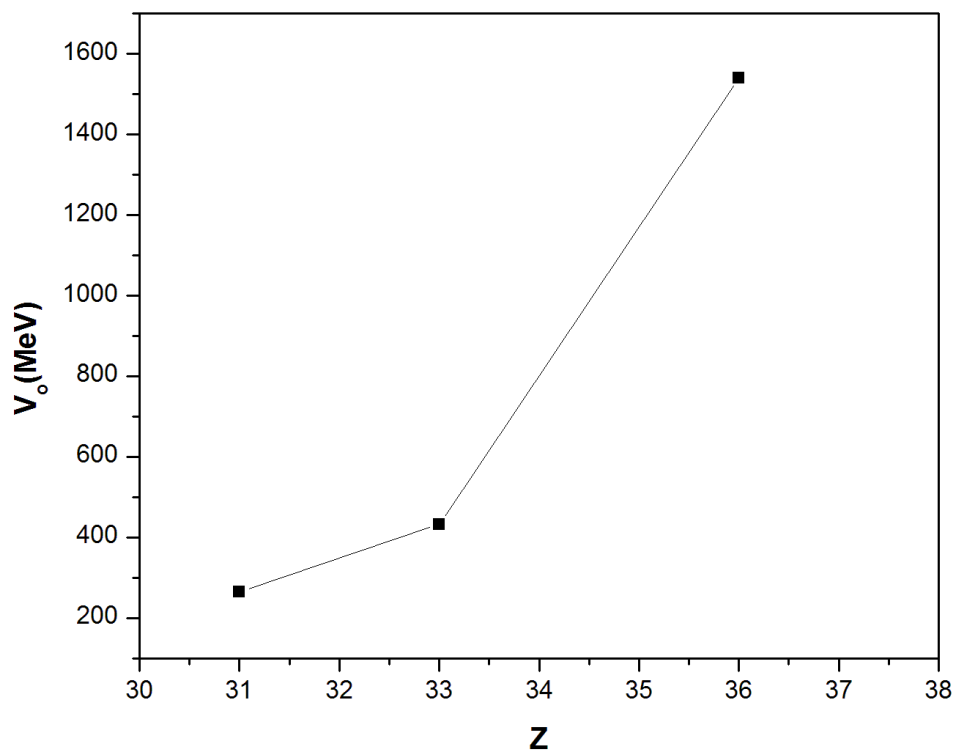


Figure 6. Graph showing variation of V_0 (MeV) against Z for Isobars with Mass number $A=75$ from results of Table 8.

From Table 8, the value of V_0 increased with a decreased in the neutron number (N). This can be explained by the fact that reduction in neutron number makes the nuclei to become bound because the ratio of $N:Z$ reduces. This means the nuclei approaches the stability line (Ishkhanov *et al.*, 2010).

4.2.3. Heavy Nuclei

4.2.3.1 Isotopes

Table 9: Isotopes of Hafnium with $Z=72$ from table 1.

Nuclei	A	N	Z	A-2Z	$V_0(\text{MeV})$
Hf	174	102	72	30	681.094
Hf	176	104	72	32	638.990
Hf	177	105	72	33	619.820
Hf	178	106	72	34	601.815
Hf	179	107	72	35	584.794
Hf	180	108	72	36	568.755

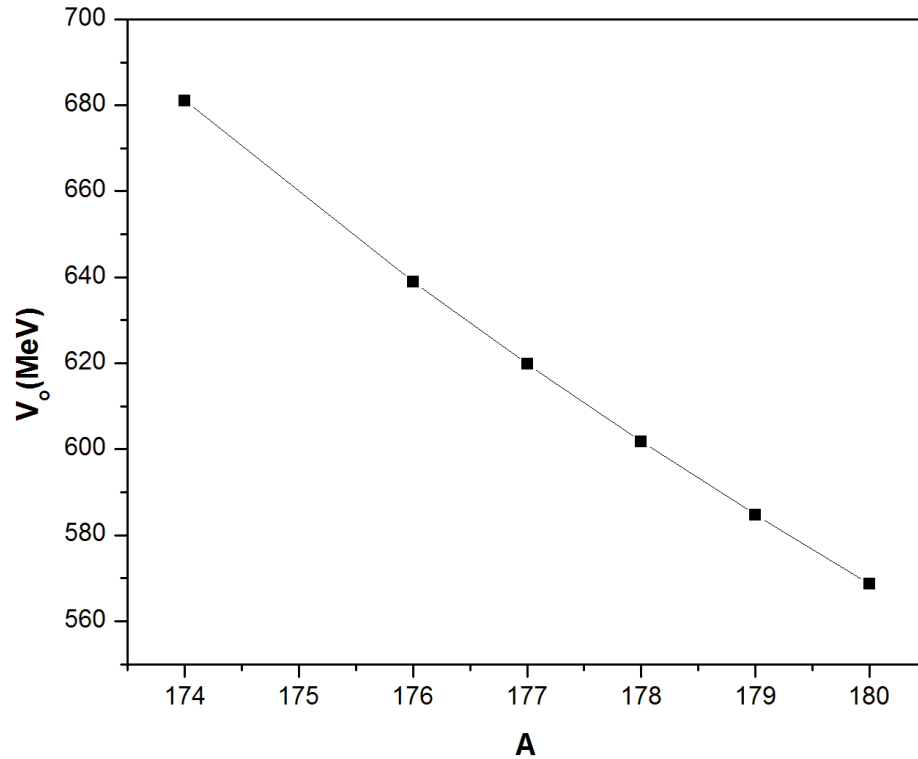


Figure 7: Graph showing variation of V_0 (MeV) against A for Hafnium Isotopes with Proton number $Z=72$ from results of Table 9.

As the neutron number changed by unity in the surface region, V_0 changed by a margin between 16 MeV and 44 MeV and it is a linear variation, or more or less a constant. Here V_0 decreases as N increased keeping Z constant. Binding between the core and surface neutrons decreased as N increased. The ratio of $N:Z$ increased with increase in A , this shows the nuclei drifted more from the stability line because of increased unpaired neutrons in the neutron skin (Kazuhiro *et al.*, 2010).

4.2.3.2 Isotones

Table 10: Isotones with Neutron number $N=90$ from table 1.

Nuclei	Symbol	A	N	Z	A-2Z	V_0 (MeV)
Xenon	Xe	144	90	54	36	296.768
Cesium	Cs	145	90	55	35	316.657
Barium	Ba	146	90	56	34	337.960
Cerium	Ce	148	90	58	32	385.200
Neodymium	Nd	150	90	60	30	439.704
Samarium	Sm	152	90	62	28	503.031

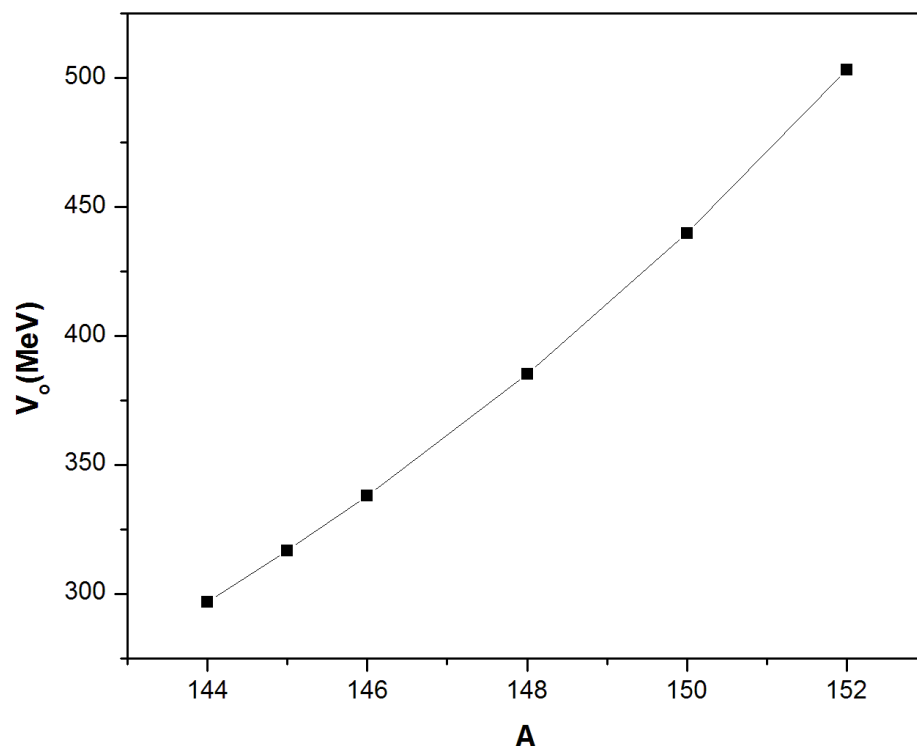


Figure 8: Graph showing variation of V_0 (MeV) against A for Isotones with Neutron number $N=90$ from results of Table 10.

Here V_0 increased as Z increased keeping N constant, interaction (binding) energy variation V_0 hovers between 20 MeV and 64 MeV as a proton is added. Binding between the core and surface neutrons increased as Z increased. Table 9 shows that increase in neutron reduced V_0 , and Table 10 shows that increase in proton number increased V_0 . This can lead to stability of transuranic elements in the “Island of stability” as presented by Hansen (1992).

4.2.3.3 Isobars

Table 11: Isobars with A=120 from Table 1.

Nuclei	Symbol	A	N	Z	A-2Z	V_0 (MeV)
Cadmium	Cd	120	72	48	24	349.969
Indium	In	120	71	49	22	397.690
Tin	Sn	120	70	50	20	486.417
Tellurium	Te	120	68	52	16	656.254
Xenon	Xe	120	66	54	12	883.795
Cesium	Cs	120	65	55	10	1099.580
Barium	Ba	120	64	56	8	1424.388

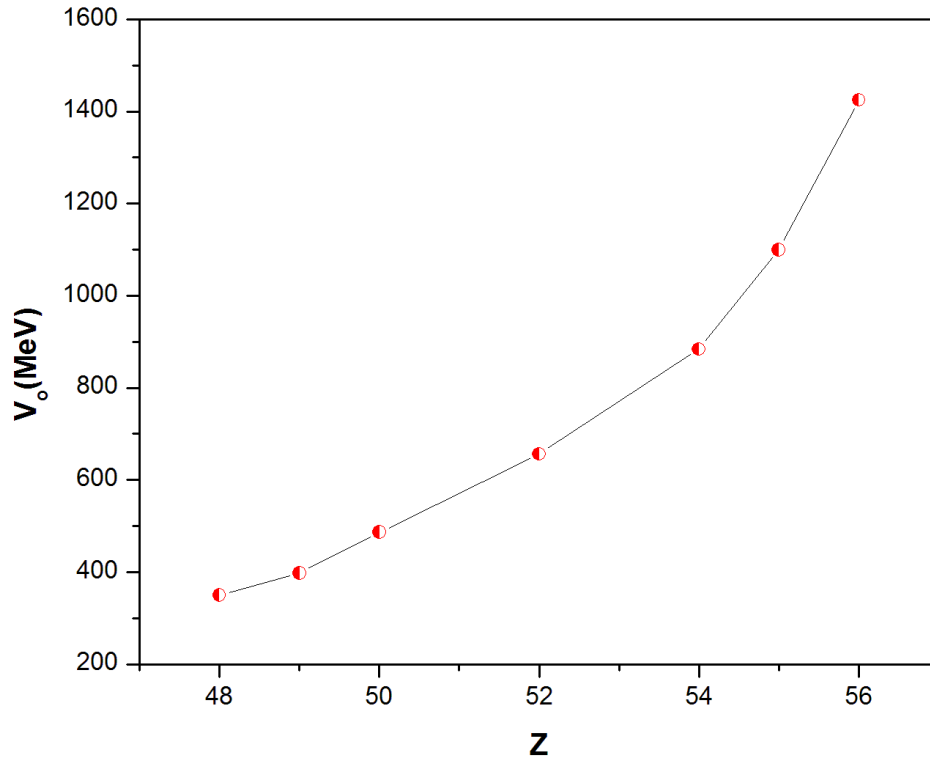


Figure 9: Graph showing variation of V_0 (MeV) against Z for Isobars with Mass number $A=120$ from results of Table 11.

From Table 11, the average interaction potential increased by a large quantity though the increase is not uniform. The nuclei becomes more bound because the number of unpaired neutrons in the neutron skin reduced by forming NZ pairs. This means the nuclei approaches the stability line. These trend of results conforms to the theories presented by Ishkhanov *et al.*, (2014) and in Hyperphysics (2020).

4.2.4 Super Heavy Elements (SHE)

4.2.4.1 Isotopes

Table 12: Calculated values of V_0 for Isotopes of $^{234-238}_{92}\text{U}$, $^{238-242}_{94}\text{Pu}$ and $^{270-274}_{110}\text{Ds}$, from Table 2.

A-2Z	V_0 (MeV)		
	$^{234-238}_{92}\text{U}$	$^{238-242}_{94}\text{Pu}$	$^{270-274}_{110}\text{Ds}$
50	671.688	701.323	961.345
51	658.621	687.682	942.628
52	646.081	674.583	924.655
53	633.988	661.954	907.315
54	622.361	649.812	890.647

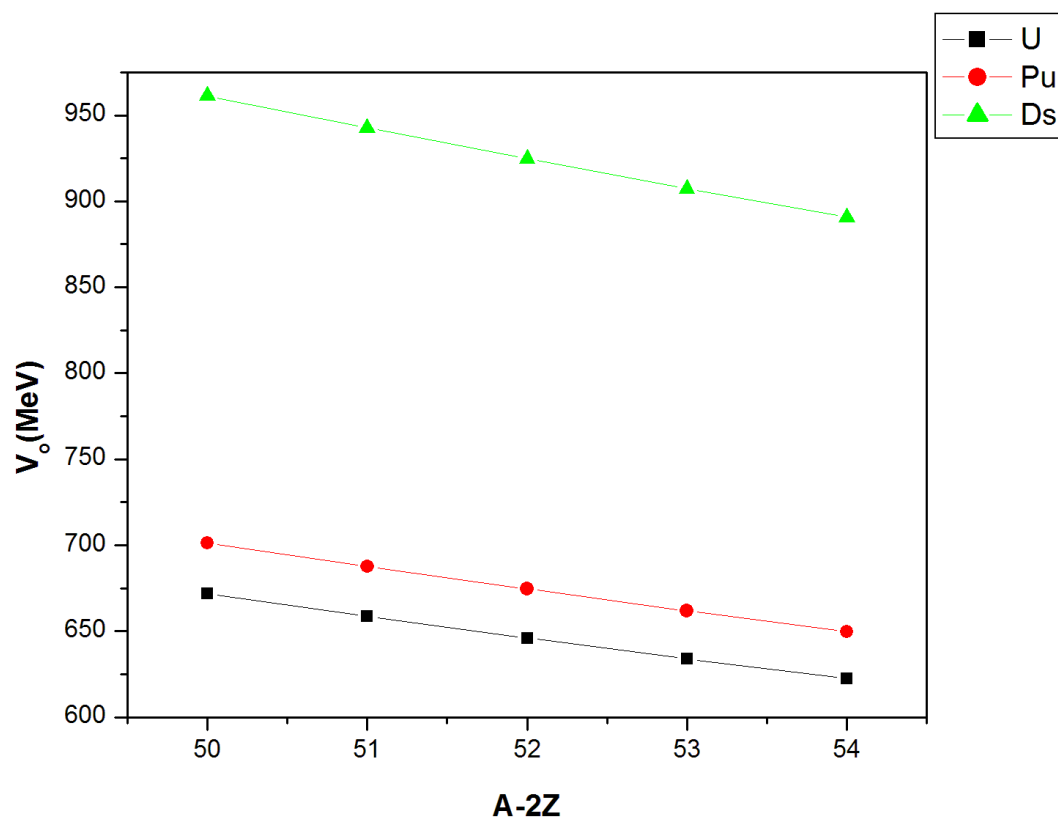


Figure 10: Graph showing variation of $V_o(\text{MeV})$ against $A-2Z$, from results of Table 12.

From Table 12, as the neutron number (N) increased at constant proton number (Z), the value of V_o reduced for the respective isotopes as the neutron number increased by unity. The reduction in the value of V_o is small when neutron number increased by unity. In this nuclei, coulomb separation energy does not increases when more neutrons were added because proton number was constant. The nuclei tends to become unbound because more unpaired neutrons resided in the neutron skin, this trend is similar to the neutron dripline theory (Kazuhiro *et al.*, 2010).

4.2.4.2 Isotones

Table 13: Values of V_0 for Isotones with Neutron number $N=142$ from Table 2.

Nuclei	A	N	Z	A-2Z	V_0 (MeV)
U	234	142	92	50	671.688
Np	235	142	93	49	700.295
Pu	236	142	94	48	730.276
Am	237	142	95	47	761.672
Cm	238	142	96	46	794.628
Bk	239	142	97	45	829.187
Cf	240	142	98	44	865.524
Es	241	142	99	43	903.692
Fm	242	142	100	42	943.901

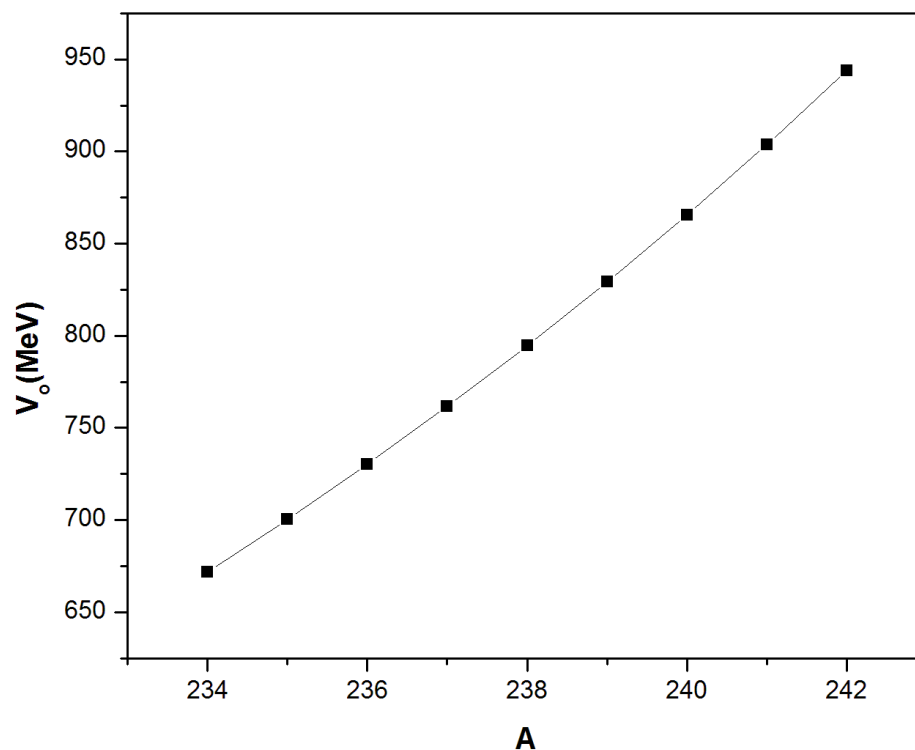


Figure 11: Graph showing variation of V_o (MeV) against A for Isotones with Neutron number $N=142$ from results of Table 13.

From Table 13, As the proton number (Z) increased with constant Neutron number (N), the value of V_o increased. This means that the excess neutrons on the surface region reduced by moving to the core region and forming more $N-Z$ pairs, this increased the interaction between the core region and surface region. Increased coulomb energy also plays a role on the nuclei such that it reduces its stability (Mahdi and Reza, 2018).

4.2.4.3 Isobars

Table 14: Values of V_0 for Isobars with mass number $A=238$ from Table 2.

Nuclei	A	N	Z	V_0 (MeV)
U	238	146	92	622.361
Np	238	145	93	660.232
Pn	238	144	94	701.323
Am	238	143	95	745.934
Cm	238	142	96	794.628
Bk	238	141	97	847.851

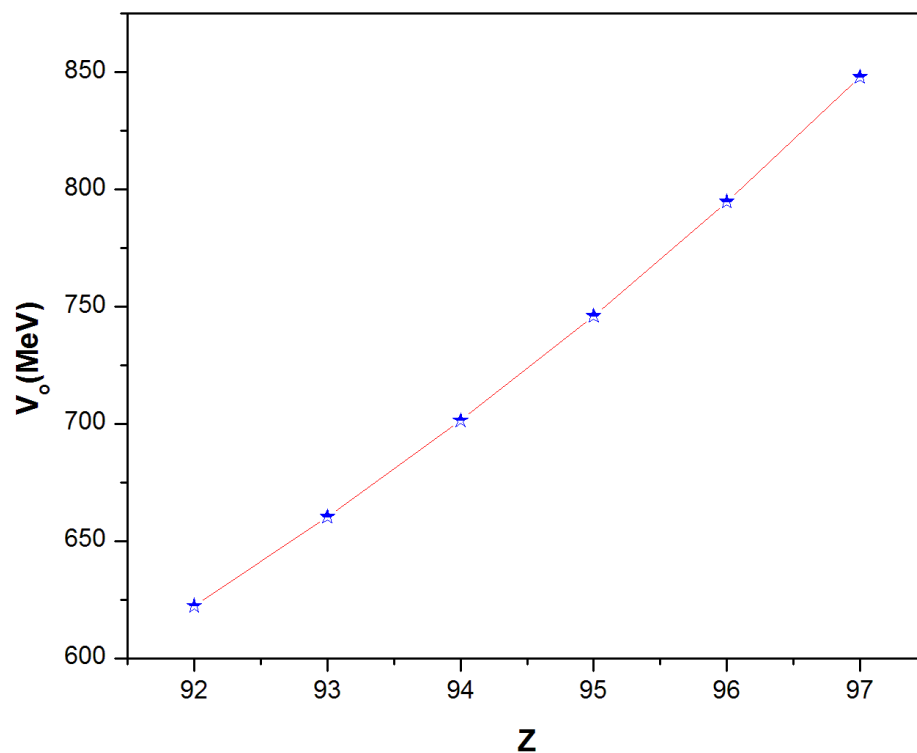


Figure 12: Graph showing variation of V_o (MeV) against Z for Isobars with Mass number $N=238$ from results of Table 14.

From Table 14, as the proton number (Z) increased with reducing neutron number (N) at constant value of mass number, the value of V_o increases, this indicates that the ratio of $N:Z$ decreased with an increase in proton number (Z), meaning that interaction between the core region and the surface region increased since there was an increase in repulsion energy. This leads the nuclei to become unbound because of increasing coulomb repulsion due to increased proton number and reduced neutron number (Ishkhanov *et al.*, 2014).

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

For the isotopes, interaction potential energy V_o generally decreased with an increased in A . This is evident for both light and heavy nuclei. When the ratio $N:Z$ increased as N increased, V_o decreased. Binding between the core and the surface decreased as N increased. The variation in V_o is different for different isotopes as the neutron number changed in the surface region of the nuclei by unity. For heavy nuclei ($99 < A < 234$) the maximum change in V_o is of the order of 40 MeV when the neutron number in the surface region changed by unity. For instance, the value of V_o for $^{234}_{92}\text{U}$ is 671.688 MeV, but for $^{235}_{92}\text{U}$, $V_o = 658.621$ MeV. And for light nuclei ($0 < A < 50$), the change in V_o for a unit change in neutron number is considerably higher. For instance, the value of V_o for $^{17}_8\text{O}$ is 137.249 MeV, but for $^{18}_8\text{O}$, $V_o = 72.646$ MeV. Similarly the value V_o for $^{21}_{10}\text{Ne}$ is 257.931 MeV and that for $^{22}_{10}\text{Ne}$ is 134.147 MeV. Table 3 indicates that as there is an increase in neutron number N by unity, V_o decreases. Here the decrease is quite large compared to the decrease in heavy nuclei as contained in Table 9. Since the addition of a neutron carries attractive energy into the surface region, the value of V_o decreased accounting for the appropriate nucleus binding energy.

For Isotones the interaction potential energy V_o generally increases with increase in Z . This is evident for both light, medium, heavy and super heavy nuclei. Thus as $N:Z$ decrease with increase in Z , V_o increases. From Table 4, Table 7, Table 10 and Table 13, it is evident that as Z increased by unity, V_o increased. This means that the interaction energy between the core of the nucleus and surface region increased. This is due to the

fact that as Z increased, repulsion interaction energy increased leading to increase in the value of V_o .

For Isotopes the value of V_o reduced as there was no increase in proton number (Z). The value of V_o in Isobars increased with a decrease in neutron number (N). For Isotones the value of V_o increased with an increase in proton number. From Table 3, Table 6 and Table 9, it is observed that as the ratio of $N:Z$ increases the value of V_o reduces meaning the interaction between the core region and surface region interaction reduces. Whereas as the ratio of $N:Z$ reduced the value of V_o increased meaning the interaction between the core region and surface region increased since here Z increased. Therefore we can conclude that the surface region neutrons (N) affect interaction between the core region and the surface region, from Table 3, Table 6 and Table 9, an increase in the neutrons number (N) in the surface region at constant proton number (Z) lowers the interaction between the core region and the surface region, meaning the system becomes unbound when more neutrons are added, This is in agreement with the neutron dripline theory (Kazuhiro *et al.*, 2010). Also from Table 4, Table 7 and Table 10 an increase in the proton number (Z) at constant neutron number (N) increased the interaction between the core region and surface region. This means as we added more protons (Z) to the nuclei, there was an increase of $N-Z$ pairs in the core region thus reducing neutron's in the surface region, this makes the system to become more bound because it approaches the stability line where $N=Z$ (Hansen, 1992).

From ${}_{92}^{234}\text{U}$ onwards as the neutron was added one by one, the value of V_o decreased, and this indicates the march towards neutron dripline as more neutrons were added keeping Z constant. This means that addition of a neutron to the nucleus keeping Z constant results

in the reduction of the value of the average potential V_o in which the neutrons in the neutron skin region moved. This trend is maintained for all the nuclei and their isotopes as we go from ${}^{234}_{92}\text{U}$ to ${}^{295}_{118}\text{Ei}$ (Table 2 and Table 12).

In the case of Isotones when the neutron number was kept constant at $N=142$, the value of Z increased from $Z=92$ to $Z=100$. Such a change led to different nuclei, namely from U to Fm (Table 13). The value of V_o increased as Z increased. In this model, the added proton would enter the core region, and to maintain the $N=Z$ condition in the core, A reduction of neutrons would be in the neutron surface or skin region because a neutron would shift from the neutron skin region to the core. Number of $N-P$ pairs in the core would increase resulting in the increase of the core size, and the number of neutrons in the neutron skin would decrease, Hence the average potential, V_o , created by the core would be increased.

In the case of Isobars, A is kept constant at $A=238$. To keep A constant, N and Z will have to be changed. When we go from U to Bk, Z increased from 92 to 97, and consequently N decreased from 146 to 141. This situation is somewhat similar to what we have in Isotones except that in Isotones N is constant and Z increased. The crucial variation is the value of Z that decided the Coulomb interaction which determined the Coulomb stability for super heavy nuclei (SHN) in the “Island of Stability” of super heavy elements (SHE). The validity of our concepts and calculations is confirmed by the fact that as Z increased both in Isotones and isobars, V_o increased.

Table 12 shows that V_o decreased linearly for all the isotopes, but its value is large for higher A and Z values.

Table 13 shows that the value of V_o increased linearly for the Isotones for which $A=238$. This means that even in super heavy nuclei, the variation in interaction between the nucleons is governed by the laws that leads to super heavy nuclei stability. The trend in variation of V_o which was evident across all the nuclides and the obtained values of V_o are within the expected theoretical values and leads to the concept of dripline (Hansen, 1992; Kazuhiro, *et. al*, 2010), The variation of V_o is in agreement with the Woods and Saxon, (1954) potential .

5.2 Recomendations

In the calculations of the binding energy of light, medium, heavy and super heavy nuclei, the value of V_o changed as A , Z and N changed for different types of nuclei. Since V_o is the average potential created by the core of the nuclei, and the surface neutrons move in this potential (like Hartree-Fock potential), it would be interesting to develop a theory to calculate the value of V_o using the fundamentals of quantum mechanics or the many-body theory based on the principles of second quantization. This will be a complex theory to develop, but this will be attempted in future. For ${}^{138}_{56}\text{Ba}$, $V_o < B$, and this could be the reason that ${}^{235}_{92}\text{U}$ undergoes fission to ${}_{56}\text{Ba}$ and ${}_{36}\text{Kr}$. One can try to look for nuclei for which $V_o < B$, and try for a fission reaction in which such nuclei appear as products of fission reaction. One can look for a new nuclei that can be used as a fuel in the nuclear reactor.

With the recent discovery of muon at the large hadron collider (LHC) as an integral part of the nuclear core, and the nuclei have finite neutron skin, a brand new many body theory of the finite nuclei will have to be developed. The core of the nuclei will have one

type of magnetic moment, and the neutron skin will have a different magnetic moment, such an amazing and brand new nuclear theory will be attempted in future.

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APPENDICES

APPENDIX I

CALCULATED VALUES OF V_0 FOR LIGHT, MEDIUM AND HEAVY NUCLEI
USING EQ (8).

Nucleus	Symbol	A	Z	B (Exp)	$V_0(\text{MeV})$	N-Z
Hydrogen	H	1	1	0	29.02853562	-1
Hydrogen	H	2	1	2.225	0	0
Hydrogen	H	3	1	8.482	-20.54653562	1
Helium	He	3	2	7.718	41.66771369	-1
Helium	He	4	2	28.296	0	0
Lithium	Li	6	3	31.994	0	0
Lithium	Li	7	3	39.244	-21.98991857	1
Beryllium	Be	9	4	58.165	-6.475	1
Boron	B	10	5	64.751	0	0
Boron	B	11	5	76.205	16.5598599	1
Carbon	C	12	6	92.162	0	0
Carbon	C	13	6	97.108	50.82960282	1

Nitrogen	N	14	7	104.659	0	0
Nitrogen	N	15	7	115.492	90.93014005	1
Oxygen	O	16	8	127.619	0	0
Oxygen	O	17	8	131.763	137.2498601	1
Oxygen	O	18	8	139.807	72.64693006	2
Fluorine	F	19	9	147.801	191.6542449	1
Neon	Ne	20	10	160.645	0	0
Neon	Ne	21	10	167.406	257.9310268	1
Neon	Ne	22	10	177.77	134.1475134	2
Sodium	Na	23	11	186.564	332.0556407	1
Magnesium	Mg	24	12	198.257	0	0
Magnesium	Mg	25	12	205.588	414.3318485	1
Magnesium	Mg	26	12	216.681	212.7124242	2
Aluminium	Al	27	13	224.952	505.2254722	1
Silicon	Si	28	14	236.537	0	0
Silicon	Si	29	14	245.011	605.0841978	1

Silicon	Si	30	14	255.62	307.8465989	2
Phosphorous	P	31	15	262.917	711.0534268	1
Sulphur	S	32	16	271.781	0	0
Sulphur	S	33	16	280.422	824.879162	1
Sulphur	S	34	16	291.839	418.148081	2
Sulphur	S	36	16	308.714	213.2927905	4
Chlorine	Cl	35	17	298.21	947.2399179	1
Chlorine	Cl	37	17	317.101	322.0436393	3
Argon	Ar	36	18	306.717	0	0
Argon	Ar	38	18	327.343	544.5963243	2
Argon	Ar	40	18	343.811	276.4151622	4
Potassium	K	39	19	333.724	1216.63569	1
Potassium	K	40	19	341.524	612.2178449	2
Potassium	K	41	19	351.619	411.51023	3
Calcium	Ca	40	20	342.052	0	0
Calcium	Ca	42	20	361.896	687.0538554	2

Calcium	Ca	43	20	369.829	460.680237	3
Calcium	Ca	44	20	380.96	348.2929277	4
Calcium	Ca	46	20	398.769	235.1634518	6
Scandium	Sc	45	21	387.848	512.531225	3
Titanium	Ti	46	22	398.193	846.8514031	2
Titanium	Ti	47	22	407.073	567.5276021	3
Titanium	Ti	48	22	418.7	428.5524516	4
Titanium	Ti	49	22	426.842	344.4703612	5
Titanium	Ti	50	22	437.781	288.881801	6
Vanadium	V	50	23	434.794	471.0736403	4
Vanadium	V	51	23	445.845	379.0691123	5
Chromium	Cr	50	24	435.049	1023.381803	2
Chromium	Cr	52	24	456.349	517.0159014	4
Chromium	Cr	53	24	464.289	415.2007211	5
Chromium	Cr	54	24	474.008	347.6204343	6
Manganese	Mn	55	25	482.075	452.8447585	5

Iron	Fe	54	26	471.763	1216.281573	2
Iron	Fe	56	26	492.258	613.2645365	4
Iron	Fe	57	26	499.905	492.1410292	5
Iron	Fe	58	26	509.949	411.7915243	6
Cobalt	Co	59	27	517.313	532.9957688	5
Nickel	Ni	58	28	506.459	1424.601103	2
Nickel	Ni	60	28	526.846	717.3973016	4
Nickel	Ni	61	28	534.666	575.4818412	5
Nickel	Ni	62	28	545.262	481.334201	6
Nickel	Ni	64	28	561.758	363.0626508	8
Copper	Cu	63	29	551.385	619.4829363	5
Copper	Cu	65	29	569.212	445.0345259	7
Zinc	Zn	64	30	559.098	829.1552093	4
Zinc	Zn	66	30	578.136	555.9431396	6
Zinc	Zn	67	30	585.189	477.5302625	7
Zinc	Zn	68	30	595.387	419.1137297	8

Zinc	Zn	70	30	611.087	336.8609837	10
Gallium	Ga	69	31	601.996	511.3166216	7
Gallium	Ga	71	31	618.951	399.5745945	9
Germanium	Ge	70	32	610.521	635.9401667	6
Germanium	Ge	72	32	628.686	479.22575	8
Germanium	Ge	73	32	635.469	426.7321111	9
Germanium	Ge	74	32	645.665	385.0785	10
Germanium	Ge	76	32	661.598	322.2265	12
Arsenic	As	75	33	652.564	454.8648507	9
Selenium	Se	74	34	642.891	721.4013797	6
Selenium	Se	76	34	662.073	543.4487848	8
Selenium	Se	77	34	669.492	483.8899198	9
Selenium	Se	78	34	679.99	436.5507278	10
Selenium	Se	80	34	696.866	365.1986065	12
Selenium	Se	82	34	712.843	314.1685913	14
Bromine	Br	79	35	686.321	513.8146562	9

Bromine	Br	81	35	704.37	422.0346278	11
Krypton	Kr	78	36	675.578	812.3794422	6
Krypton	Kr	80	36	695.434	611.7665816	8
Krypton	Kr	82	36	714.274	491.2972653	10
Krypton	Kr	83	36	721.737	447.3123321	11
Krypton	Kr	84	36	732.258	410.9130544	12
Krypton	Kr	86	36	749.235	353.4238324	14
Rubidium	Rb	85	37	739.283	473.3510642	11
Rubidium	Rb	87	37	757.856	401.9565158	13
Strontium	Sr	84	38	728.906	684.1944145	8
Strontium	Sr	86	38	748.928	549.3577316	10
Strontium	Sr	87	38	757.356	500.1823014	11
Strontium	Sr	88	38	768.469	459.4265263	12
Yttrium	Y	89	39	775.538	527.7678905	11
Zirconium	Zr	90	40	783.893	610.7248517	10
Zirconium	Zr	91	40	791.087	555.8584106	11

Zirconium	Zr	92	40	799.722	510.2564597	12
Zirconium	Zr	94	40	814.677	438.4308941	14
Zirconium	Zr	96	40	828.996	384.5219698	16
Niobium	Nb	93	41	805.765	584.6139006	11
Molybdenum	Mo	92	42	796.508	841.4140551	8
Molybdenum	Mo	94	42	814.256	674.9060441	10
Molybdenum	Mo	95	42	821.625	614.2208583	11
Molybdenum	Mo	96	42	830.779	563.7986201	12
Molybdenum	Mo	97	42	837.6	520.9541878	13
Molybdenum	Mo	98	42	846.243	484.3605315	14
Molybdenum	Mo	100	42	860.458	424.7039026	16
Ruthenium	Ru	96	44	826.496	925.6850126	8
Ruthenium	Ru	98	44	844.79	742.37741	10
Ruthenium	Ru	99	44	852.255	675.5671909	11
Ruthenium	Ru	100	44	861.928	620.0760084	12
Ruthenium	Ru	101	44	868.73	572.9010846	13

Ruthenium	Ru	102	44	874.844	532.4162929	14
Ruthenium	Ru	104	44	893.083	467.0041938	16
Rhodium	Rh	103	45	884.163	599.8082626	13
Palladium	Pd	102	46	875.212	813.1095246	10
Palladium	Pd	104	46	892.82	679.0586038	12
Palladium	Pd	105	46	899.914	627.3690189	13
Palladium	Pd	106	46	909.474	583.2398033	14
Palladium	Pd	108	46	925.239	511.3201404	16
Palladium	Pd	110	46	940.207	455.338347	18
Silver	Ag	107	47	915.263	655.5278687	13
Silver	Ag	109	47	931.727	569.2217529	15
Cadmium	Cd	106	48	905.14	887.063129	10
Cadmium	Cd	108	48	923.402	740.7411075	12
Cadmium	Cd	110	48	940.646	636.1526636	14
Cadmium	Cd	111	48	947.622	594.2075527	15
Cadmium	Cd	112	48	957.016	557.6567056	16

Cadmium	Cd	113	48	963.556	525.2380759	17
Cadmium	Cd	114	48	972.599	496.5605717	18
Cadmium	Cd	116	48	987.44	447.6465645	20
Indium	In	113	49	963.094	619.7101337	15
Indium	In	115	49	979.404	547.762471	17
Tin	Sn	112	50	953.532	805.1108537	12
Tin	Sn	114	50	971.574	691.3837317	14
Tin	Sn	115	50	979.121	645.7946163	15
Tin	Sn	116	50	988.684	606.0301403	16
Tin	Sn	117	50	995.627	570.7897202	17
Tin	Sn	118	50	1004.955	539.5974025	18
Tin	Sn	119	50	1011.438	511.5387497	19
Tin	Sn	120	50	1020.546	486.4172122	20
Tin	Sn	122	50	1035.53	442.8785566	22
Tin	Sn	124	50	1049.963	406.5733852	24
Antimony	Sb	121	51	1026.325	532.5018863	19

Antimony	Sb	123	51	1042.097	482.5384685	21
Tellurium	Te	120	52	1017.282	656.2547911	16
Tellurium	Te	122	52	1034.333	584.2848699	18
Tellurium	Te	123	52	1041.263	553.8977715	19
Tellurium	Te	124	52	1050.686	526.6740329	20
Tellurium	Te	125	52	1057.256	501.9071742	21
Tellurium	Te	126	52	1066.369	479.507439	22
Tellurium	Te	128	52	1081.439	440.1764024	24
Tellurium	Te	130	52	1095.941	406.8744484	26
Iodine	I	127	53	1072.577	521.6726474	21
Xenon	Xe	124	54	1046.257	708.5455359	16
Xenon	Xe	126	54	1063.909	630.7989208	18
Xenon	Xe	128	54	1080.743	568.5607287	20
Xenon	Xe	129	54	1087.651	541.8153607	21
Xenon	Xe	130	54	1096.907	517.608117	22
Xenon	Xe	131	54	1103.512	495.3905902	23

Xenon	Xe	132	54	1112.448	475.1216489	24
Xenon	Xe	134	54	1127.435	439.1502528	26
Xenon	Xe	136	54	1141.878	408.2981991	28
Cesium	Cs	133	55	1118.528	514.1343715	23
Barium	Ba	130	56	1092.722	679.0856934	18
Barium	Ba	132	56	1110.038	612.0429241	20
Barium	Ba	134	56	1126.696	557.1598401	22
Barium	Ba	135	56	1133.668	533.2386296	23
Barium	Ba	136	56	1142.775	511.3998117	24
Barium	Ba	137	56	1149.681	491.2200593	25
Barium	Ba	138	56	1158.293	472.6582108	26
Lanthanum	La	138	57	1155.774	529.9590994	24
Lanthanum	La	139	57	1164.551	509.1118155	25
Cerium	Ce	136	58	1138.792	657.1312641	20
Cerium	Ce	138	58	1156.035	598.175831	22
Cerium	Ce	140	58	1172.692	549.0218867	24

Cerium	Ce	142	58	1185.29	507.2739724	26
Praseodymium	Pr	141	59	1177.919	592.6306615	23
Neodymium	Nd	142	60	1185.142	640.6656478	22
Neodymium	Nd	143	60	1191.266	613.0768805	23
Neodymium	Nd	144	60	1199.083	587.8577189	24
Neodymium	Nd	145	60	1204.838	564.5736101	25
Neodymium	Nd	146	60	1212.403	543.150202	26
Neodymium	Nd	148	60	1225.028	504.8046519	28
Neodymium	Nd	150	60	1237.448	471.5650084	30
Samarium	Sm	144	62	1195.737	752.1778509	20
Samarium	Sm	147	62	1217.251	655.0030877	23
Samarium	Sm	148	62	1225.392	628.0505007	24
Samarium	Sm	149	62	1231.263	603.1633207	25
Samarium	Sm	150	62	1239.25	580.2719238	26
Samarium	Sm	152	62	1253.104	539.3187149	28
Samarium	Sm	154	62	1266.94	503.8253339	30

Europium	Eu	151	63	1244.141	622.9344393	25
Europium	Eu	153	63	1258.998	577.3414068	27
Gadolinium	Gd	152	64	1251.485	669.5943552	24
Gadolinium	Gd	154	64	1266.627	618.6694817	26
Gadolinium	Gd	155	64	1273.062	595.9941305	27
Gadolinium	Gd	156	64	1281.598	575.013483	28
Gadolinium	Gd	157	64	1287.958	555.4047422	29
Gadolinium	Gd	158	64	1295.896	537.1558508	30
Gadolinium	Gd	160	64	1309.29	504.0021726	32
Terbium	Tb	159	65	1302.027	573.0525795	29
Dysprosium	Dy	156	66	1278.021	712.5164589	24
Dysprosium	Dy	158	66	1294.046	658.3238467	26
Dysprosium	Dy	160	66	1309.455	611.8510362	28
Dysprosium	Dy	161	66	1315.909	590.9752763	29
Dysprosium	Dy	162	66	1324.106	571.5493338	30
Dysprosium	Dy	163	66	1330.4	553.3145488	31

Dysprosium	Dy	164	66	1338	536.2627817	32
Holmium	Ho	165	67	1344.3	570.3439787	31
Erbium	Er	162	68	1320.7	699.2037309	26
Erbium	Er	164	68	1336.4	649.8230002	28
Erbium	Er	166	68	1351.6	607.0056335	30
Erbium	Er	167	68	1358	587.6324195	31
Erbium	Er	168	68	1365.8	569.5117501	32
Erbium	Er	170	68	1379	536.4010884	34
Thulium	Tm	169	69	1371.4	605.1707817	31
Ytterbium	Yb	168	70	1362.8	688.9369741	28
Ytterbium	Yb	170	70	1378.1	643.5190758	30
Ytterbium	Yb	171	70	1384.7	622.9737508	31
Ytterbium	Yb	172	70	1392.8	603.7564461	32
Ytterbium	Yb	173	70	1399.1	585.6537356	33
Ytterbium	Yb	174	70	1406.6	568.6481551	34
Ytterbium	Yb	176	70	1419.3	537.4090354	36

Lutetium	Lu	175	71	1412.1	602.6121934	33
Lutetium	Lu	176	71	1418.4	585.0732466	34
Hafnium	Hf	174	72	1403.9	681.0943945	30
Hafnium	Hf	176	72	1418.8	638.9907761	32
Hafnium	Hf	177	72	1425.2	619.8208738	33
Hafnium	Hf	178	72	1432.8	601.8151422	34
Hafnium	Hf	179	72	1438.9	584.794681	35
Hafnium	Hf	180	72	1446.3	568.7555788	36
Tantalum	Ta	180	73	1444.7	618.7213799	34
Tantalum	Ta	181	73	1452.2	601.2601119	35
Tungsten	W	180	74	1444.6	675.2363728	32
Tungsten	W	182	74	1459.3	635.9503215	34
Tungsten	W	183	74	1465.5	617.9571694	35
Tungsten	W	184	74	1472.9	600.9975814	36
Tungsten	W	186	74	1485.9	569.7067876	38
Rhenium	Re	185	75	1478.3	634.8737195	35

Rhenium	Re	187	75	1491.9	600.922059	37
Osmium	Os	184	76	1469.9	712.4866876	32
Osmium	Os	186	76	1484.8	671.0135295	34
Osmium	Os	187	76	1491.1	652.0214287	35
Osmium	Os	188	76	1499.1	634.1316668	36
Osmium	Os	189	76	1505	617.1529731	37
Osmium	Os	190	76	1512.8	601.117158	38
Osmium	Os	192	76	1526.1	571.3942251	40
Iridium	Ir	191	77	1518.1	633.6028305	37
Iridium	Ir	193	77	1532.1	601.4685828	39
Platinum	Pt	190	78	1509.9	707.0229619	34
Platinum	Pt	192	78	1524.96	668.163714	36
Platinum	Pt	194	78	1539.58	633.3817553	38
Platinum	Pt	195	78	1545.68	617.297736	39
Platinum	Pt	196	78	1553.6	602.0633426	40
Platinum	Pt	198	78	1567.01	573.7127786	42

Gold	Au	197	79	1559.39	633.3378792	39
Mercury	Hg	196	80	1551.22	703.1116349	36
Mercury	Hg	198	80	1566.49	666.5076278	38
Mercury	Hg	199	80	1573.15	649.5885604	39
Mercury	Hg	200	80	1581.18	633.5495464	40
Mercury	Hg	201	80	1587.41	618.2490697	41
Mercury	Hg	202	80	1595.17	603.7134728	42
Mercury	Hg	204	80	1608.65	576.578474	44
Thallium	Tl	203	81	1600.87	633.8982387	41
Thallium	Tl	205	81	1615.07	604.7448788	43
Lead	Pb	204	82	1607.51	665.8192118	40

APPENDIX II

CALCULATED VALUES OF V_0 FOR SUPER HEAVY NUCLEI USING EQ (12)

SYMBOL	B(Exp)MeV	A	Z	N	A-2Z	V_0 (MeV)
U	1778.565438	234	92	142	50	671.688398
U	1783.86291	235	92	143	51	658.621909
U	1790.408336	236	92	144	52	646.081977
U	1801.68856	238	92	146	54	622.361908
U	1795.534278	237	92	145	53	633.988467
U	1806.494884	239	92	147	55	611.133624
U	1812.42384	240	92	148	56	600.326398
U	1816.899	241	92	149	57	589.872868
U	1822.744	242	92	150	58	579.803422
U	1826.874	243	92	151	59	570.046245
Np	1741.7969	229	93	136	43	797.048943
Np	1756.08972	231	93	138	45	761.942163
Np	1769.90761	233	93	140	47	729.81309

Np	1775.97342	234	93	141	48	714.735022
Np	1783.14945	235	93	142	49	700.295042
Np	1788.69356	236	93	143	50	686.400024
Np	1795.270497	237	93	144	51	673.070159
Np	1800.758694	238	93	145	52	660.232045
Np	1806.974079	239	93	146	53	647.892108
Np	1812.42384	240	93	147	54	635.995027
Pu	1762.504	232	94	138	44	796.077336
Pu	1767.02307	233	94	139	45	778.487152
Pu	1774.797804	234	94	140	46	761.732534
Pu	1781.03445	235	94	141	47	745.658154
Pu	1788.387468	236	94	142	48	730.276797
Pu	1794.268224	237	94	143	49	715.493204
Pu	1801.268014	238	94	144	50	701.323336
Pu	1806.91409	239	94	145	51	687.682605
Pu	1813.4484	240	94	146	52	674.5836

Pu	1818.689871	241	94	147	53	661.954503
Pu	1824.999682	242	94	148	54	649.812934
Pu	1836.053884	244	94	150	56	626.802726
Am	1791.957	237	95	142	47	761.672008
Am	1798.22804	238	95	143	48	745.934488
Am	1805.329759	239	95	144	49	730.856268
Am	1811.2824	240	95	145	50	716.358196
Am	1817.928311	241	95	146	51	702.442269
Am	1823.465886	242	95	147	52	689.040255
Am	1829.830824	243	95	148	53	676.159589
Cm	1796.424	238	96	142	46	794.62863
Cm	1802.79134	239	96	143	47	777.857114
Cm	1810.2852	240	96	144	48	761.807879
Cm	1816.378681	241	96	145	49	746.385136
Cm	1823.348032	242	96	146	50	731.596821
Cm	1829.041074	243	96	147	51	717.363413

Cm	1835.842336	244	96	148	52	703.698756
Cm	1841.362425	245	96	149	53	690.525574
Cm	1847.820144	246	96	150	54	677.857651
Cm	1852.975722	247	96	151	55	665.626704
Cm	1859.1878	248	96	152	56	653.851443
Cm	1869.7325	250	96	154	58	631.486647
Bk	1790.95	238	97	141	44	847.851025
Bk	1798.953	239	97	142	45	829.187736
Bk	1839.769435	245	97	148	51	732.43656
Bk	1845.6888	246	97	149	52	718.465075
Bk	1850.84263	247	97	150	53	705.006372
Bk	1857.768	248	97	151	54	692.078946
Bk	1864.019727	249	97	152	55	679.609361
Bk	1868.99025	250	97	147	56	667.562239
Cf	1802.4552	240	98	142	44	865.524737
Cf	1809.187	241	98	143	45	846.44045

Cf	1817.2022	242	98	144	46	828.213814
Cf	1823.715	243	98	145	47	810.730814
Cf	1831.251964	244	98	146	48	793.997609
Cf	1837.41572	245	98	147	49	777.919367
Cf	1844.782044	246	98	148	50	762.508306
Cf	1857.77668	248	98	150	52	733.43096
Cf	1863.362118	249	98	151	53	719.698026
Cf	1869.98725	250	98	152	54	706.492972
Cf	1875.094245	251	98	153	55	693.7405
Cf	1881.266688	252	98	154	56	681.462499
Cf	1886.070978	253	98	155	57	669.591302
Cf	1892.10188	254	98	156	58	658.150606
Es	1795.44	240	99	141	42	925.008261
Es	1803.885	241	99	142	43	903.692836
Es	1810.886	242	99	143	44	883.313476
Es	1819.098	243	99	144	45	863.866777

Es	1833.58	245	99	146	47	827.414616
Es	1840.08	246	99	147	48	810.312228
Es	1847.5847	247	99	148	49	793.928401
Es	1854.048	248	99	149	50	778.179099
Es	1861.026	249	99	150	51	763.057509
Es	1873.934625	251	99	152	53	734.506445
Es	1879.22448	252	99	153	54	721.002434
Es	1885.576363	253	99	154	55	708.008787
Es	1890.670082	254	99	155	56	695.456733
Es	1896.6441	255	99	156	57	683.360545
Fm	1797.86	241	100	141	41	966.711604
Fm	1806.53	242	100	142	42	943.901089
Fm	1813.752	243	100	143	43	922.117855
Fm	1822.192	244	100	144	44	901.352449
Fm	1829.17	245	100	145	45	881.477461
Fm	1837.12062	246	100	146	46	862.487747

Fm	1843.608	247	100	147	47	844.274973
Fm	1876.35312	248	100	148	48	827.368101
Fm	1858.002891	249	100	149	49	810.108544
Fm	1865.5225	250	100	150	50	794.056765
Fm	1871.71202	251	100	151	51	778.608388
Fm	1878.919812	252	100	152	52	763.773761
Fm	1884.459621	253	100	153	53	749.46746
Fm	1890.975644	254	100	154	54	735.7091
Fm	1896.150165	255	100	155	55	722.426653
Fm	1902.535168	256	100	156	56	709.640195
Fm	1907.503087	257	100	157	57	697.277523
Fm	1913.844	258	100	158	58	685.364823
Fm	1918.413	259	100	159	59	673.825911
Fm	1924.52	260	100	160	60	662.697263
Md	1845.616	248	101	147	46	879.858157
Md	1874.376	252	101	151	50	810.044704

Md	1906.314462	257	101	156	55	736.984976
Md	1911.694344	258	101	157	56	723.920599
Md	1922.96	260	101	159	58	699.152055
No	1856.5	250	102	148	46	897.457323
No	1863.173	251	102	149	47	878.504465
No	1871.3016	252	102	150	48	860.371634
No	1877.883898	253	102	151	49	842.947362
No	1885.58932	254	102	152	50	826.242524
No	1891.5798	255	102	153	51	810.15915
No	1898.63424	256	102	154	52	794.714829
No	1904.278765	257	102	155	53	779.82671
No	1911.006	258	102	156	54	765.510053
No	1916.6	259	102	157	55	751.693397
No	1923.22	260	102	158	56	738.388515
No	1934.87	262	102	160	58	713.127704
Lr	1864.548	252	103	149	46	915.171658

Lr	1872.959	253	103	150	47	895.878878
Lr	1879.6	254	103	151	48	877.353089
Lr	1887.6579	255	103	152	49	859.612371
Lr	1890.67008	256	103	153	50	842.480367
Lr	1901.029	257	103	154	51	826.16426
Lr	1907.136	258	103	155	52	810.393928
Lr	1914.01	259	103	156	53	795.233175
Lr	1919.58	260	103	157	54	780.609783
Lr	1926.441	261	103	158	55	766.541623
Lr	1931.988	262	103	159	56	752.952433
Rf	1867.14	253	104	149	45	953.676289
Rf	1875.536	254	104	150	46	933.126717
Rf	1882.41	255	104	151	47	913.419213
Rf	1890.67008	256	104	152	48	894.561731
Rf	1897.0969	257	104	153	49	876.436528
Rf	1904.69274	258	104	154	50	859.059715

Rf	1910.643	259	104	155	51	842.332078
Rf	1918.02	260	104	156	52	826.27525
Rf	1923.92757	261	104	157	53	810.796614
Rf	1930.94	262	104	158	54	795.911722
Rf	1936.732	263	104	159	55	781.545909
Rf	1960.314	267	104	163	59	728.959441
Db	1883.648	256	105	151	46	951.196057
Db	1892.034	257	105	152	47	931.136268
Db	1898.364	258	105	153	48	911.869471
Db	1906.33324	259	105	154	49	893.422528
Db	1912.82	260	105	155	50	875.683812
Db	1920.177	261	105	156	51	858.657796
Db	1926.224	262	105	157	52	842.261435
Db	1933.576	263	105	158	53	826.508427
Db	1952.174	266	105	161	56	782.563297
Db	1958.712	267	105	162	57	768.948818

Db	1963.904	268	105	163	58	755.780597
Db	1969.887	269	105	164	59	743.072163
Db	1974.78	270	105	165	60	730.769177
Sg	1894.236	258	106	152	46	969.496015
Sg	1901.06	259	106	153	47	949.013632
Sg	1909.0656	260	106	154	48	929.409298
Sg	1915.67997	261	106	155	49	910.576748
Sg	1923.38916	262	106	156	50	892.519397
Sg	1929.631	263	106	157	51	875.141406
Sg	1937.232	264	106	158	52	858.457936
Sg	1943.245	265	106	159	53	842.37407
Sg	1950.312	266	106	160	54	826.90542
Sg	1956.309	267	106	161	55	811.979813
Sg	1979.655	271	106	165	59	757.326029
Sg	1985.872	272	106	166	60	744.807545
Sg	1990.443	273	106	167	61	732.67252

Bh	1901.38	260	107	153	46	987.897887
Bh	1909.737	261	107	154	47	967.056592
Bh	1916.53	262	107	155	48	947.0511
Bh	1924.634	263	107	156	49	927.888915
Bh	1931.16	264	107	157	50	909.461656
Bh	1938.74	265	107	158	51	891.777702
Bh	1945.258	266	107	159	52	874.753477
Bh	1952.571	267	107	160	53	858.386676
Bh	1971.27	270	107	163	56	812.735586
Bh	1982.88	272	107	165	58	784.910393
Bh	1989.078	273	107	166	59	771.711878
Bh	1994.172	274	107	167	60	758.934914
Bh	2000.075	275	107	168	61	746.590128
Hs	1918.585	263	108	155	47	985.276204
Hs	1926.76968	264	108	156	48	964.92013
Hs	1933.5036	265	108	157	49	945.36531

Hs	1941.33982	266	108	158	50	926.614728
Hs	1947.765	267	108	159	51	908.571796
Hs	1962.086	269	108	161	53	874.556275
Hs	1969.65	270	108	162	54	858.500862
Hs	1975.048	271	108	163	55	842.989901
Hs	1998.425	275	108	167	59	786.234264
Hs	2009.635	277	108	169	61	760.639862
Mt	1933.82	266	109	157	48	982.892115
Mt	1948.628	268	109	159	50	943.872591
Mt	1963.17	270	109	161	52	907.849453
Mt	1989.24	274	109	165	56	843.468599
Mt	1995.675	275	109	166	57	828.783799
Mt	2001.276	276	109	167	58	814.590992
Mt	2007.696	277	109	168	59	800.893179
Mt	2012.998	278	109	169	60	787.633326
Ds	1934.949	267	110	157	47	1022.20609

Ds	1950.25	269	110	159	49	980.79566
Ds	1958.5179	270	110	160	50	961.345105
Ds	1965.292	271	110	161	51	942.628026
Ds	1973.36	272	110	162	52	924.655718
Ds	1978.977	273	110	163	53	907.315365
Ds	1986.226	274	110	164	54	890.647469
Ds	2016.612	279	110	169	59	815.683887
Ds	2028.82	281	110	171	61	789.140284
Rg	1965.744	272	111	161	50	978.926951
Rg	1980.198	274	111	163	52	941.553875
Rg	2006.604	278	111	167	56	874.771563
Rg	2013.543	279	111	168	57	859.54643
Rg	2019.36	280	111	169	58	844.826957
Rg	2026.291	281	111	170	59	830.625331
Rg	2031.81	282	111	171	60	816.873559
Cn	1995.785	277	112	165	53	940.686505

Cn	2016.56	280	112	168	56	890.663567
Cn	2022.357	281	112	169	57	875.139592
Cn	2029.554	282	112	170	58	860.175065
Cn	2035.053	283	112	171	59	845.68903
Cn	2042.528	284	112	172	60	831.718796
Cn	2047.725	285	112	173	61	818.169258
Ed	1996.596	278	113	165	52	975.871569
Ed	2024.196	282	113	169	56	906.659314
Ed	2031.374	283	113	170	57	890.87894
Ed	2037.416	284	113	171	58	875.623131
Ed	2044.59	285	113	172	59	860.903654
Ed	2050.334	286	113	173	60	846.650993
Ed	2057.216	287	113	174	61	832.884288
Fl	2047.474	286	114	172	58	891.249219
Fl	2053.198	287	114	173	59	876.240317
Fl	2060.64	288	114	174	60	861.760345

Fl	2066.061	289	114	175	61	847.721995
Ef	2048.893	287	115	172	57	922.77645
Ef	2055.168	288	115	173	58	906.974701
Ef	2062.304	289	115	174	59	891.723197
Ef	2068.28	290	115	175	60	876.960744
Ef	2075.121	291	115	176	61	862.696486
Lv	2057.391	289	116	173	57	938.934527
Lv	2064.8	290	116	174	58	922.873743
Lv	2070.756	291	116	175	59	907.332764
Lv	2078.164	292	116	176	60	892.334018
Lv	2083.523	293	116	177	61	877.793444
Eh	2072.032	292	117	175	58	938.871545
Eh	2079.421	293	117	176	59	923.083705
Eh	2085.342	294	117	177	60	907.79766
Ei	2073.561	293	118	175	57	971.663981
Ei	2081.52	294	118	176	58	955.048378
Ei	2087.42	295	118	177	59	938.961117

APPENDIX III

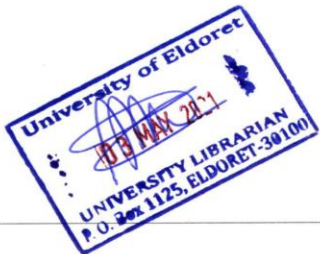
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