MATHEMATICAL MODELING OF FLOW OF WATER IN AN OPEN CHANNEL OF PARABOLIC CROSS-SECTION

BY

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DECLARATION

Declaration by the student

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DEDICATION

To my loving parents, Mr. Paul Kiplagat and Mrs. Esther Kiprono, and my dear siblings Lillian, Kenneth, and Jane for their tireless support throughout my study. Further, I dedicate this thesis to my love Joyline and daughter Felicia for their support and care throughout the study period.

ABSTRACT

This study investigates open channel flows with a parabolic cross-section. The objectives of this study were to examine the effects of top width, channel slope, and energy coefficient on flow velocity. The methodology used to solve the objectives of the study was continuity and momentum equations. Because of its stability, convergence, and precision, the governing equations are solved by the finite-difference approximation approach. Using MATLAB software, the result is presented graphically. The findings are that; an increase in the channel slope and energy coefficient has been shown to lead to increased velocity of flow. While a decrease in top width leads to an increase in velocity. The findings of this study are useful in the flood control, construction of channels such as canals, and crop irrigation.

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ABBREVIATIONS, ACRONYMS, AND SYMBOLS

Symbol	**meaning**
Re	Reynolds number (dimensionless)
g	acceleration due to gravity (ms^{-2})
q	specific discharge (m^2s^{-1})
t	time (s)
y	flow Depth (m)
z	co-ordinate direction, bed level relative to adatum
A	area of cross-section of flow (m^2)
C	coefficient of the flow (Chézy coefficient)
D	Hydraulic depth (m)
Fr	Froude number (dimensionless)
L	length of the channel (m)
Q	Discharge (m^3s^{-1})
s	slope of the channel bottom
s_f	friction slope
x	distance along the main flow direction (m)
T	Top width of the free surface (m)

v Mean flow velocity (m/s)

m dimension of length

n The Manning's coefficient of roughness ($sm^{-1/3}$)

r radius of the conduit

α Energy coefficient

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CHAPTER ONE

INTRODUCTION

1.1 Background Information

Floods in Kenya have been experienced in most parts of the country for several years. Areas most affected by the floods include; Baringo, Budalangi, Tana River, West Pokot, Makueni, and Machakos. Flooding leads to the destruction of houses, bridges, and other land structures. It also leads to the death of people and animals. The soil, crops, and plantations are destroyed in times of flooding. In addition, roads made impassable and transport paralyzed. Moreover, flooding water is a breeding ground for mosquitoes that causes malaria.

Handling of such unexpected amounts of water is a challenge for open channel designers in Kenya and around the world. The solution to such disasters is the design of channels of different cross-sections such as circular, triangular, rectangular, trapezoidal, elliptical, and parabolic. Currently, floods are still a challenge in Kenya and there is a need to design an efficient channel that would transport the maximum amount of water in flood areas to the areas needed for crop irrigation and hydroelectric power generation. The current Government of Kenya focuses on four main agendas: Food security and nutrition, universal health coverage, and affordable housing, and manufacturing.

This study cuts across the four agendas. Firstly, food security, irrigation of crops from floodwaters. Secondly, the Universal Health Survey floods lead to health hazards such as cholera. Thirdly, manufacturing, flooding of water to the hydroelectric power plant for the generation of electricity for the manufacturing industry. Finally, affordable housing, flood control, housing, and roads will not be destroyed. The parabolic open channel cross-section has an advantage over other channels in that it can maintain a greater velocity at a low volume of fluid which reduces the tendency to deposit sediments on the channel bed. Furthermore, discharge with lower velocity can carry floating debris easily than a flat-bottomed channel.

The goal of this study is to investigate parabolic open channels as an efficient channel that would allow excess water out of the flooded areas and direct the excess water to crop irrigation and hydroelectric power generation for industrial and domestic use.

1.2 Basic Concepts: Definition of Terms

1.2.1 Fluids

Liquid and gases are two types of fluids. The molecules in a liquid, such as water or oil are spaced further apart, the intermolecular forces are weaker and the molecules have more freedom of movement than in solid. Fluid is a material that continually changes shape when shearing stress of any size acts on it. The study of motion of fluids is called hydrodynamics

1.2.2 Open channel flow

In research, engineering, and daily life, flows in conduits or channels are of interest. Closed conduits or channels, such as pipes or air ducts, are surrounded by rigid boundaries. Open-channel flows, on the other hand, have a boundary that is not completely made of a solid and rigid material; the other half of the boundary can be made of another fluid or nothing at all. Rivers, tidal waves, drainage canals, and sheets of water flowing over the ground surface after rain are examples of important open-channel flows.

Rivers, lakes, artificial channels, irrigation ditches, and partially filled pipes are examples of open channel flow. A flow with a free surface that is exposed to atmospheric pressure is known as an open channel flow. The pressure gradient at the atmospheric interface is zero, and the open channel flows are solely dominated by gravity. Flow in structures with open tops, such as rivers, streams, sewers, and drainage channels, is referred to as open channel flow.

An open-channel flow is when a liquid flows into a channel or conduit that is not completely filled up. Between the moving fluid (usually water) and the fluid above it, there is a free surface (usually the atmosphere). Gravity pushes the fluid to flow downhill, which is the main driving force for such flows. Majority of open-channel flow findings are focused on model and full-scale experiment correlations. Various analytical and computational efforts will provide additional knowledge. Open-channel flow is exemplified by the natural drainage of water into various streams and river systems. The flow of rainwater in our gutters, the flow in canals, irrigation ditches, sewers, and gutters along roads, the flow of small rivulets and sheets of water through fields, and the flow in chutes of water rides in the amusement park are all examples of open channel flows.

1.2.3 Types of open channels

Open channel flows are branded by occurrence of a liquid-gas interface called the free surface. Open channel is classified as either natural or artificial. Natural channels are irregular in shape and made of diverse materials. Natural open channels include streams and rivers. Artificial channels or man-made channels are regular in shape and made of concrete, steel, and earth. Man-made open channels include; culverts sewers, irrigation canals, spillways, and drainage ditches.

The prismatic channels are open channels in which the shape, size of the cross-section, and slope of the bed remain constant. Non-prismatic channels are open channels in which the shape, cross-section size, and slope of the bed vary. Non-prismatic channels are examples of natural channels, whereas prismatic channels are examples of man-made open channels.

1.2.4 Flow classification

Open channel flow is classified following to change of flow depth to space and time.

i. Steady uniform and unsteady flow

For a steady uniform flow, the depth and velocity of flow are constant along with the flow and over the cross-section while the unsteady flow is a flow in which depth and velocity of flow vary along the flow channel and over the cross-section.

The terms steady-state and unsteady-state have the same meanings in open channel flow as they do in a variety of other flowing fluid applications. At a given channel cross-section, there are no shifts in velocity patterns or magnitude with time for steady-state flow. Unsteady state flow, on the other hand, has a velocity that changes over time at a given cross-section. When there is a shifting flow rate, such as in a river during a rainstorm, an unsteady state of open channel flow occurs. When a constant flow rate of liquid passes through a pipe, it is called steady-state open channel flow. Steady-state open channel flow occurs when a constant flow rate of liquid passes through the channel. Many practical open channel flow situations have steady-state or nearly steady-state conditions. This course's equations and calculations will be for steady-state flow. Steady flow is one in which the variation of depth of flow (y) with respect to time is constant

$$\frac{\partial y}{\partial t} = 0 \quad (1.1)$$

If the flow characteristic at a given flow section remains constant, the flow characteristics remain constant with respect to time, the flow is said to be steady while at a given flow section the flow characteristics vary with time, the flow is said to be unsteady. Unsteady flow is one in which the variation of depth of flow y with respect to time varies.

$$\frac{\partial y}{\partial t} \neq 0 \quad (1.2)$$

ii. Uniform and non-uniform flow

Uniform flow is a flow in which variation of depth y in space x is constant,

$$\frac{\partial y}{\partial x} = 0 \quad (1.3)$$

While Non-uniform flow is a flow in which variation of depth y in space x varies,

$$\frac{\partial y}{\partial x} \neq 0 \quad (1.4)$$

1.2.5 State of flow

Fluid flow can be categorized as laminar, transitional, or turbulent depending on its effects on viscosity concerning inertia. This idea was emphasized by Osborne Reynolds and defined the Reynolds number as the ratio of inertial forces to viscous forces.

$$Re = \frac{inertial\ force}{viscous\ force} = \frac{\rho VD}{\mu} \quad \text{.....(1.5)}$$

Laminar flow is a flow that is orderly and with smooth streamlines with Re≤2000.

Turbulent flow is a flow with fluctuating velocity and highly disordered motion with Re≥4000.

Transitional flow is a flow fluctuation between laminar and turbulent flows with 2000≤Re≤ 4000.

Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress. Turbulent flow is characterized by random and rapid fluctuations of the swirling region of fluid flow known as eddies. In laminar flow, fluid particles flow in an orderly manner along path lines, and momentum and energy are transferred across the fluid of flow.

Froude Number is defined as the ratio of inertial forces to gravity forces.

$$Fr = \frac{v}{\sqrt{gL}} \quad \text{.....(1.6)}$$

The fluid flow may be categorized as super-critical flow and critical flow and sub-critical flow depending on the effect of viscosity on gravity.

$Fr < 1$ Subcritical flow, Fr=1 critical flow, and $Fr >$1 supercritical flow.

1.2.6 Type of channels

i. Prismatic and Non-prismatic channels.

Prismatic channels have a cross-sectional shape, size, and bed slope are constant e.g. Man-made channels while Non-prismatic channels have a cross-sectional shape, size and bed slope varies e.g. natural channels.

ii. Newtonian and Non-Newtonian fluid

Newtonian fluids show a linear relationship between the shear stress and the strain rate. A graph of stress and the strain rate gives a straight line passing through the origin, the slope gives the coefficient of viscosity, and example is water while Non-Newtonian fluids the relationship between the strain rate and shear stress is Non-linear, a graph of stress and the strain rate does not give a straight line through the origin, a constant coefficient of viscosity cannot be defined. Examples are blood, starch suspensions, and paint.

1.2.7 Streamline flow

Streamline flow is the motion of a fluid in which every particle in the fluid follows the same path past a specific point as the particles before it.

A stream line flow is one where, at a given point, each and every particles of the fluid travel in the same direction and the same velocity.

1.2.8 Geometric properties of parabolic open channel flow

A channel section is defined as the cross-section that is taken perpendicular to the main flow direction

a. The Top width, T

   This is the free surface width of the channel section.

$$T = \frac{3}{2}\,Ah$$

b. The flow Area, A

   This is the flow’s cross-sectional area normal to the flow direction

$$A = \frac{2}{3}Th$$

c. The wetted Perimeter, P

This is the length of the line of intersection of the channel wetted surface with a cross-sectional plane normal to the direction of flow.

d. The hydraulic Radius R=$\frac{A}{P}$

This is the proportion of water area to its wetted perimeter.

$$R = \frac{Flow\ area}{wetted\ perimeter}$$

e. The hydraulic Depth, D = $\frac{A}{T}$

This is the ratio of the water area to the Top width.

$$D_h = \frac{2}{3}h$$

f. Flow depth, y.

This is the perpendicular distance from the channel bottom to the free surface.

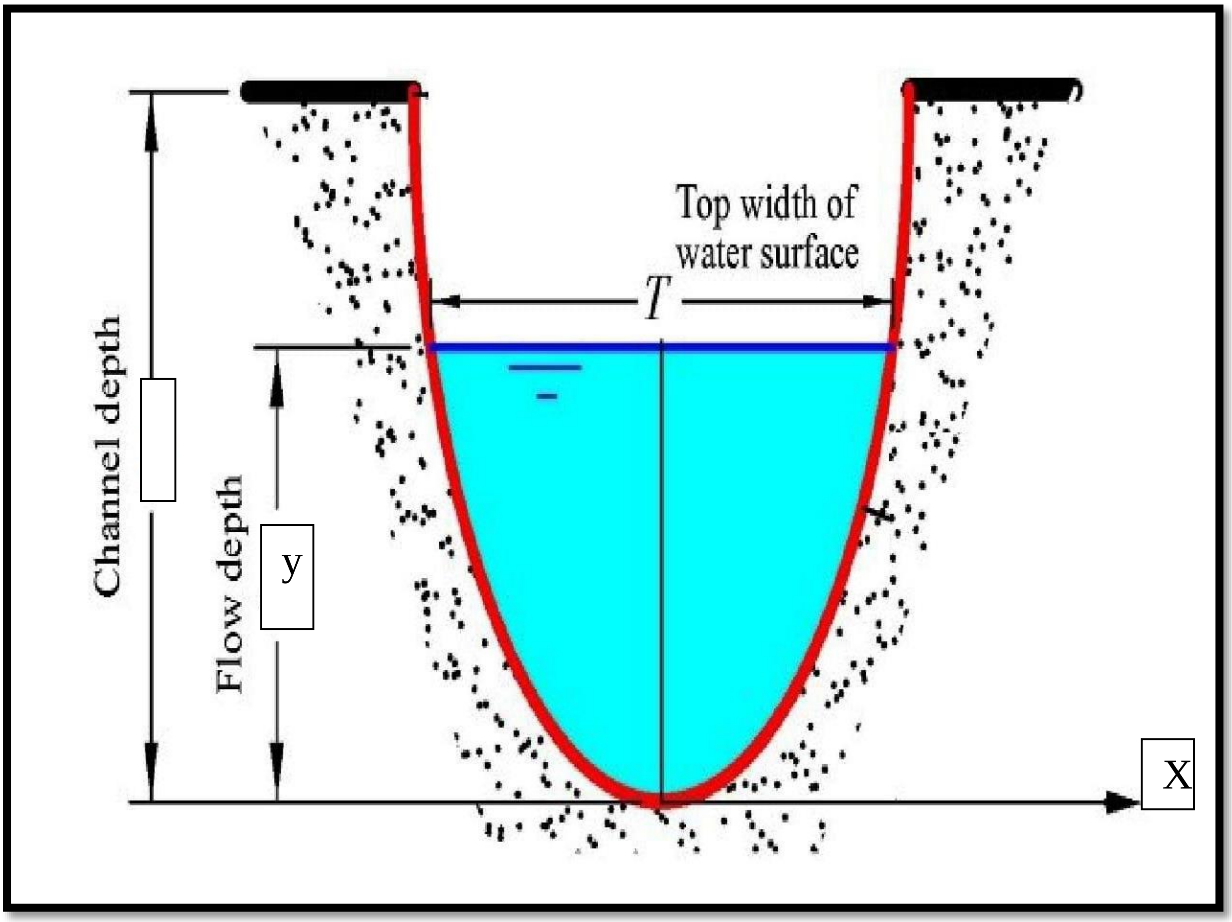

Figure 1.1: Geometric Profile **Ali, (2005)**

1.3 Statement of the Problem

It has been a challenge even to the engineers to come up with or design a channel that can convey the maximum amount of water efficiently. The effects of flooding on health and the environment have been extensively discussed and these range from obstruction of traffic, submerging roads, disruption of economic activities, coastal erosion; loss of property to loss of lives, displacement of people, water pollution, and diseases. Farmers who grow crops by irrigation can also benefit a lot if a channel that can hold maximum discharge is put in place.

Due to the effects of flooding, there is a need to model water flow channels to control flooding and enhance maximum water discharge to farms because knowing the extent to which flood can affect food security as well as mapping the flood vulnerability and food insecurity hotspots would help in suggesting the optimal adaptation strategies against such events. It would also assist policymakers in designing sustainable food security policies and flood emergency programs. The findings of the research serves as a baseline for comparative studies related to flood and food security.

1.4 Research Objectives

1.4.1 General Objective

The main objective of this research was to determine the effects of various parameters on the velocity of water flow in an open channel flow of parabolic cross-section.

1.4.2 Specific Objectives

The specific objectives were as follows;

i. To determine the effects of energy coefficient on the velocity of fluid flow in parabolic channels.

ii. To investigate the effect of varying top-width on the velocity of fluid flow in parabolic channels.

iii. To investigate the effect of varying channel slopes on the velocity of fluid flow in parabolic channels.

1.5 Significance of the Study

This study will be useful in areas such as mathematics, engineering, agriculture and energy. Below are some of the discussions of areas mentioned above**; (**Mukuna *et al.*, 2020).

1.5.1 Mathematics

The findings of this study will be useful in the field of applied mathematics in adding knowledge on open channel flows.

1.5.2 Engineering

Designers of open channel flows have made attempt to control flooding by trying different methods without success since there has been no open channel construction that conveys the maximum amount of water to the required areas for several purposes. This study will be useful in the design of open channel with parabolic cross-section.

1.5.3 Agriculture

Water is important for humans, animals, and plants to exist, water is life for any society to be and flourish, and it needs clean water for use. Excess water can cause destruction and death as a result of floods. To convey water to ponds and waterways, human beings have constructed canals and channels. With this much effort, the challenge of flooding is there, mainly during heavy rainfall.

1.5.4 Energy

This study will add value to the generation of electricity from water flowing in open channel. An effective design of open channels with parabolic cross-section has to be designed to solve the problem. This study made use of a mathematical model that will be used in the building of parabolic channels that will increase the volume of water conveyed to irrigation farms, generation of hydroelectric power, and in draining water from flood-stricken areas.

CHAPTER TWO

LITERATURE REVIEW

2.1 Related Literature Review

The study of open channel flow is common research area with studies carried out on Natural channels like rivers and man-made channels such as irrigation canals. In open-channel flows, gravity, viscosity, and inertia are the main forces at work, individually playing a key function. For a long time, studies on open channels have been a subject of discussion with the Chézy equation as one of the oldest uniform flow equations developed for the computation of average velocity of a uniform flow. Chézy formula provided unsatisfactory results to the designers of open channels. Henderson (1966). Manning formula has been proven to be the most used formula in the study of the open channel; this formula was developed through studies conducted by Manning in 1895. The Manning equation makes uses of the coefficient of roughness called Manning constant in the open channel flow. This has made the equation very reliable and more desired for the design of open channels. The Manning coefficient considers the degree of irregularity of the channel, channel size, bed material, and variation in shape and comparative effect of channel obstruction, meandering, and growth of vegetation in a channel (Chadwick & Morfeit, 1993).

Shao *et al* (2003) did an investigation on Numerical modeling of turbulent flow in curved channels of compound cross-section a mathematical model is developed based on a curvile orthogonal coordinate system using several algebraic stress models for the simulation of secondarily spiraled currents. Measurements in curved open channels with a simple rectangular cross-section are used to validate the model. Secondary flows were reproduced using the LY and NR algebraic stress models, as well as the SY nonlinear k– model, which were driven by both centrifugal force and turbulence stresses. The effects of changes in cross-cutting and canal curvature configurations on secondary motion have been discussed and the relationship between secondary stream pattern and different driving forces analyzed. In composition curved channels, predicted secondary currents have been compared to quality measures published on the right channel.

Kwanza *et al.*, (2007) investigated the effects of the slope of the channel, the width of the channel, and channel discharge for both trapezoidal and rectangular channels. The findings were shown that trapezoidal open channel flows are efficient hydraulically than the rectangular cross-sectional open channels. They noted that the volume of flow increases when identified factors are varied upwards.

Tsombe *et al,.(*2011) investigated fluid flow in open channels with a circular cross-section. He found out that increasing the flow depth, causes a reduction of fluid velocity. Further, increasing the channel slope results in an increase in velocity of flow. Also, increasing the radius of the channel results in a drop in velocity of flow. In addition, the findings were that a decrease in the slope of the channel results in a drop in the flow velocity because the slope and the flow velocity are directly proportional. Furthermore, for a fixed flow area, the flow velocity increases with increasing depth from the channel bottom to the free stream, with the maximum velocity occurring just below the free surface.

Thiong'o *et al* (2011) investigated fluid flow in an open rectangular and triangular channel. The findings were that open channels with rectangular cross-sections are efficient hydraulically than open channels with triangular cross-sections. Further findings were that for both triangular and rectangular channels, an increase in energy coefficient, Top width, and slope of the channel results in to arise in velocity of flow. Also, the velocity of flow increases as depth increases and becomes maximum slightly below the free surface. The velocity profile for both rectangular and triangular channels indicates that the channel that is rectangular moves more water at a faster rate than an open triangular channel at constant depth and width.

Ma *et al* (2012) did an investigation on iterative algorithm of conjugate depth for semi-cubic parabolic open channels. According to the cross-section geometric features of semi-cubic parabolic channels and the hydraulic jump equation of general prism channels, the iterative calculating formula of semi-cubic parabolic channels was deduced, and the convergence of the corresponding iterative formula was theoretically proven. The calculation formula for the initial iteration value of conjugate depth was obtained by calculating the sequent depth in the condition of different discharge Q, cross-section shape parameter p, and appropriate fitting formula. The hydraulic jump equation of semi-cubic parabolic channels was deduced, and the iterative calculating formula of the initial depth was obtained. The hydraulic jump equation of semi-cubic parabolic channels was deduced, and the iterative calculating formula of the initial depth and subsequent depth was obtained. They also used the direct calculation formula to calculate the initial iteration value, which we then substituted into the iterative formula of conjugate depth. After several iterations, the conjugate depth value was obtained with high precision. Conclusion: The iterative calculating formula of conjugate depth for semi-cubic parabolic channels had a clear physics concept, was simple to calculate, had high precision, and covered a wide range, and could meet the needs of engineering practice.

Thiong'o (2013) did a research focusing on open rectangular and triangular channel flows. The goal is to determine which of the open rectangular and triangular channels is more hydraulically efficient. The laws of conservation of mass and momentum have resulted in non-linear partial differential equations. Because analytical methods cannot be used to solve such equations, the finite difference method was employed. The velocity and depth of flow are important factors in determining discharge. The effects of changing various parameters on velocity have been studied. The variation of fluid velocity with depth has also been investigated.

The velocity profiles obtained by varying parameters such as channel slope, energy coefficient, channel top width, and roughness coefficient have been graphed. More graphs of velocity variation with depth and velocity profile comparison for both open rectangular and triangular channels have been plotted. It is discovered that the velocity of flow increases with depth, with the maximum velocity occurring slightly below the free surface. Furthermore, increasing the channel slope, energy coefficient, and top-width increases flow velocity, whereas increasing the roughness coefficient decreases flow velocity. It is also discovered that an open rectangular channel is more hydraulically efficient than an open triangular channel for a fixed flow depth and width. This research will help with flood control, irrigation, and the construction of channels such as house gutters.

Macharia *et al.,(*2014) studied the flow of fluids in an open rectangular channel with lateral inflow channels and discovered that increasing the channel's lateral inflow angle does not increase the velocity of the fluid in the core channel. The flow speed in the main channel is reduced as the cross-sectional area of the lateral inflow is increased. The flow velocity in the open rectangular channel increases as the lateral inflow channel velocity increases, while the velocity in both channels decreases as the lateral inflow channel length increases.

Ojiambo *et al.*,(2014) the study looked into a Mathematical model of fluid flow in an open channel with a circular cross-section, the findings of the study were that for a static area of flow, the velocity of flow increases as the depth of flow increase from the lowest part of the channel to the free stream and that maximum velocity is attained just below the free surface. and the results showed that decreasing the cross-sectional area of the channel and flow depth results in an increase in flow velocity. The velocity of flow increases as the lateral inflow rate per unit length of the channel decreases.

Jomba *et al.,* (2015) investigated a mathematical model of fluid flow in an open channel with a Horseshoe cross-section. From the study, the findings were that as the velocity of flow increases the depth increases for a fixed flow area, towards the free stream. Also, it was established that an increase in hydraulic radius and roughness coefficient results in a reduction of velocity due to increased shear stresses. A decrease in the slope of the channel results in a drop in flow velocity since flow velocity and slope of the channel are directly proportional. Increasing the cross-sectional area of flow leads to a drop in the flow velocity.

Longo et al, (2016) did an on the propagation of viscous gravity currents of non-Newtonian fluids in channels with varying cross section and inclination A model for the laminar propagation of gravity currents in rheological complex fluids over natural slopes is presented in this paper. The study is motivated by the common occurrence of gravity currents in environmental applications that are confined by channels that widen and have reduced slopes in the flow direction; mud and lava flows are typical examples. Many fluids exhibit nonlinear relationships between shear stress and shear rate in these applications, with or without the appearance of a yield stress. The variations in the channel shape and slope in the flow direction are captured using a power-law equation. We investigate the motion of these fluids' constant and time-dependent volumes on smoothly varying topographies. Constant volume and constant influx tests were carried out in a channel with a widening parabolic cross-section and a decreasing downstream inclination from 7° to 3.2°. The front position was measured continuously over time, and the current thickness and surface velocity were measured in some cross sections for a subset of experiments.

Marangu *et al.*, (2016) did an investigation on a model of open channel fluid flow with trapezoidal cross-section and a segment base. The purpose of this research was to look into the relevance of trapezoidal cross-sections with segment bases in drainage system design. The analysis took into account a constant, uniform open channel flow. The finite-difference approximation method was used to solve the saint-Venant partial differential equations of continuity and momentum that control free surface flow in open channels. The flow velocity is investigated concerning the channel radius, cross-section area, flow depth, and manning coefficient. The flow parameters are cross-section area of flow, channel radius, channel slope, and manning coefficient, and the flow variables are velocity and flow depth. The study discovered that increasing the flow's cross-section area causes a decrease in flow velocity. Furthermore, an increase in cross-section and the channel radius of flow causes a reduction in flow velocity, and increasing the roughness coefficient results in a reduction in flow velocity. The results of the study were that the flow velocity reduces as a result of increasing the radius of the circle forming the segment. The findings were that increase in depth of flow, channel radius, and the cross-sectional area produces a corresponding decrease in fluid velocity. Also, the results were that an increase in the bed slope of the waterway resulted in an increasing flow velocity.

Han *et al* (2017) did a study on new and improved three and one-third parabolic channels and most efficient hydraulic section the findings of the study were that, the literature contains several parabolic-shaped open channel sections, including quadratic and semi-cubic parabolic sections. This paper presents a three-and-one-third parabolic cross-section with superior properties to previous parabolic-shaped sections. The section characteristics are presented, along with two approximate formulas for the wetted perimeter and a simple iterative formula for the normal water depth. The precise solution for the most efficient hydraulic section is found. The width–depth ratio for the most efficient hydraulic section is found to be 2.1273. Practical applications of the proposed most efficient hydraulic section are presented, including direct discharge formulas and explicit normal and critical depth formulas. The results show that the proposed section outperforms other parabolic and trapezoidal sections in terms of hydraulic properties.

Omari *et al.,* (2018) Modeled circular closed channels for sewer lines. The result showed that increasing the area of cross-sectional sewer flow results in a decrease in the sewer depth. It was observed that decreasing the friction slope results in an increasing sewer flow velocity. Also, it was found out that an increase in tunnel angle of inclination results in an increase in sewer velocity.

Karimi (2018) studied flow in an open rectangular channel with a lateral inflow channel is investigated in this study. An incompressible Newtonian fluid is explored as it flows through a man-made open rectangular channel with a lateral inflow channel. The effects of angle (which ranges from zero to ninety degrees), cross-sectional area, velocity, and length of the lateral inflow channel on flow velocity in the main open rectangular channel have all been investigated. Because the discharge is related to the flow velocity, increasing the flow velocity also increases the discharge, and vice versa. The flow is governed by the continuity and momentum equations of motion, which are highly nonlinear and cannot be solved exactly. As a result, the finite difference method is used to calculate an approximate solution to these partial differential equations. Because of its precision, consistency, stability, and convergence, the finite difference method is employed to solve these equations. MATLAB software is used to generate the results, which are then graphed. The results show that at zero degrees of the lateral rectangular channel, the results are comparable to previous research. It is also discovered that increasing the area and length of the lateral inflow channel results in a decrease in velocity, whereas increasing the velocity of this channel results in an increase in the velocity of the main channel. Finally, increasing the angle of the lateral inflow channel does not always result in increased velocity in the main channel.

Nazir (2019)In this paper, we investigate the saint venant equations for analyzing water flow in various channels. The comparison of various open channels under various conditions is established. We investigate some novel findings concerning the non-uniform and unsteady flow of water in open channels. Some numerical experiments are also presented to demonstrate the validity of the main findings.

Mose *et al.,* (2019) investigated Mathematical modeling of the flow of fluid in an open channel with an elliptical cross-section. The findings showed an increased hydraulic radius, which results in an increasingly fluid depth. The depth of fluid flow reduces along the channel due to the accumulation of eroded particles which consequently reduces the fluid velocity. Variation of friction slope also affects flow velocity. When friction is raised, the flow velocity is reduced. Friction arises from the shear forces on the walls and channel bed which offers resistance to the smooth flow of water.

Although a lot of research has been done in the last two decades on open channels with a different cross-sectional area no research has been made on parabolic channels. The problem of flooding persists in the current years and a channel that can convey maximum discharge on flooded areas into irrigational land has to be designed, and this is what this research strives to explore.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Assumptions and Approximations

In this study, the following assumptions will be utilized.

1. The fluid is flowing in one direction.
2. Newtonian fluid is considered.
3. Forces due to gravity cause the fluid to flow.
4. Incompressible flow is considered.

3.2 Governing Equations

The study of the flow of open channels considers the main equations as; momentum and continuity. This governing equation is used in the study of one-dimensional flow and used to solve partial differential equations in this research. The Navier-stokes equation derives the continuity equation while Newton's Second Law of motion derives the equation of momentum.

3.2.1 Continuity Equation

In the study of uniform flow, the continuity equation is regarded as one of the important principles used. The principle is derived from the concept that mass is conserved always in fluid systems flowing in any direction and flow complexity.

The discharge Q is obtained as;

$$Q = AV$$

For a given pair of regions, the discharge Q is expressed as;

$$Q = AV \quad \text{(3.1)}$$

Where Q= discharge

A= area of cross-section fluid flow

V = mean velocity rate

The continuity equation governing unsteady flow in an open channel of general shape is

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \quad (3.2)$$

If we substitute equation (3.1) into equation (3.2) above, then we differentiate partially with respect to x gives:

$$V\frac{\partial A}{\partial x} + \frac{\partial A}{\partial t} + A\frac{\partial V}{\partial X} - q = 0 \quad (3.3)$$

Expressing the derivatives of A as a function of y since the area flow is assumed to be a depth function.

$$\frac{\partial A}{\partial x} = \frac{dA}{dy}\frac{\partial y}{\partial x} = T\frac{\partial y}{\partial x} \quad (3.4)$$

$$\frac{\partial A}{\partial t} = \frac{dA}{dy}\frac{\partial y}{\partial t} = T\frac{\partial y}{\partial t} \quad (3.5)$$

For this research $T = \frac{dA}{dy}$

If we substitute equation (3.4) and (3.5) in equation (3.3) yields:

$$VT\frac{\partial y}{\partial x} + A\frac{\partial V}{\partial x} + T\frac{\partial y}{\partial t} - q = 0 \quad (3.6)$$

Dividing equation (3.6) by T we obtain;

$$\frac{\partial y}{\partial t} + \frac{A}{T}\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} - \frac{q}{T} = 0 \quad (3.7)$$

3.2.2 Momentum Equation

Newton's second law of motion derives from the Momentum equations.

The momentum equation governing unsteady flow in open channels of general shape is:

$$\frac{\partial V}{\partial t} + \alpha V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} = g(s_o - s_f) \quad (3.8)$$

The channel bottom slope s_o can be conveniently expressed as:

$$s_o = -\frac{dz}{dx} \quad (3.9)$$

Where z is the bed level or channel bottom elevation relative to a datum. The term $\frac{dz}{dx}$ is the change of elevation of the bottom of the channel with respect to distance or the bottom slope.

The friction slope, s_f also known as the friction term due to bed's roughness is expressed as:

$$s_f = -\frac{dH}{dx} \quad (3.10)$$

Where H is the total energy at any cross-section of the channel. The term $\frac{dH}{dx}$is the change of energy with longitudinal distance or the friction slope.

Rearranging equation (3.8) yields:

$$\frac{\partial V}{\partial t} + \alpha V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} - g\left(s_0 - s_f\right) = 0 \quad (3.11)$$

Equation (3.7) and (3.11) are first-order partial differential equation which is a non-linear type and would be solved by the use of finite difference method and MATLAB program.

3.2.3 Chézy Equation

Chézy equation was developed by Antoine Chézy a French Engineer in 1768 while designing an open canal for the supply of water.

$$V = c\sqrt{R_h S_f} \quad (3.12)$$

3.2.4 Manning Equation

V= $\frac{1}{n}R^{2/3}{S_o}^{1/2}$ ....(3.13)

Discharge is given by;

Q=AV..(3.14)

Substituting equation (3.14) into equation (3.13) we obtain,

Q= $\frac{1}{n}AR^{2/3}{S_o}^{1/2}$ ....(3.15)

Finite difference equations for parabolic channel are;

Discretization of derivatives

$$\frac{\partial y}{\partial t} = \frac{y(i,j+1) - y(i,j)}{\Delta t}$$

$$\frac{\partial v}{dt} = \frac{v(i,j+1) - v(i,j)}{\Delta t}$$

$$\frac{\partial y}{\partial x} = \frac{y(i+1,j) - y(i-1,j)}{2\Delta x}$$

$$\frac{\partial v}{\partial x} = \frac{v(i+1,j) - v(i-1,j)}{2\Delta x}$$

From equation 3.7

$$\frac{y(i,j+1) - y(i,j)}{\Delta t} + \frac{A}{T}\left(\frac{v(i+1,j) - v(i-1,j)}{2\Delta x}\right) + v(i,j)\left(\frac{y(i+1,j) - y(i-1,j)}{2\Delta x}\right) = \frac{q}{T}$$

$$y(i,j+1) = \Delta t\left[-\frac{A}{T}\left(\frac{v(i+1,j)-v(i-1,j)}{2\Delta x}\right) - v(i,j)\left(\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}\right) + \frac{q}{T}\right] + y(i,j)$$

....(3.12)

From equation 3.11

$$\frac{v(i,j+1)-v(i,j)}{\Delta t}+\alpha\, v(i,j)\left(\frac{v(i+1,j)-v(i-1,j)}{2\Delta x}\right)+g\left(\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}\right)$$

$$= g(s_o - s_f$$

$$v(i,j+1) = \Delta t\left[-\alpha\, v(i,j)\left(\frac{v(i+1,j)-v(i-1,j)}{2\Delta x}\right) - g\ \left(\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}\right) + g\ (s_o - s_f)\right] + v(i,j).$$

....(3.13)

Equations 3.12 and 3.13 are the momentum and continuity equations of an open parabolic channel in finite difference form.

Conditions of flow for parabolic channel in finite difference form.

The initial conditions as per the program in finite difference form are;

v (0,t) = 10,   y(i,o) = 15

The boundary conditions as per the program in finite-difference forms are;

v (x_o, j) = 20,   y(x_o,j) = 15

v (x_n, j) =20   y(x_n, j) = 30

Where i denote the distance along the channel,

j denotes time.

x_o *and* x_n denote the entry point and the exit point respectively of the section of the channel.

CHAPTER FOUR

RESULTS

The equations 3.12 and 3.13 are solved using the MATLAB program. The effects of energy coefficient, Top width, and channel slope on the flow velocity are represented graphically as shown in figure 4.1 – 4.3

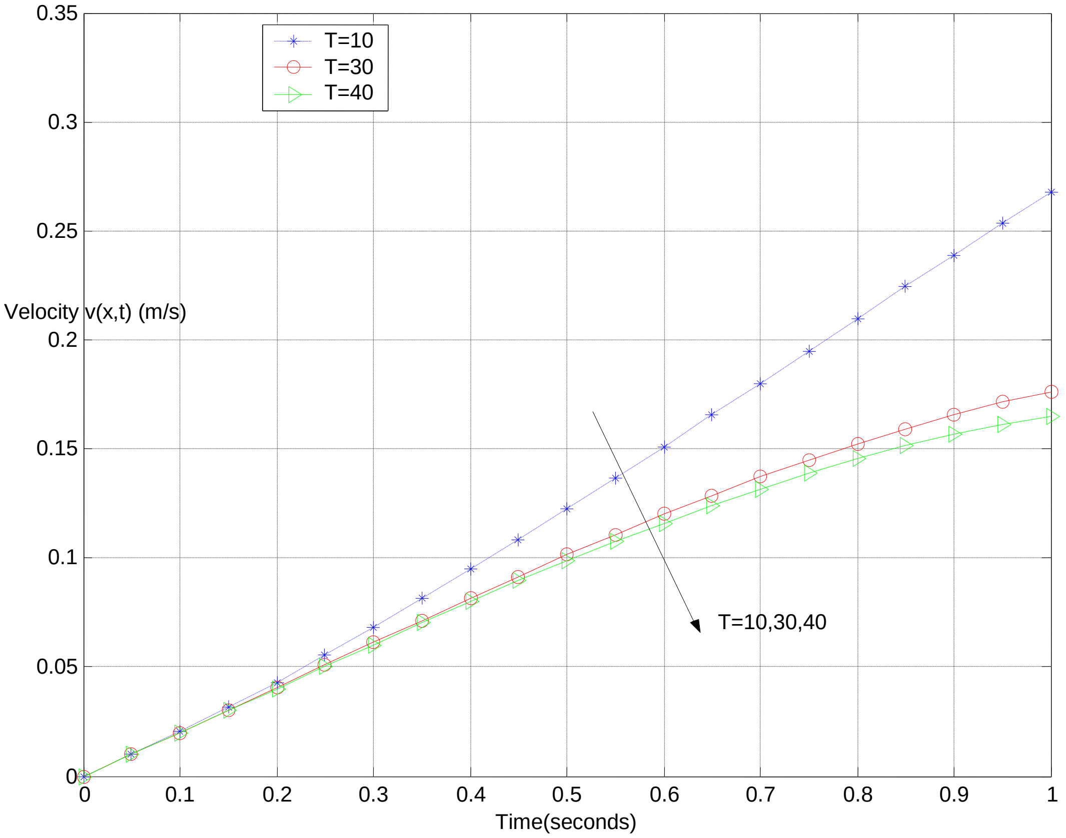

Figure 4.1: Effect of Top-Width, T on velocity

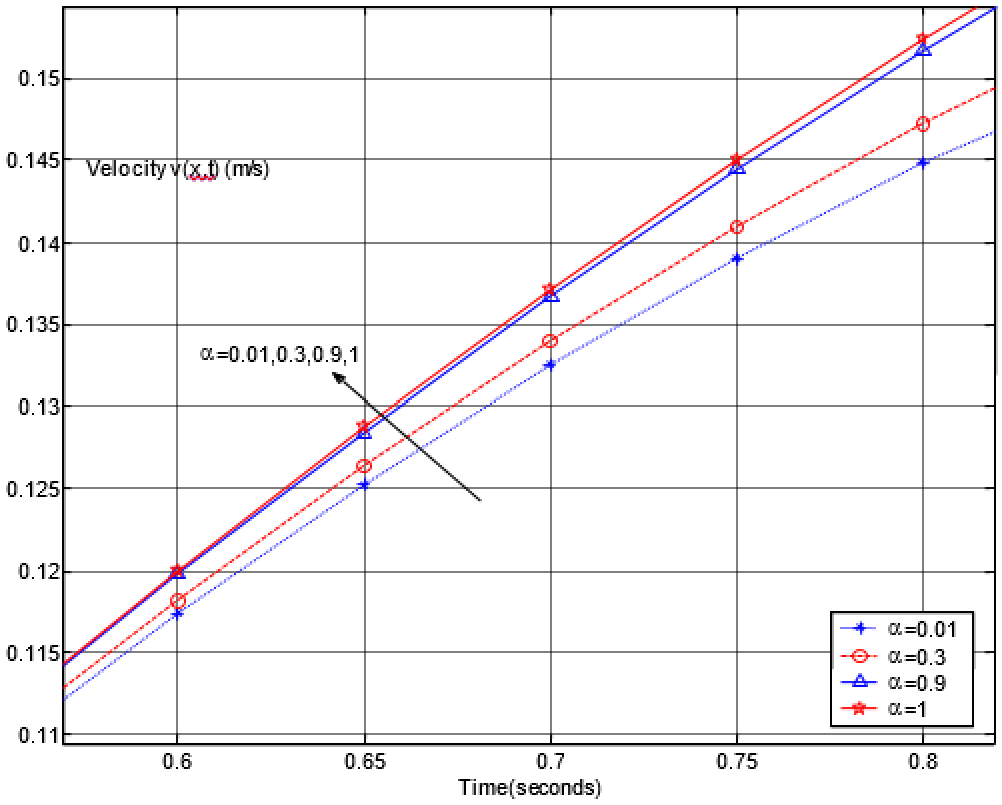

Figure 4.2: Effects of varying energy coefficient (α) on the velocity of flow

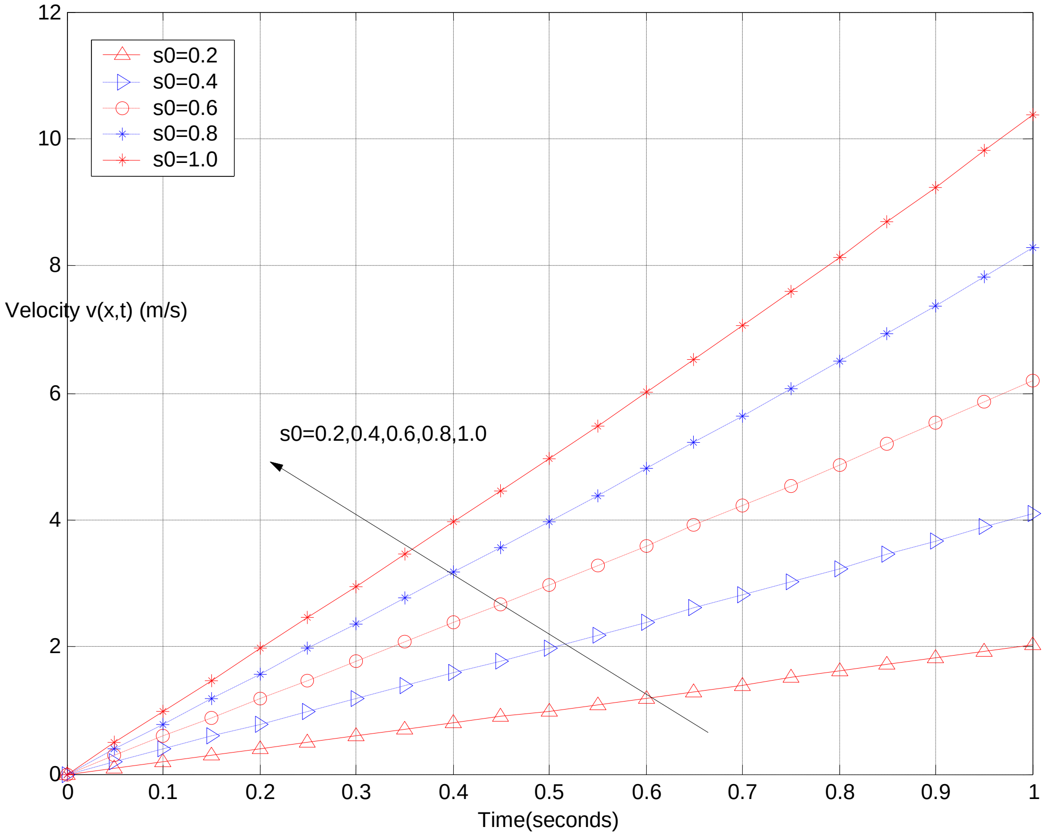

Figure 4.3: Effect of channel slope, so on velocity

CHAPTER FIVE

DISCUSSIONS

From figure 4.1 it is observed that as the energy coefficient increases from 0.01 to 1.0 the velocity increases, hence when the energy coefficient increases, the velocity increases. According to the Kinetic theory of matter fluid molecules possesses kinetic energy (energy in motion) which is reflected clearly from the graph. Increasing the energy of the flow leads to kinetic energy increasing of the particles hence particles move faster.

From figure 4.2 it is noted that reduction in Top width increases the velocity of fluid flow in parabolic channels. The Top width of 10m yields a higher velocity as compared to the higher values up to 100m.

From figure 4.3 for the same depth decreasing the channel slope from 1.0 to 0.2 decreases the velocity of flow in a parabolic channel. Hence, the velocity value when the channel slope is 0.2 is lower than when the channel slope is 1.0

According to Manning's velocity formula, velocity is directly proportional to the slope, and therefore as the slope increases, velocity also increases.

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The study was conducted over the effects of energy coefficient, channel slope, and Top width on the fluid flow velocity of an open channel with a parabolic cross-section. From the analysis, it is clear that the energy coefficient, Top width, and channel slope for parabolic open channel affect the velocity of flow of fluid. The following conclusions were made from the results obtained:

i. Increasing the energy coefficient increases the velocity of flow.
ii. Reduction in the channel Top width, results in an increased velocity of the channel.
iii. Increasing the channel slope of flow leads to an increase in the velocity of flow in the parabolic channel since flow velocity is directly proportional.

5.2 Recommendations

We recommend that future research should be done on;

i. In this research, the fluid flow was 1-dimensional the same research can be carried out by considering 2-D, 3-D flows
ii. Further research should be carried out by keeping other parameters constant other than depth.
iii. Comparison of fluid flow in an elliptical and parabolic channel.

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APPENDICES

Appendix I: Tables

Table 1:Velocity versus time: alpha =0.01,0.3,0.9,1

	Velocity(m/s)			
Time(s)	alpha=0.01	alpha=0.3	alpha=0.9	alpha=1
0	0	0	0	0
0.05	0.0098	0.0098	0.0098	0.0098
0.1	0.0199	0.0199	0.0199	0.0199
0.15	0.0301	0.0301	0.0301	0.0301
0.2	0.0404	0.0404	0.0404	0.0404
0.25	0.0507	0.0507	0.0508	0.0508
0.3	0.0609	0.061	0.0611	0.0611
0.35	0.0711	0.0712	0.0714	0.0714
0.4	0.081	0.0812	0.0815	0.0816
0.45	0.0907	0.091	0.0915	0.0916
0.5	0.1	0.1004	0.1012	0.1013
0.55	0.1089	0.1095	0.1106	0.1108
0.6	0.1173	0.1182	0.1197	0.12
0.65	0.1252	0.1263	0.1284	0.1288
0.7	0.1325	0.1339	0.1367	0.1371
0.75	0.139	0.1409	0.1445	0.145
0.8	0.1448	0.1473	0.1518	0.1525
0.85	0.1498	0.1528	0.1585	0.1593
0.9	0.1538	0.1576	0.1646	0.1656

0.95	0.1569	0.1615	0.17	0.1713
1	0.1589	0.1645	0.1748	0.1763

Table 2:Velocity versus time: channel slope =0.02,0.04,0.06,0.08,1

	velocity(m/s)				
Time(s)	s_0=0.02	s_0=0.04	s_0=0.06	s_0=0.08	s_0=1
0	0	0	0	0	0
0.05	0.0982	0.1964	0.2946	0.3928	0.491
0.1	0.1967	0.3931	0.5895	0.7859	0.9824
0.15	0.2954	0.5902	0.8849	1.1797	1.4745
0.2	0.3944	0.7877	1.1811	1.5744	1.9677
0.25	0.4937	0.9859	1.4781	1.9702	2.4624
0.3	0.5934	1.1848	1.7762	2.3676	2.9591
0.35	0.6933	1.3844	2.0755	2.7668	3.458
0.4	0.7936	1.5849	2.3764	3.168	3.9598
0.45	0.8943	1.7865	2.679	3.5718	4.4648
0.5	0.9953	1.9892	2.9835	3.9783	4.9735
0.55	1.0968	2.1931	3.2901	4.3879	5.4864
0.6	1.1986	2.3983	3.5991	4.801	6.004
0.65	1.3009	2.6049	3.9107	5.218	6.5268
0.7	1.4036	2.8132	4.2251	5.6393	7.0554
0.75	1.5068	3.0231	4.5426	6.0652	7.5904
0.8	1.6104	3.2348	4.8635	6.4961	8.1323
0.85	1.7146	3.4485	5.188	6.9326	8.6818
0.9	1.8193	3.6642	5.5164	7.375	9.2395
0.95	1.9246	3.8822	5.8489	7.8239	9.8061
1	2.0305	4.1025	6.186	8.2797	10.3823

Table 3:Velocity versus time Top width: T=10,30,40

	velocity(m/s)		
Time(s)	T=10	T=30	T=40
0	0	0	0
0.05	0.0098	0.0098	0.0098
0.1	0.0203	0.0199	0.0198
0.15	0.0315	0.0301	0.0299
0.2	0.0432	0.0404	0.0401
0.25	0.0554	0.0508	0.0502
0.3	0.0681	0.0611	0.0603
0.35	0.0812	0.0714	0.0702
0.4	0.0946	0.0816	0.08
0.45	0.1084	0.0916	0.0895
0.5	0.1224	0.1013	0.0987
0.55	0.1366	0.1108	0.1076
0.6	0.151	0.12	0.1161
0.65	0.1655	0.1288	0.1242
0.7	0.1802	0.1371	0.1317
0.75	0.1948	0.145	0.1388
0.8	0.2095	0.1525	0.1453
0.85	0.2242	0.1593	0.1512
0.9	0.2389	0.1656	0.1565
0.95	0.2535	0.1713	0.161
1	0.268	0.1763	0.1649

Appendix II: MATLAB Code

```
%% ####################################################################

% solve y_t+ v*y_x+(A/T)v_x=  for q/T 0 <= x <= xf, 0 <= t <= tfinal

%       v_t+ alpha*v*v_x+g*y_x-g(s0-sf)=0 for  0 <= x <= xf, 0 <= t <= tfinal

% Initial Condition: y(x,0)= ity0(x)

%                    v(x,0)= itv0(x)

%

% Boundary Conditions: y(0,t) = g0(t)=by0(t)  (left  BC)

%                      y(xf,t)= g1(t)=byf(t) (right BC)

%                      v(0,t) = h0(t)=bv0(t)  (left  BC)

%                      v(xf,t)= h1(t)=bvf(t) (right BC)

clc,clf,clearall,close all% clear screen,clearfigure,clear all declared variables, close all figures

alpha=1;

s0=0.02;

A=15;       % Area

q=0.3;      % Discharge

g=9.82;     % gravitational force

n=0.01;R=1.1;

t0=0;              % initial time

x0=0;              % initial distance
```

xfinal=100; % maximum distace final distance/distance at which concetration is to be calculated

tfinal=1; % maximum time (final time)

M=5; % N=20; %

dx = (xfinal-x0)/M ; % distance interval

x = [0:M]'*dx; % values of distance (x)

dt = (tfinal-t0)/N; % time interval

t = [0:N]*dt; % values of time (t)

% initial and boundary conditions formulae definitions (next three lines)

ity0=inline ('0','x','t'); itv0=inline ('0','x','t');

by0=inline ('10','x','t');bv0=inline('15','x','t');

byf=inline('20','x','t'); bvf=inline('30','x','t');

```
for i = 1:M + 1,
    y(i,1) = ity0(x(i),t(1));        % initial condition evaluation
    v(i,1) = itv0(x(i),t(1));        % initial condition evaluation
end
for j = 1:N + 1
    y(1,j) = by0(x(1),t(j));    % boundary conditions evaluations
    y(M+1,j)=byf(x(M+1),t(j));  % boundary conditions evaluations
    v(1,j) = bv0(x(1),t(j));    % boundary conditions evaluations
```

```
    v(M+1,j)= bvf(x(M+1),t(j)); % boundary conditions evaluations
end
% if gt(dt/(2*dx),1)                      % stability condition: 2*D_L*dt/dx^2 <= 1
%      error('stability condition not satisfied')     % error message if stability condition is not satisfied
%     return                               % return control to command line
% end                                      % end of stability condition loop
for T=[10 30 40 ]
 for j = 1:N                               % start of time loop
  for i = 2:M                              % start of distance loop
  y(i,j+1)=dt*(-v(i,j).*((y(i+1,j)-y(i-1,j))/(2*dx))-(A/T)*((v(i+1,j)-v(i-1,j))/(2*dx))+q/T)+y(i,j)
  v(i,j+1)=dt*(-alpha*v(i,j).*((v(i+1,j)-v(i-1,j))/(2*dx))-g*((y(i+1,j)-y(i-1,j))/(2*dx))-
g*(n^2*(v(i,j).^2/R^(4/3))-s0))+v(i,j)
  end                                      % end of distance loop
 end                                       % end of time loop
plot(t,v(end-2,:),'*:')                    % plot velocity against time
hold on
end
xlabel('Time(seconds)')                    % label x-axis
ylabel('Velocity v(x,t) (m/s)')            % label y-axis
```

```
title('Effect of Top Width,T on Velocity ')           % title of 2D graph

grid                                    % insert grid lines to graph

legend('T=10','T=30','T=40',0)
```

```
% Numerical Study of Flow of Fluid in an open parabolic Channel

% ################################################################

% solve y_t+ v*y_x+(A/T)v_x=  for q/T 0 <= x <= xf, 0 <= t <= tfinal

%       v_t+ alpha*v*v_x+g*y_x-g(s0-sf)=0 for  0 <= x <= xf, 0 <= t <= tfinal

% Initial Condition: y(x,0)= ity0(x)

%                   v(x,0)= itv0(x)

%

% Boundary Conditions: y(0,t) = g0(t)=by0(t)  (left  BC)

%                     y(xf,t)= g1(t)=byf(t) (right BC)

%                     v(0,t) = h0(t)=bv0(t)  (left  BC)

%                     v(xf,t)= h1(t)=bvf(t) (right BC)

clc,clf,clearall,close all% clear screen,clearfigure,clear all declared variables, close all figures

alpha=1;

s0=0.02;

A=15;       % Area

q=0.3;      % Discharge

g=9.82;     % gravitational force

n=0.01;R=1.1;

t0=0;               % initial time

x0=0;               % initial distance
```

xfinal=100;   % maximum distace final distance/distance at which concetration is to be calculated

tfinal=1;   % maximum time (final time)

M=5;   % M = # of subintervals along x(distance) axis

N=20;   % N = # of subintervals along t(time) axis

dx = (xfinal-x0)/M ;   % distance interval

x = [0:M]'*dx;   % values of distance (x)

dt = (tfinal-t0)/N;   % time interval

t = [0:N]*dt;   % values of time (t)

% initial and boundary conditions formulae definitions (next three lines)

ity0=inline('0','x','t'); itv0=inline('0','x','t');

by0=inline('10','x','t');bv0=inline('15','x','t');

byf=inline('20','x','t'); bvf=inline('30','x','t');

for i = 1:M + 1,

    y(i,1) = ity0(x(i),t(1));   % initial condition evaluation

    v(i,1) = itv0(x(i),t(1));   % initial condition evaluation

end

for j = 1:N + 1

    y(1,j) = by0(x(1),t(j));   % boundary conditions evaluations

    y(M+1,j)=byf(x(M+1),t(j)); % boundary conditions evaluations

```
    v(1,j) = bv0(x(1),t(j));    % boundary conditions evaluations

    v(M+1,j)= bvf(x(M+1),t(j)); % boundary conditions evaluations

end

% if gt(dt/(2*dx),1)                    % stability condition: 2*D_L*dt/dx^2 <= 1

%      error('stability condition not satisfied')     % error message if stability condition is not
satisfied

%     return                            % return control to command line

% end                                   % end of stability condition loop

for T=[10 30 40 ]

 for j = 1:N                            % start of time loop

  for i = 2:M                           % start of distance loop

  y(i,j+1)=dt*(-v(i,j).*((y(i+1,j)-y(i-1,j))/(2*dx))-(A/T)*((v(i+1,j)-v(i-1,j))/(2*dx))+q/T)+y(i,j)

  v(i,j+1)=dt*(-alpha*v(i,j).*((v(i+1,j)-v(i-1,j))/(2*dx))-g*((y(i+1,j)-y(i-1,j))/(2*dx))-
g*(n^2*(v(i,j).^2/R^(4/3))-s0))+v(i,j)

  end                                   % end of distance loop

 end                                    % end of time loop

plot(t,v(end-2,:),'*:')                 % plot velocity against time

hold on

end

xlabel('Time(seconds)')                 % label x-axis
```

```
ylabel('Velocity v(x,t) (m/s)')                    % label y-axis

title('Effect of Top Width,T on Velocity ')             % title of 2D graph

grid                                           % insert grid lines to graph

legend('T=10','T=30','T=40',0)
```

```
% Numerical Study of Fluid Flow in an open parabolic Channel

% ################################################################

% solve y_t+ v*y_x+(A/T)v_x=  for q/T 0 <= x <= xf, 0 <= t <= tfinal

%       v_t+ alpha*v*v_x+g*y_x-g(s0-sf)=0 for  0 <= x <= xf, 0 <= t <= tfinal

% Initial Condition: y(x,0)= ity0(x)

%                    v(x,0)= itv0(x)

%

% Boundary Conditions: y(0,t) = g0(t)=by0(t)  (left  BC)

%                      y(xf,t)= g1(t)=byf(t) (right BC)

%                      v(0,t) = h0(t)=bv0(t)  (left  BC)

%                      v(xf,t)= h1(t)=bvf(t) (right BC)

clc,clf,clearall,close all% clear screen,clearfigure,clear all declared variables, close all figures

A=15;       % Area

T=30;       % Top width

q=0.3;      % Discharge

g=9.82;     % gravitational force

s0=0.02;

n=0.01;R=1.1;

t0=0;              % initial time

x0=0;              % initial distance
```

```
xfinal=100;          % maximum distace final distance/distance at which concetration is to be calculated

tfinal=1;          % maximum time (final time)

M=5;               % M = # of subintervals along x(distance) axis

N=20;              % N = # of subintervals along t(time) axis

dx = (xfinal-x0)/M ;    % distance interval

x = [0:M]'*dx;         % values of distance (x)

dt = (tfinal-t0)/N;     % time  interval

t = [0:N]*dt;          % values of time (t)

% initial and boundary conditions formulae definitions (next three lines)

ity0=inline ('0','x','t'); itv0=inline ('0','x','t');

by0=inline('10','x','t');bv0=inline('15','x','t');

byf=inline('20','x','t'); bvf=inline('30','x','t');

for i = 1:M + 1,

    y(i,1) = ity0(x(i),t(1));               % initial condition evaluation

    v(i,1) = itv0(x(i),t(1));               % initial condition evaluation

end

for j = 1:N + 1

    y(1,j) = by0(x(1),t(j));    % boundary conditions evaluations

    y(M+1,j)=byf(x(M+1),t(j));  % boundary conditions evaluations
```

```
    v(1,j) = bv0(x(1),t(j));    % boundary conditions evaluations

    v(M+1,j)= bvf(x(M+1),t(j)); % boundary conditions evaluations

end

% if gt(dt/(2*dx),1)                    % stability condition: 2*D_L*dt/dx^2 <= 1

%      error('stability condition not satisfied')     % error message if stability condition is not
satisfied

%     return                            % return control to command line

% end                                   % end of stability condition loop

for alpha=[ 0.01 0.3 0.9 1]

 for j = 1:N                            % start of time loop

  for i = 2:M                           % start of distance loop

  y(i,j+1)=dt*(-v(i,j).*((y(i+1,j)-y(i-1,j))/(2*dx))-(A/T)*((v(i+1,j)-v(i-1,j))/(2*dx))+q/T)+y(i,j)

  v(i,j+1)=dt*(-alpha*v(i,j).*((v(i+1,j)-v(i-1,j))/(2*dx))-g*((y(i+1,j)-y(i-1,j))/(2*dx))-
g*(n^2*(v(i,j).^2/R^(4/3))-s0))+v(i,j)

  end                                   % end of distance loop

 end                                    % end of time loop

plot(t,v(end-2,:),'*:')                 % plot velocity against time

hold on

end

xlabel('Time(seconds)')                 % label x-axis
```

```
ylabel('Velocity v(x,t) (m/s)')                    % label y-axis

title('Effect of \alpha on Velocity ')             % title of 2D graph

grid                                               % insert grid lines to graph

legend('\alpha=0.01','\alpha=0.3','\alpha=0.9','\alpha=1',0)
```

Appendix III : Similarity Report

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