

**MODELLING TRAFFIC FLOW BEFORE AND AFTER ROUNDABOUT USING
NAVIER-STOKES AND ADVECTION-DIFFUSION EQUATIONS**

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DECLARATION

Declaration by the Student

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DEDICATION

This thesis is dedicated to my Mother Fridah Mogire, Father Momanyi Nyaega, Spouse Purity Mageto, my brothers Cyrus Sure and christopher Ongeru and my coleagues George Ogutu and Peter Kipchirchir.

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ABSTRACT

Traffic congestion remains a persistent challenge in urban areas, with roundabouts playing a pivotal role in enhancing road safety, improving traffic flow, and minimizing congestion. Understanding traffic dynamics before and after roundabout implementation is critical for optimizing urban infrastructure. Traditional traffic models lack the precision to account for the complex interactions and flow disruptions associated with roundabouts. These limitations hinder accurate predictions of traffic patterns, requiring more advanced mathematical approaches to model flow dynamics effectively. This study aims to model traffic flow around roundabouts using Navier-Stokes and advection-diffusion equations. Specific objectives include formulating mathematical models, analyzing the influence of roundabout geometry on traffic flow, evaluating disruption and diffusion effects, and identifying critical factors impacting flow stability. The study employed fluid mechanics principles, utilizing the Navier-Stokes and advection-diffusion equations to model traffic as a fluid-like system. Numerical simulations were conducted using the Finite Volume Method and Crank-Nicolson scheme, with Matlab R2023b facilitating sensitivity analyses to evaluate various scenarios. Findings indicate that roundabouts significantly improve traffic flow efficiency by reducing congestion and enhancing speed regulation. The geometric design of roundabouts and their capacity to handle disruptions were identified as key factors influencing performance. Sensitivity analysis revealed optimal configurations for minimizing delays and maximizing output. The integration of roundabouts enhances urban traffic dynamics by mitigating congestion and optimizing vehicle movement. Mathematical models provide a robust framework for analyzing these effects, ensuring informed urban planning. Policymakers should incorporate advanced mathematical modeling in roundabout designs, emphasizing scenario-specific analyses to address diverse traffic conditions. Future research should integrate behavioral and environmental factors to refine predictive capabilities and practical applications.

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LIST OF NOTATION AND ABBREVIATIONS

Abbreviation	Description	Unit
$\mathbf{F}_{\text{obstacle}}$	Obstacle force vector	N
k_{obs}	Constant determining strength of obstacle force	N m^{-1}
\mathbf{r}	Vector from vehicle to obstacle	m
\mathbf{v}	Velocity vector in 2D (v_x, v_y)	m s^{-1}
∇	Del operator or gradient	m^{-1}
p	Pressure term	Pa
ν	Kinematic viscosity	$\text{m}^2 \text{s}^{-1}$
∇^2	Laplacian operator	m^{-2}
\mathbf{F}	External forces	N
ρ	Traffic density	kg m^{-3} or m^{-1}
D	Diffusion coefficient	$\text{m}^2 \text{s}^{-1}$
S	Additional terms for disruptions (e.g., accidents, jams)	Varies by model
ANSYS	Analysis System	
ADE	Advection Diffusion Equation	
CFD	Computational Fluid Dynamics	
FVM	Finite Volume Method	
QUICK	Quadratic Upstream Interpolation for Convective Kinematics	
UGM	Urgent-Gentle Class Traffic Flow Mode	

CHAPTER ONE

INTRODUCTION

1.1 Background Information

Traffic flow on a road can be visualized as the flow of a fluid, like water in a river. The Navier-Stokes equations are a set of mathematical rules initially developed to describe the motion of fluids, like air or water (Cao and Cao, 2021; Ershkov et al., 2021; Sreenivasan and Yakhot, 2021). They can also be applied to describe the motion of vehicles on the road, treating traffic as if it were a kind of traffic fluid. Advection-diffusion equations used to model the transport and dispersion of substances within a fluid (Gautam et al., 2022; Sun et al., 2022).

Traffic acceleration refers to the changes in speed and direction of vehicles in traffic modeled using fluid dynamics principles (Kessels et al., 2019). The Navier-Stokes equations, which describe the motion of fluids, can be used to understand how and why vehicles accelerate or decelerate in traffic. Traffic flow can be treated as a two-dimensional fluid flow, and the interactions between cars are modeled as traffic pressure similar to pressure in fluids (Shao, 2023). The advection-diffusion equation relates the concentration of vehicles as the variable and the diffusivity as the diffusion coefficient (Dwivedi et al., 2020). The velocity field that the quantity is moving with is a function of time and space. The traffic pressure arises from the interactions between vehicles when they get close together, and it affects their movement. The traffic pressure can be calculated using mathematical models that take into account factors such as vehicle density, speed, and the coefficient of viscous resistance (Sun et al., 2020). The use of fluid dynamics principles to model traffic flow has led to the development of mathematical models that can predict traffic patterns and optimize traffic control

strategies (Kachroo et al., 2018; Kang et al., 2022; Xiao and Duan, 2020). In this work, advection refers to the transport of vehicle density by the velocity field, while diffusion accounts for spreading due to random interactions or disturbances.

1.2 Basic Concepts

Suppose the traffic density (ρ), traffic velocity ($\mathbf{v} = (u, v)$), and diffusion coefficient (D) as functions of both space (x, y) and time (t), then the basic Navier-Stokes and advection-diffusion equation for traffic flow in 2D is represented by (1.1) and (1.2), respectively.

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0 \\ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} &= -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v}\end{aligned}\tag{1.1}$$

where ∇ is the del operator. The Advection-Diffusion Equation in 2D

$$\frac{\partial p}{\partial t} + \nabla \cdot (\mathbf{v} p) = D \nabla^2 p\tag{1.2}$$

The pressure p in traffic can manifest in several ways. For example, if the vehicles are close, the space available for maneuvering reduces the space available for each vehicle to maneuver (he et. al 2022). This can result in bottlenecks, where the traffic is held up because single vehicles cannot change lanes, switch directions, or flow smoothly. The pressure coming between vehicles affects their speeds and following distances [1]. In congested traffic, drivers might have to slow down and speed up more often, and staying well behind the vehicle in front of you is especially important in order to prevent accidents. Local conditions of high traffic flow can impact the overall traffic flow and vice versa [Delgado, Rodriguez and Chavez, 2021]. It may lead to pockets of congestion, where vehicles move at lower speeds and occasionally in stop-and-go patterns. This lateral pressure between vehicles can lead to a creation of traffic waves - fluctuations in vehicles' density and speed. As one vehicle slows-down the effect is passed backwards through the line of traffic causing periodic variations in

speed. Such data is vital for road planners and engineers to handle the burden of traffic. In an effort to address traffic, the traffic stakeholders often use and utilize mathematical models incorporating elements of fluid dynamics such as pressure to optimize traffic flow and in the design of road systems to reduce congestion and maximise efficiency.

Viscosity ν characterizes the internal friction or resistance to flow within a fluid (Khan and Alzahrani, 2020). It is a measure of how thick or sticky a liquid is. This property determines how easily molecules within the fluid can move past each other. High viscosity implies greater resistance to flow, making the fluid thicker and less prone to rapid movement. Traffic flow is analogous to viscosity as the resistance that vehicles experience when navigating the road. Various factors, such as road surface conditions, the density of vehicles, and the overall state of traffic influence this resistance. Viscosity in fluids results in resistance to flow. Vehicles on the road encounter resistance to their movement (Sener and Aksu, 2022). This resistance is particularly noticeable in congested or densely populated traffic conditions, where vehicles may experience delays, reduced speeds, and challenges in freely navigating the traffic stream. Similar to how high viscosity in fluids corresponds to a more crowded and restricted flow, higher traffic density can lead to increased vehicle resistance (Suzuki and Marumo, 2018). As the number of vehicles on a stretch of road rises, each vehicle must contend with limited space and reduced maneuverability, contributing to greater resistance. Higher resistance can result in lower speeds and more cautious driving behaviors (Stogios et al., 2019). It may also affect the ease with which vehicles can change lanes or make turns.

1.2.1 Navier-Stokes Equations in Traffic Flow

The Navier-Stokes equations are foundational in fluid dynamics, representing the conservation of momentum for a viscous fluid (Łukaszewicz and Kalita, 2016). In order to understand these concepts, let's consider the form of the Navier-Stokes equations in fluid dynamics:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} + \mathbf{f}, \quad (1.3)$$

where \mathbf{v} is the velocity field, ρ is the fluid density, p is the pressure, ν is the kinematic viscosity, and \mathbf{f} represents external forces. The following adaptations are made in order to make equations suitable in traffic flow. The velocity field \mathbf{v} is replaced with the traffic velocity v along the direction of traffic flow (assumed to be 1D for simplicity). The density ρ is analogous to the traffic density $\rho(x, t)$, representing the number of vehicles per unit length. The pressure p can represent the internal forces due to density variations, influencing speed adjustments as drivers attempt to maintain safe distances. Thus, the adapted form of the Navier-Stokes equation for traffic flow can be written as:

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 v}{\partial x^2} + f(x, t), \quad (1.4)$$

where $f(x, t)$ may represent external influences on traffic, such as signals, road conditions, or sudden braking. The advection-diffusion equation describes how a quantity is transported through a medium, with advection accounting for the bulk movement and diffusion representing the spread. For traffic flow, the advection-diffusion equation can describe how the density $\rho(x, t)$ of vehicles changes over time:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = D \frac{\partial^2 \rho}{\partial x^2} + S(x, t), \quad (1.5)$$

where D is the diffusion coefficient, representing the dispersion effect due to the variability in drivers' speeds and interactions. $S(x, t)$ is a source term, representing sources or sinks in the vehicle density, such as entry and exit ramps.

Advection captures the motion of vehicles along a road, while diffusion accounts for fluctuations in vehicle concentration due to changes in speed or lane usage (Zhu et al., 2018). The term $\frac{\partial}{\partial x}(\rho v)$ represents the advection of vehicles, whereas the term $D \frac{\partial^2 \rho}{\partial x^2}$ models the diffusive spread of density due to random driver behaviors and other factors.

1.2.1.1 Continuity Equation

The continuity equation is a fundamental concept in both fluid dynamics and traffic flow, representing the conservation of mass (or vehicles) within a defined segment of a road (Delle Monache, 2014). The equation expresses the fact that mass cannot be created or destroyed within a control volume, ensuring the continuity of flow. Similarly, in traffic flow, the continuity equation ensures that the number of vehicles in a segment changes only due to the flow of vehicles entering and exiting that segment. The general form of the continuity equation is:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1.6)$$

where ρ represents the density of the fluid, and \mathbf{v} is the velocity vector. This equation states that the rate of change of density ρ within a given volume, combined with the divergence of the mass flux $\rho \mathbf{v}$, must be zero. For traffic flow, a 1-dimensional model was considered where vehicles move along a single lane. $\rho(x, t)$ represents the traffic density (vehicles per unit length), and $v(x, t)$ denotes the traffic velocity along the x -axis. In this case, the continuity equation reduces to:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0. \quad (1.7)$$

The term $\frac{\partial \rho}{\partial t}$ represents the temporal rate of change of traffic density at a given location x . The term $\frac{\partial}{\partial x}(\rho v)$ represents the spatial rate of change of vehicle flow, where ρv is the flow rate or flux, indicating the number of vehicles passing through a point per unit time. $\frac{\partial}{\partial x}(\rho v)$ can be expanded using the product rule to obtain

$$\frac{\partial}{\partial x}(\rho v) = \rho \frac{\partial v}{\partial x} + v \frac{\partial \rho}{\partial x}, \quad (1.8)$$

which highlights two contributions: changes in velocity with respect to position and changes in density with respect to position. The continuity equation thus provides a dynamic picture

of traffic flow, showing how density variations propagate through changes in velocity and density gradients. It is a foundational equation for any traffic flow model, as it captures how vehicle density evolves in response to vehicle flow, congestion, and external influences.

1.2.2 Momentum Equation

In fluid flow, the momentum equation is expressed by the Navier-Stokes equation (1.3). For traffic, Eq. (1.4) was adapted, accounting for internal and external forces influencing traffic flow.

1.2.3 The 2D Advection-Diffusion Equation in Traffic Flow

The Advection-Diffusion Equation (ADE) is a mathematical framework used to describe the transport and dispersion of traffic density over time and space (Kundu, 2018). Originally rooted in fluid dynamics, the ADE has been adapted to simulate the movement of vehicle density across a two-dimensional (2D) space, such as a network of roads or urban areas with multi-lane and multi-directional flows. The equation integrates advection, which models the directed flow of traffic, and diffusion, which captures the spreading of traffic density due to fluctuations in vehicle speed, lane changing, and external perturbations. The general form of the 2D ADE is given by (Szymkiewicz and Gkasiorski, 2021):

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = D \nabla^2 \rho + S, \quad (1.9)$$

where $\rho(x, y, t)$ represents the traffic density (vehicles per unit area) as a function of the spatial coordinates x and y and time t , $\mathbf{u} = (u, v)$ is the advection velocity vector, with components $u(x, y, t)$ and $v(x, y, t)$ denoting vehicle speeds in the x - and y -directions, respectively, D is the diffusion coefficient, which quantifies the rate at which traffic density spreads across the area due to stochastic fluctuations, $S(x, y, t)$ represents a source or sink term that can model the addition or removal of vehicles due to external factors such as entry/exit ramps, intersections, or traffic signals.

The advection term, $\mathbf{u} \cdot \nabla \rho$, captures the directional flow of vehicles, where the density of traffic is transported according to the velocity vector \mathbf{u} . In 2D traffic models, this term is essential for representing vehicle movement along the x - and y -axes simultaneously. For example, in urban traffic settings, advection reflects vehicles moving along a road network with different velocities influenced by factors such as road capacity, traffic lights, and speed limits (Szymkiewicz and Gkasiowski, 2021). The term $\mathbf{u} \cdot \nabla \rho$ is expanded as:

$$\mathbf{u} \cdot \nabla \rho = u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y}, \quad (1.10)$$

which demonstrates how density changes in each spatial direction due to advection.

The diffusion term, $D \nabla^2 \rho$, represents the spreading of vehicle density due to random variations in traffic conditions. Diffusion models the dispersal of traffic density, accounting for the effect of lane changes, minor speed variations, and external disruptions, which tend to smoothen high-density regions. The Laplacian operator $\nabla^2 \rho$ in 2D is defined as:

$$\nabla^2 \rho = \frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2}, \quad (1.11)$$

which captures how density gradients in both directions influence the spreading of traffic. A higher diffusion coefficient D leads to more rapid dispersion, reducing peak densities in congested areas.

The source term $S(x, y, t)$ allows the 2D ADE to accommodate additional complexities in traffic flow, such as entry and exit points, or incidents that alter vehicle density. A positive S indicates a source (e.g., vehicles entering from an on-ramp), while a negative S indicates a sink (e.g., vehicles leaving at an off-ramp). By varying S across the traffic network, researchers can model realistic conditions like intersections, bottlenecks, and disruptions that influence the overall flow dynamics.

Numerous studies have employed the 2D Advection-Diffusion Equation to model complex traffic patterns in urban environments (Chaari et al., 2024; Kanda et al., 2013; Matin et al.,

2023; Mollier et al., 2018). For instance, research on multi-lane roadways and intersections uses the ADE to simulate congestion and flow distribution across networked roads (Roy et al., 2019). The ADE's adaptability allows it to model high-density scenarios like rush hour, where both advection (vehicle movement along main directions) and diffusion (due to lane changing and speed variations) play significant roles. Additionally, the ADE is instrumental in modeling pedestrian and mixed-traffic flows, where interactions across x - and y -axes need simultaneous consideration.

The 2D Advection-Diffusion Equation offers a versatile framework for understanding and simulating traffic flow in complex networks. By integrating both advection and diffusion processes, the equation provides a realistic model that accounts for both directed and random influences on vehicle density. The inclusion of source terms further enhances its applicability in real-world scenarios, enabling researchers to represent entry and exit points within traffic models. The ADE thus plays a critical role in modern traffic flow analysis, offering insights for urban planning, congestion management, and transportation policy.

1.2.4 Applications of Navier-Stokes and Advection-Diffusion Equation in Traffic Flow

The Navier-Stokes and advection-diffusion equations help model various aspects of traffic flow:

1.2.4.1 Congestion Modeling

In traffic flow studies, congestion refers to the phenomenon where high vehicle density causes reduced speeds, leading to periodic stop-and-go waves (Suh and Yeo, 2016). These waves are typically observed in heavy traffic conditions, where vehicles move in short spurts followed by abrupt halts. The Navier-Stokes equation, adapted from fluid dynamics, is employed in traffic modeling to better capture the complex interactions of congestion. By treating high-density traffic zones as areas of increased "pressure" (analogous to pressure in fluid flow), the adapted Navier-Stokes equation can accurately simulate the characteristics of stop-and-

go waves.

In this adapted model, vehicle density ρ functions as a pressure-like term, influencing vehicle velocity \mathbf{v} . The modified Navier-Stokes equation in one dimension can be expressed as:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \frac{\partial \mathbf{v}}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 \mathbf{v}}{\partial x^2}, \quad (1.12)$$

where $p = f(\rho)$ represents a pressure term dependent on traffic density, modeling the buildup of congestion, and ν is the viscosity term, capturing the tendency for vehicles to adjust their speeds due to nearby traffic.

As traffic density ρ increases, the pressure p also increases, creating resistance to flow. This higher “pressure” leads to decelerations, contributing to congestion waves that propagate through the traffic. When density reduces, vehicles accelerate again, producing a wave-like pattern. This cycle of deceleration and acceleration is analogous to pressure waves in fluids.

In two-dimensional models, this approach is extended by using a velocity vector $\mathbf{v} = (u, v)$ to simulate traffic in both x - and y -directions (Liu, 2019). The 2D Navier-Stokes equation becomes:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v}. \quad (1.13)$$

This model enables a more detailed analysis of urban congestion, particularly at intersections or multi-lane roads, where stop-and-go waves are prevalent. Using the Navier-Stokes framework, traffic engineers can simulate congestion dynamics, assess roadway capacities, and propose interventions to reduce delays in high-density traffic networks.

1.2.4.2 Traffic Stability Analysis

Traffic stability analysis is crucial in understanding how small disturbances in traffic flow can either dissipate over time or escalate into larger disruptions, potentially leading to traffic jams (Tao et al., 2024). Advection-diffusion models provide a valuable framework for this type of analysis by incorporating the effects of both advection (movement of vehicles) and

diffusion (spread of disturbances). In traffic flow, advection represents the propagation of the average velocity of vehicles along a road, while diffusion describes how fluctuations or minor disturbances spread through the traffic flow.

In these models, a small disturbance, such as a vehicle suddenly braking, creates a localized disruption in the traffic density and velocity (Chowdhury et al., 2000). This disruption propagates backward through the flow as other vehicles adjust their speed in response. The advection-diffusion model assesses whether this disturbance diminishes as it travels along the traffic or if it amplifies (Bellomo and Dogbe, 2011). Stability in traffic flow implies that small disturbances tend to dissipate over time, allowing the system to return to its previous, steady-state condition. On the other hand, if the disturbance grows as it propagates, it may lead to instability, potentially triggering stop-and-go waves or even full-scale traffic jams.

The stability analysis of traffic flow is often conducted mathematically by linearizing the traffic model around a steady state and examining the resulting equations (Sun et al., 2018). The behavior of these equations, particularly the sign and magnitude of eigenvalues in the solution, indicates the stability of the traffic flow. Positive eigenvalues suggest that disturbances will grow, leading to unstable conditions, while negative eigenvalues imply stability.

By incorporating parameters like vehicle speed, density, and reaction times, advection-diffusion models help predict the conditions under which traffic flow remains stable or unstable (Bellomo and Dogbe, 2011). This analysis is critical for traffic management and control strategies aimed at reducing congestion and improving overall flow efficiency. For instance, adaptive speed limits and coordinated braking could be implemented based on insights from traffic stability analysis to prevent minor disturbances from causing larger disruptions.

1.2.4.3 Propagation of Traffic Waves

The propagation of traffic waves is a fundamental concept in traffic flow theory, describing how fluctuations in vehicle density and speed spread through a stream of traffic (Gazis, 1972). Mathematical models, particularly those based on partial differential equations (PDEs), help

simulate these density waves, providing insights into various phenomena, such as traffic shockwaves.

Traffic waves typically form when there is a sudden change in speed among vehicles—often caused by a vehicle braking sharply or encountering an obstacle (Perumal et al., 2021). This abrupt change in speed leads to a localized increase in vehicle density, which travels backward through the flow of traffic as each following driver reduces speed to avoid a collision. These backward-propagating disturbances, or shockwaves, illustrate how traffic behaves similarly to a fluid, where compression waves emerge due to disruptions.

In mathematical models, these shockwaves are represented through advection-diffusion equations, where advection accounts for the directed movement of vehicles and diffusion captures the spread of fluctuations across the traffic flow (Chock, 1991). More sophisticated models, such as the Lighthill-Whitham-Richards (LWR) model, treat traffic as a continuous flow. They analyze conditions under which density waves either dissipate or remain persistent based on factors like traffic density, flow rates, and vehicle speed.

The mathematical analysis of shockwaves often involves examining changes in traffic density over time and space (Saxena and Jain, 2017). For instance, when the density reaches a critical threshold, small disturbances can amplify and sustain themselves, leading to persistent traffic waves. Conversely, if the density is low enough, disturbances dissipate quickly, restoring smoother flow. These models provide valuable insights for traffic management systems. By simulating how density waves propagate under various conditions, engineers can design interventions, such as variable speed limits or adaptive braking systems, to reduce the likelihood of shockwaves escalating into traffic jams (Yuan and Zeng, 2024). Such applications aim to mitigate the impact of traffic waves, enhancing flow stability and reducing congestion, especially during peak hours or in high-density traffic regions.

1.2.4.4 Effect of Road Conditions

The effect of road conditions on traffic flow can be effectively studied by modifying specific terms in advection-diffusion models, particularly the diffusion term D and external forces $f(x, t)$ (Pérez et al., 2021). In traffic modeling, these terms represent the spread of disturbances and the impact of external influences, respectively. By adjusting these components, models can simulate diverse real-world conditions, including road quality, lane changes, and intersections, which significantly affect traffic patterns.

The diffusion term D in the model represents how disturbances (like sudden braking or acceleration) spread through traffic. Higher values of D imply greater "smoothing" effects, where fluctuations in vehicle density or speed quickly dissipate across the flow (Mohammadian et al., 2023). This term is sensitive to factors such as road surface quality or weather conditions. For instance, wet or icy roads reduce traction, increasing the likelihood of erratic vehicle behavior and reducing the natural diffusion of disturbances, as drivers tend to brake more cautiously. By adjusting D to reflect these conditions, models can simulate how poor road conditions might lead to more persistent traffic waves or delays.

The external force term $f(x, t)$, on the other hand, represents location-specific factors or events affecting traffic, such as intersections, traffic signals, or on-ramps (Rios-Torres and Malikopoulos, 2016). At intersections or lane changes, vehicles must often decelerate or accelerate, creating local disturbances. By modeling $f(x, t)$ as a periodic function (e.g., to represent traffic lights) or a spatially varying factor (e.g., representing a road narrowing), the influence of these factors were considered during the formation and dissipation of traffic waves. At intersections, for example, stopping and starting cycles can create recurring density waves that propagate upstream, impacting traffic flow beyond the immediate area.

Incorporating these road conditions into traffic models allows transportation planners to predict how traffic flow will respond to specific roadway designs or conditions (Boukerche et al., 2020). It enables the design of interventions like optimized signal timings, lane configurations, and road maintenance strategies to minimize traffic disruptions. Ultimately, these

models contribute to better traffic management by anticipating and mitigating conditions that could lead to congestion or instability in traffic flow.

1.2.5 Navier-Stokes Equation Analog for Traffic

We derive a fluid dynamics equation representing traffic flow dynamics, incorporating acceleration and deceleration terms.

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} + \mathbf{F}_{\text{obstacle}} \quad (1.14)$$

where \mathbf{v} is the velocity vector in 2D (v_x, v_y) , $-\frac{1}{\rho} \nabla p$ is the pressure gradient force contribution to the motion of the fluid, ∇ is the del operator, p is the pressure term, ν is the kinematic viscosity, ∇^2 is the Laplacian operator, $\mathbf{F}_{\text{obstacle}}$ represents external forces. In this study the external force as a vehicle package within the round-about due to mechanical failure was considered. This external force is considered repulsive and increases as vehicles get closer to obstacles. This can be modelled as

$$\mathbf{F}_{\text{obstacle}} = k_{\text{obs}} \cdot \frac{\mathbf{r}}{\|\mathbf{r}\|} \quad (1.15)$$

Therefore, the study model equation can be written based on (1.14)-(1.15) as

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} + k_{\text{obs}} \frac{\mathbf{r}}{\|\mathbf{r}\|} \quad (1.16)$$

The description of parameters in Eq. (1.15) and Eq. (1.16) are described in Table 1.2.

1.2.6 Modified Advection-Diffusion Equation for Traffic Flow

Modeling the flow of traffic in a roundabout, especially under conditions like accidents or traffic jams, involves adapting the advection-diffusion equation. These modifications should consider the unique characteristics of traffic flow in a roundabout, including the circular motion, interactions between vehicles, and the potential for disruptions such as accidents or

Table 1.1: Parameter descriptions for (1.16)

Symbol	Description	Units
$\mathbf{F}_{\text{obstacle}}$	Obstacle force vector	N (newton)
k_{obs}	Obstacle force constant	N/m
\mathbf{r}	Vector from vehicle to obstacle	m
\mathbf{v}	Velocity vector in 2D (v_x, v_y)	m/s
∇	Del operator / Gradient	m^{-1}
p	Pressure term	N/m^2 (Pa)
ν	Kinematic viscosity	m^2/s
∇^2	Laplacian operator	m^{-2}
\mathbf{F}	External force vector	N (newton)
t	time	sec

congestion. The advection-diffusion equation describes how a quantity (in this case, traffic density or flow) is advected (transported) and diffused (spread out) in a medium. In the context of traffic flow in a roundabout, this equation can be modified to account for circular motion and potential disruptions as in Eq. (1.17):

$$\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho = D \nabla^2 \rho + S \quad (1.17)$$

The description of parameters are described in Table 1.2.

Table 1.2: Parameters description for (1.17)

Symbol	Description
ρ	traffic density.
\mathbf{v}	Velocity vector, accounting for the circular flow in the roundabout.
D	Diffusion coefficient, representing the spreading or dispersion of traffic.
S	Any additional terms accounting for disruptions, such as accidents or traffic jams.

1.2.7 Proposed Navier-Advection diffusion equation for modelling traffic flow

Combining the Navier-Stokes Equation (1.16)) and the modified advection-diffusion Equation (1.17)) into a single equation requires integrating both equations and incorporating relevant terms. To simplify, let's assume the 2D scenario mentioned earlier. The combined equation can be expressed as follows:

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho &= D \nabla^2 \rho + S \\ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} &= -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} + k_{\text{obs}} \frac{\mathbf{r}}{\|\mathbf{r}\|},\end{aligned}\tag{1.18}$$

Equation (1.18) captures the interaction of traffic density, velocity, diffusion, external forces, and obstacle forces. The specific values of parameters and terms might will need further calibration to suit the traffic flow modeling.

1.2.8 Basic Assumptions

Several assumptions were made to simplify the mathematical representation and facilitate the simulation process, as outlined below:

- (i) Steady-state conditions were assumed, particularly in the analysis of the long-term behavior of traffic flow. This implied a constant average flow rate over time.
- (ii) Traffic was considered homogeneous, meaning all vehicles were assumed to be similar in size, speed, and behavior. This simplification enabled a more generalized representation.
- (iii) Traffic flow was primarily modeled in one dimension along the road axis. This simplification was appropriate for analyzing traffic behavior on straight-road segments and reduced the complexity of the governing equations. However, in this study, a two-dimensional (2D) approach was adopted to better capture the dynamics of traffic flow at roundabouts. This study models a single-lane modern roundabout, commonly applied in suburban arterial intersections.
- (iv) The road geometry, apart from the roundabout itself, was assumed to remain constant. Variations in curvature or slope along the studied segment were considered negligible.

1.3 Problem statement

The design and implementation of roundabouts in urban traffic management are crucial components for enhancing road safety, improving traffic efficiency, and minimizing congestion. Understanding the dynamics of traffic flow before and after roundabouts is essential for effective transportation planning and infrastructure development.

Traditional traffic models often face limitations in representing the dispersion and movement of traffic density under various conditions (Pinto et al., 2020; Ravish and Swamy, 2021). The use of advection-diffusion equation offers platform for addressing these challenges by describing the transport and spreading of traffic density. Additionally, the increasing complexity of urban road networks and the growing demand for efficient traffic management necessitate a detailed understanding of traffic dynamics before and after the implementation of roundabouts. Traditional traffic models often fall short in capturing the intricate interactions and the impact of geometric changes, such as the introduction of roundabouts, on traffic flow (Campi et al., 2023; Demir and Demir, 2020; Zhou et al., 2021; Žura, 2022). This study aims to address these limitations by employing advanced mathematical models based on the Navier-Stokes and advection-diffusion equations to simulate and analyze traffic flow dynamics in the vicinity of roundabouts.

Traffic flow modeling, particularly in roundabouts, requires a thorough understanding of fluid dynamics, mainly through applying Navier-Stokes and advection-diffusion equations (Albi et al., 2019). These equations, fundamental in fluid mechanics, can be used in traffic flows to predict patterns and behaviors resulting from changes in traffic infrastructure, such as roundabouts. However, the complexity of traffic flow, characterized by nonlinear and chaotic behaviors, poses significant challenges to directly applying these equations. Empirical validation and calibration of these models are crucial to ensure their relevance and applicability to real-world scenarios. Given the critical role of roundabouts in urban planning, there is a need for more research in this area to develop more sophisticated models that can predict traffic flow changes with higher accuracy. This research contributes to traffic engineering by

providing a methodological framework for fluid dynamics in traffic flow modeling.

1.4 Objectives

1.4.1 Main Objective

The main objective of the study is to model traffic flow before and after roundabout using Navier-stokes and advection-diffusion equations.

1.4.2 Specific Objectives

The specific objectives of the study were to:

- (i) To formulate equations to model traffic dynamics around roundabouts.
- (ii) To investigate the specific influence of roundabouts on traffic flow patterns and vehicle velocity.
- (iii) To evaluate the influence of disruption and diffusion on traffic flow patterns on a roundabout.

1.5 Significance of the study

This study aims to critically assess the effectiveness of roundabouts as pivotal traffic control measures, offering a valuable contribution to optimizing vehicle movement and alleviating congestion. The research will delve into the intricate dynamics of traffic flow by employing sophisticated mathematical models based on the Navier-Stokes and advection-diffusion equations. This exploration, explicitly considering the influence of roundabouts, holds the potential to advance mathematical models associated with traffic flow modeling. The study will provide crucial insights for urban planners and traffic engineers, shedding light on the implications of integrating roundabouts into road network design. This information will play

a key role in facilitating informed decision-making for upcoming infrastructure projects, contributing to developing safer and more efficient road networks.

A comprehensive assessment of the impact of roundabouts on the safety and efficiency of traffic flow will be undertaken. This evaluation is crucial in addressing the need for a balanced approach in urban transportation planning, where safety and efficiency are paramount considerations. The research will also delve into the adaptability of the proposed mathematical model under various conditions. This includes exploring its responsiveness to different roundabout designs, varying traffic volumes, and environmental factors. The study aims to provide a versatile and applicable model that can be effectively employed in diverse real-world scenarios, ensuring its relevance and utility for transportation planning professionals.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

The chapter presents beginning with theoretical review followed by empirical review. The empirical review contain existing studies on modelling traffic flow via fluid mechanics concepts. Subsequently, summary of the literature gap is presented.

2.2 Theoretical review

The use of Navier-Stokes and advection-diffusion equations in modeling traffic flow before and after a roundabout is complex and specialized (Woodward et al., 2022). The governing equations are included in the numerical fluid dynamics (CFD) analysis, which is applied to the analysis of fluid flows, including traffic flow. The Navier-Stokes equations are essential in the modeling of fluid substances, and the advection-diffusion equations are employed in the transport of a quantity by the flow of a fluid substance [1], [2]. These equations are adequate to simulate the motion of vehicles in a traffic flow. However, the specific use of these equations to traffic flow around a roundabout would involve a detailed knowledge of the roundabout and traffic dynamics and the physical characteristics of such a facility. The search results give hints on the use of Navier-Stokes and advection-diffusion equations in various fluid flows applications such as urban air pollution dispersion and two-phase flows. Management of the traffic flow is one of the major characteristics of urban planning and transportation engineering which plays a vital role in order to make the road network safe, effective, and sustainable (Haque et al., 2013). This includes incorporating roundabouts, circular

intersections that can be used to control traffic and allow vehicles to move more smoothly. The modeling of traffic before and after the implementation of roundabouts becomes a critical methodology to gain an understanding of the impact of such geometric changes on the vehicle behavior and on traffic flow overall.

Finally, roundabouts, which have proven to be an effective way to reduce the amount of traffic congestion and the chances of serious accidents compared to signalized intersections, have emerged as one of the focus points of modern traffic management (for a review on 40 years of research on roundabouts see Aziz et al. (2018)). Besides their physical changes, these changes also entail new rules and new factors for drivers to take into account at the intersections. Traffic modeling serves as a powerful tool for predicting, analyzing, and optimizing vehicle behavior on road networks (Alghamdi et al., 2022). Through mathematical equations and simulations, traffic engineers can gain insights into how changes, such as roundabout introductions, influence parameters like traffic density, vehicle speed, and overall flow efficiency.

Studying traffic dynamics before and after roundabout installation enables a comparative analysis of the road network's performance (Demir and Demir, 2020). This approach identifies potential bottlenecks, improvements in traffic flow, and changes in driver behavior resulting from the roundabout's presence. Key considerations in modeling include assessing traffic density and flow patterns to understand vehicle movement through the network, accounting for roundabout geometry, analyzing driver behavior changes, and evaluating safety implications, including accident frequency and severity.

2.3 Empirical Review

The current state of traffic before and after a roundabout shows significant improvements in traffic flow and safety in the advent of new road technologies (Deluka Tibljaš et al., 2018). Roundabouts are designed to promote a continuous, circular flow of traffic, reducing delays and congestion on approaching roads (Davidović et al., 2021). Mohebifard and Hajbabaie

(2021); Zakeri and Choupani (2021) noted that increasing the number of lanes in and out of roundabout increases, the average delay was reduced from 28 seconds before the roundabouts to 13 seconds after, with a maximum delay reduced from 118 seconds to 40 seconds. Additionally, traffic volumes increased, but the number of annual crashes dropped significantly, and injury crashes were reduced as lanes in roundabouts were increased. However, these observations have not been supported by mathematical modelling concepts due to a lack of such studies depicting similar scenarios.

Damaskou et al. (2022) posits that the effectiveness of a roundabout can be impacted by factors such as the design of the roundabout, driver confusion, and the presence of traffic light intersections too close to the roundabout exits. Dangers and traffic jams arise because of the distraction from the flow of traffic caused by the driver attempting to understand the signal that will control the traffic circle or because of break in continuity of the traffic circle. It is important that roundabouts are designed correctly and that drivers understand how to use them to gain the most from them and ensure safety. In order to prove that the well-designed circles can really improve traffic flow, decrease the delays and also make robust enough environment for the traffic, the proposed study consists of modeling the traffic flow before and after the circle. However, other factors of signalised intersections that affect their effectiveness, such as driver behavior and the configuration of surrounding intersections have usually been marginalised in studies (cheah2022modeling; kabanga2022evaluating; kalavsova2020comparison). The proposed study introduces such factors as external forces and assesses the impact of the model. In their work, Shao et al. (2023) derived a two-lane traffic model of complete RNN, which considers the mass conservation law and acceleration equation. The fundamental traffic flow characteristics on two lanes were examined through the elementary waves derived successfully. The Riemann problem in the 1-D case was solved which gave a constructive understanding of the traffic wave behaviour. A Godunov-type numerical scheme with exact Riemann solver was developed. Basically, the study was concerned with the one-dimensional representation, but it was recognized by the authors that they could further increase the ap-

plicability of the method to the multi-dimensional situation according to real road networks. The study did not simulate the reality of traffic flow, but did have to include more problems such as different traffic conditions and driver characteristics. However, it is still possible to extend further the understanding of the applicability of the model in a wide variety of traffic conditions, taking into account the road geometries, intersections and including realistic driver behaviour aspects.

Kang et al. (2022) highlighted the inadequacy of existing traffic flow models, often relying on constant coefficient differential models, which exhibit poor adaptability to dynamic changes. To address this, the study simplified and refined a viscoelastic traffic flow model, introducing a fractional viscoelastic traffic flow model. Incorporating the Bass model's principles and leveraging fractional calculus in viscoelastic fluids, the study introduced conformable fractional derivatives and the fractional grey model, resulting in a fractional grey viscoelastic traffic flow model capturing time-varying characteristics. While the study successfully surpassed traditional models' limitations, gaps exist. It focused on the model introduction and application without addressing real-world complexities like varying road conditions and driver behaviours. A literature gap emerges, urging a deeper understanding of traffic flow models' interaction with real-world scenarios, considering factors such as road geometries, intersections, and diverse driver behaviours for enhanced practical applicability and adaptability.

Sun et al. (2020) introduced a car-following model incorporating driver memory and derived a corresponding macroscopic traffic flow model, preventing the wrong-way travel phenomenon. Driver memory, considering past traffic conditions and headway, introduced a viscosity parameter in the macroscopic traffic flow equation, proportional to a unique quantity defined by delay time and kinematic wave velocity at jam density. Conducting linear and nonlinear stability analyses using the perturbation method, the study found that macroscopic models derived from microscopic models, with driver memory and viscosity parameters, are more realistic than those directly based on Navier–Stokes equations. However, lacking em-

pirical validation and practical applications with real-world traffic data, the study's utility is limited. A literature gap exists, urging further empirical validation and diverse application scenarios to comprehensively understand the proposed models' practicality and limitations in real-world traffic situations.

Cheah and Yeak (2022) modelled roundabouts, starting with a three-arm roundabout validated against an existing model and expanding to a four-arm roundabout. The study emphasized the modern intelligent transportation system's need for effectiveness in handling non-linear, time-varying, and congested traffic flow. The roundabout was modelled as a 2x2 junction circuit with a main and secondary lane, rotating clockwise. Mathematical modelling employed one-dimensional hyperbolic conservation laws represented by non-linear partial differential equations. The study utilized the Godunov method and Courant-Friedrichs-Levy condition for computation, stability, and accuracy. Optimization for Total Travel Time and Total Waiting Time with various parameters provided results for roundabout assessment. Comparisons with existing models showed higher but reasonable readings, suggesting the model's flexibility and realism. The study, while comprehensive, lacks specific details on real-world validation and application scenarios. A literature gap exists in exploring the model's adaptability to diverse traffic conditions and geometries.

Kabanga et al. (2022) addressed the critical role of roundabouts in the traffic network and emphasizes the need for effective management to improve traffic flow. The study is dedicated to mathematically modelling traffic dynamics at circular roundabouts, conducted in two parts. Firstly, a model for a circular roundabout with four entry and exit lanes was developed, analyzing its performance under various factor levels. Secondly, parameters enhancing roundabout efficiency, such as the roundabout radius (r), turning movement gradient θ , and the number of connected lanes (n), were explored using a fluid flow analogy. The effective roundabout design, crucial for maximum vehicle throughput capacities, was evaluated. Numerical analysis in MATLAB with periodic boundary conditions provided insights into the parameters' effects on traffic velocities. While the research contributes valuable information

for road engineers, it lacks specific details on real-world validation and broader application scenarios. A literature gap exists in exploring the model's adaptability to diverse traffic conditions and geometries.

Storani et al. (2021) compared a hybrid traffic flow model, combining the macroscopic cell transmission model with the microscopic cellular automata, with benchmark macroscopic and microscopic models. Three applications were considered: a link with a signalised junction, a signalised artery, and a grid network with signalised junctions. The hybrid model was compared to the Krauß model, intelligent driver model, cellular automata, Cell Transmission Model, and Cell Transmission Model with dispersion. Numerical simulations indicated that the hybrid model produced acceptable results, showing behaviour similar to microscopic models, particularly in terms of travel times. However, it exhibited lower values of queue propagation compared to microscopic models, aligning more closely with the enhanced macroscopic cell transmission model and the cell transmission model with dispersion. The study validates the model by analysing wave propagation at the boundary region. Despite the contributions, the research lacks a detailed exploration of real-world applications and a comprehensive literature review on hybrid traffic flow models. A literature gap exists in understanding the adaptability of the hybrid model to diverse traffic scenarios and the practical implications of its results in real-world traffic management.

Zhang et al. (2018) developed an urgent-gentle class traffic flow model (UGM) with viscoelastic and ramp effects to estimate travel time on a ring road. The traffic flow is categorized into urgent and gentle classes based on the urgency of reaching the destination. The urgent class has time-sensitive demands, while the gentle class doesn't. For the sake of simplicity, the proposed model assumes that both classes have the same instantaneous speed. Validation is performed in the form of the Navier-Stokes-like model but extended for this purpose. Numerical simulations on an 80 km ring road with initially assumed jams show the important influence of on/off ramp flows on traffic flow patterns, as in case of initially assumed jams. The behavior of the urgent density fraction is seen to be different from that of traffic den-

sity and speed in that, during propagation, it moves at the local traffic speed. The average travel time is found out to be more as the initial ring road density is increased, this brings out the importance of rational management of road functioning. Despite these contributions, the research lacks an investigation of real-world applications and an extensive literature review about traffic flow models with urgent-gentle class distinctions. A literature gap exists regarding the understanding of the practical implications of the model proposed with respect to different traffic scenarios and the potential to be applied in traffic management.

Xue et al. (2019) introduced a full angular velocity difference model, considering angular velocity and displacement on curved roads. It deduces the transformation relation from microscopic angular variables to a macroscopic continuum traffic flow model on curved roads. The critical condition for steady traffic flow is obtained, and the stability condition is compared with microscopic and lattice hydrodynamic traffic models. The KdV–Burgers equation describing density waves near the neutral stability line is derived through nonlinear analysis. Numerical simulations validate the macroscopic traffic model, showing that the radius of curved roads significantly influences traffic jam formation. Traffic flow instability leads to local clustering effects and stop&go traffic jams. This paper notes that the unstable region shrinks with the strength of angular velocity difference. But there are no practical applications, real-life verifications, and comparisons with other models of traffic flows in the research. There is a literature gap in the comprehension of the wider implications of the proposed model and its practical application in the different traffic conditions and road geometries.

The article by Chiotis and Kolky (2019numerical) is dedicated to the modern problem of the distribution of transport flows due to the increase in the number of vehicles. Macroscopic (hydrodynamic) models, which are in existence since the 50s, describe the transport flow as a compressible liquid, which can be modeled by the Navier-Stokes equation. Hydrodynamic models are suggested by scholars, combining the transport flow and incompressible fluid dynamics. Oskolkov equation on a geometric graph taking length and width of the edges is used

to model a traffic flow. The values are considered to be dimensionless but it is advantageous to present them in a linear metric scale and the count of lanes across the traffic. The Oskolkov model leads to a non-classical multipoint initial-final value condition, which has been studied by the theory of Sobolev equations. Nevertheless, the study requires additional practical implementation or testing in real-life traffic conditions. There is a literature gap regarding the wider implications and utility of the Oskolkov model when it is applied in a different range of traffic conditions, road geometries and how it compares with other traffic flow models.

The vehicle traffic, which is akin to self-propelled particles, was examined using a generalized car-following model with multiple look-ahead by the author of the article by Qiu et al. (2013). A second-order nonlocal hydrodynamic model was constructed by assuming no skewness in velocity distribution and using an iterative procedure. Unlike two-phase fluid-dynamic models with a fundamental diagram, this model features microscopically determined relaxation time parameters. Although the rigour is slightly reduced compared to previously studied Navier–Stokes-like traffic flow models, the model successfully reproduces the phase transition from free flow to synchronized flow and from synchronized flow to wide-moving jam. It also uncovers the catch effect of synchronized flow. Simulations indicate that nonlocality in relaxation time and steady velocity, even without nonlocality in viscosity (velocity variance), provides an alternative explanation for synchronized flow. However, the study lacks empirical validation in real-world traffic scenarios, leaving a literature gap in understanding the practical implications and limitations of the proposed model.

2.4 Summary of Literature and Gap

Despite advancements in traffic flow modeling—such as multi-lane and fractional-order models, recent studies (e.g., Shao (2023), Kang et al. (2022), Sun et al. (2020)) often lack empirical validation and practical application under real-world conditions. Models focused on roundabouts (Cheah and Yeak (2022), Kabanga et al. (2022)) provide useful simulations but fall short in capturing complex geometries and dynamic traffic behaviors. This consis-

tent absence of real-world adaptability, especially for non-linear flow conditions in roundabouts, highlights a persistent literature gap. To address this, the present study proposes a fluid dynamics-based approach, leveraging Navier-Stokes and advection-diffusion formulations—to simulate traffic flow more realistically. This enhances predictive accuracy and supports informed decision-making in traffic management and infrastructure design under varied operational and environmental conditions. Summary of literature gap is presented in Table 2.1.

Table 2.1: Summary of Literature Review

Author	Research Findings	Literature Gap
Deluka Tibljaš et al. (2018)	New road technologies and roundabout designs improve traffic flow and safety and reduce delays and congestion.	Lack of mathematical modeling to support findings on roundabouts' effectiveness in improving traffic flow.
Davidović et al. (2021)	Roundabouts facilitate continuous, circular traffic flow, reducing delays on approaching roads.	Need for sensitivity analysis to identify parameters that enhance roundabouts' traffic management capabilities.
Mohebifard and Hajbabaie (2021); Zakeri and Choupani (2021)	Increasing lanes in roundabouts significantly reduces average delays and maximum delays; traffic volume increased while annual crashes decreased.	No mathematical model demonstrating roundabouts' influence on traffic patterns and vehicle speed with lane changes.

Author	Research Findings	Literature Gap
Damaskou et al. (2022)	Roundabout effectiveness is affected by design, driver confusion, and nearby intersections; disruptions can lead to congestion and accidents.	Lack of sensitivity analysis for identifying factors in roundabout design that impact flow stability and safety.
Cheah and Yeak (2022); Kabanga et al. (2022); Kalašová et al. (2020)	Proposed studies aim to model traffic flow before and after a roundabout, including driver behavior and nearby intersections.	Requires sensitivity analysis on how factors like driver behavior impact roundabout effectiveness in managing flow.
Shao (2023)	Developed a two-lane traffic model capturing elementary waves to understand traffic wave behavior, primarily in 1-D scenarios.	Model lacks application to roundabout-specific dynamics and multi-dimensional traffic flow interactions.
Kang et al. (2022)	Introduced a fractional viscoelastic traffic model improving adaptability to time-varying conditions using fractional calculus.	Limited applicability to real-world roundabout dynamics and sensitivity to key parameters for roundabout flow.
Kabanga et al. (2022)	Explored roundabout dynamics with four entry/exit lanes; evaluated parameters like radius, gradient, and lane count for effective design.	Requires sensitivity analysis on key parameters affecting traffic efficiency and safety in roundabout scenarios.

Author	Research Findings	Literature Gap
Storani et al. (2021)	Developed a hybrid traffic flow model combining macroscopic and microscopic features; showed effectiveness in queue propagation and travel times.	Requires exploration of adaptability and application to complex roundabout scenarios with real-world data.
Zhang et al. (2018)	Proposed a two-class model for urgent and gentle traffic flow, evaluating time-sensitive dynamics and travel time impact on ring roads.	Lacks application to roundabout flow management and does not assess key roundabout parameters on traffic patterns.
Xue et al. (2019)	Introduced angular velocity difference model for curved roads, deriving stability and density wave conditions. Validated via simulations.	Model not applied to roundabouts; lacks analysis of specific roundabout geometries and their impact on traffic flow.
Konkina (2019)	Utilized hydrodynamic models with Oskolkov equation on geometric graphs for traffic flow modeling, relating flow to incompressible fluid dynamics.	No exploration of application to roundabouts and requires sensitivity analysis for critical roundabout parameters.
Qiu (2013)	Developed a generalized car-following model with nonlocal hydrodynamic characteristics, capturing transitions in traffic flow states.	Lacks empirical validation for roundabout scenarios and impact on flow under diverse roundabout configurations.

The study addressed a critical gap in existing traffic flow research, particularly the lack of realistic and adaptable models capable of simulating roundabout traffic systems under diverse

operating conditions. Previous models, including those by Shao (2023), Kang et al. (2022), and Sun et al. (2020), were largely limited to one-dimensional analyses and lacked empirical validation, making them inadequate for capturing the nonlinear and dynamic behaviors characteristic of complex intersections. Similarly, studies focusing on roundabouts, such as Cheah and Yeak (2022) and Kabanga et al. (2022), offered valuable insights through simulations but did not incorporate the multi-directional nature of traffic flow or the geometric intricacies of circular junctions. To bridge this gap, the present study employed a two-dimensional fluid dynamics-based approach using the Navier–Stokes and advection–diffusion equations to simulate realistic traffic behavior at roundabouts. This approach enhanced the representation of spatial interactions, lane-changing dynamics, and flow stability while maintaining adaptability to varying road, weather, and driver conditions. Consequently, the study contributed to bridging the divide between theoretical modeling and practical traffic management applications by providing a more comprehensive and realistic simulation framework for roundabout traffic flow analysis.

CHAPTER THREE

METHODOLOGY

In this chapter, the methodology of the research is discussed. The governing equations are formulated and derived. This study employed a two-dimensional

3.1 Finite Volume Method

Applying the QUICK scheme to solve the given system of Eq. (1.18) using the finite volume method (FVM) is performed by discretizing the equations in space using finite volumes and apply the QUICK scheme to the convective terms. The FVM combined with the QUICK scheme is particularly suitable for modeling traffic flow in complex geometries such as roundabouts. In such environments, sharp spatial gradients in vehicle density and velocity often occur due to merging, diverging, and circulating traffic patterns. The FVM ensures local conservation of mass and momentum by integrating the governing equations over control volumes, which is critical in traffic modeling where vehicle conservation is essential. Meanwhile, the QUICK scheme offers higher-order accuracy for convective terms while minimizing numerical diffusion, which is crucial for capturing sharp transitions in traffic density, such as the onset of congestion or clearing of jams. This combination provides a robust and accurate numerical framework capable of resolving the nonlinear and advective-dominated behavior characteristic of vehicular flow dynamics, especially in regions influenced by geometry-induced disruptions like roundabouts. Therefore, the approach enhances predictive reliability and supports better-informed infrastructure planning and traffic management decisions. Suppose a 2D grid with cell centers (i, j) and face velocities (u, v) is used (see Figure 3.1). The

cell size is $\Delta x = \Delta y = 1$. There is no strict restriction on (Δy) , but stability is improved for smaller step sizes. The condition $(\Delta x > \frac{1}{2})$ was derived based on CFL condition for stability. Boundary conditions were set as periodic for (ρ) , and inflow/outflow for (v) . Initial condition for (ρ) assumed uniform density at $(t = 0)$. The control volume averages are represented by $\rho_{i,j}$ and $\mathbf{v}_{i,j}$. Applying the QUICK scheme on the density equation yields:

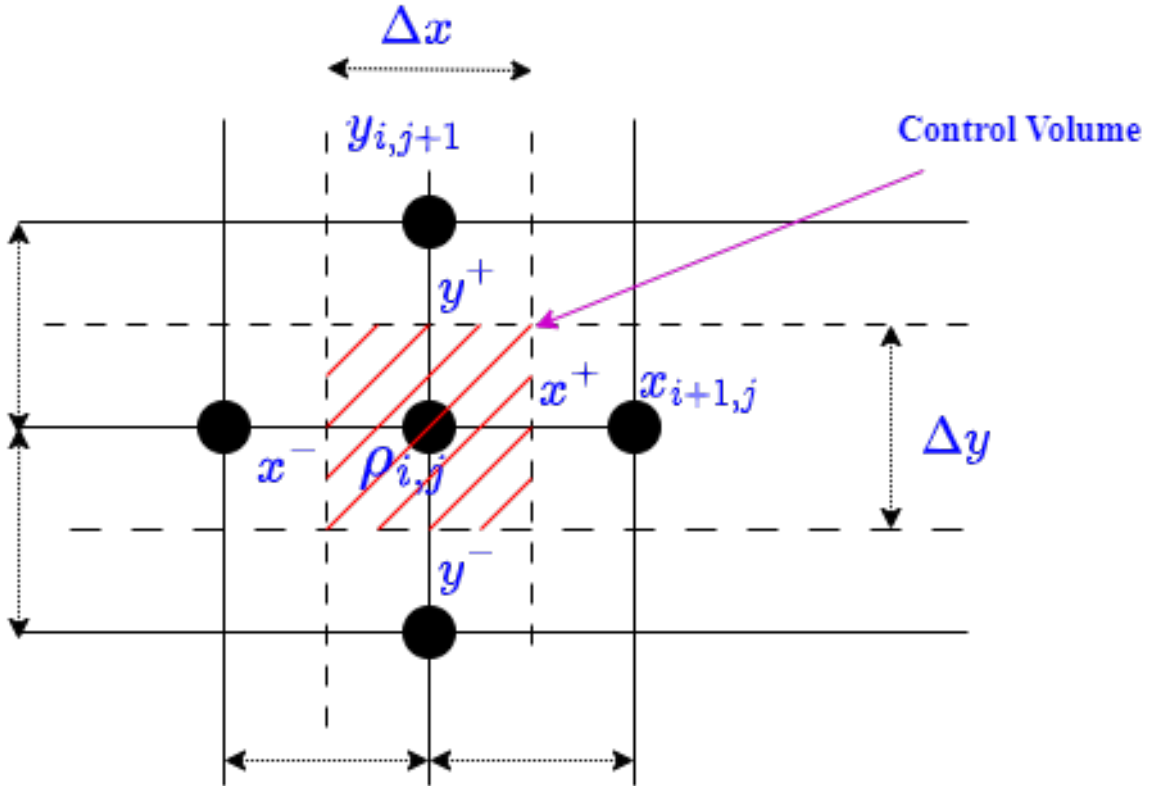


Figure 3.1: Control volume depicting the grid cells

$$\begin{aligned} \frac{\Delta \rho_{i,j}}{\Delta t} + \frac{1}{\Delta x} \left(\mathbf{v}_{i+\frac{1}{2},j} \rho_{i+\frac{1}{2},j} - \mathbf{v}_{i-\frac{1}{2},j} \rho_{i-\frac{1}{2},j} \right) \\ + \frac{1}{\Delta y} \left(\mathbf{v}_{i,j+\frac{1}{2}} \rho_{i,j+\frac{1}{2}} - \mathbf{v}_{i,j-\frac{1}{2}} \rho_{i,j-\frac{1}{2}} \right) = D \nabla^2 \rho_{i,j} + S_{i,j} \end{aligned} \quad (3.1)$$

The QUICK scheme is also applied to the convective term:

$$\begin{aligned}
& \frac{1}{\Delta x} \left(\mathbf{v}_{i+\frac{1}{2},j} \rho_{i+\frac{1}{2},j} - \mathbf{v}_{i-\frac{1}{2},j} \rho_{i-\frac{1}{2},j} \right) + \frac{1}{\Delta y} \left(\mathbf{v}_{i,j+\frac{1}{2}} \rho_{i,j+\frac{1}{2}} - \mathbf{v}_{i,j-\frac{1}{2}} \rho_{i,j-\frac{1}{2}} \right) \\
&= \frac{1}{\Delta x} \left(\frac{1}{2} (\mathbf{v}_{i,j} + \mathbf{v}_{i+1,j}) \rho_{i,j} + \frac{1}{2} (\mathbf{v}_{i-1,j} + \mathbf{v}_{i,j}) \rho_{i-1,j} + \frac{1}{6} (\nabla \rho_{i,j} - \nabla \rho_{i-1,j}) \right) \\
&+ \frac{1}{\Delta y} \left(\frac{1}{2} (\mathbf{v}_{i,j} + \mathbf{v}_{i,j+1}) \rho_{i,j} + \frac{1}{2} (\mathbf{v}_{i,j-1} + \mathbf{v}_{i,j}) \rho_{i,j-1} + \frac{1}{6} (\nabla \rho_{i,j} - \nabla \rho_{i,j-1}) \right)
\end{aligned} \tag{3.2}$$

The QUICK scheme is also applied to the velocity equation to yield:

$$\begin{aligned}
& \frac{\Delta \mathbf{v}_{i,j}}{\Delta t} + \frac{1}{\Delta x} \left((\mathbf{v}_{i+\frac{1}{2},j}^2 - \mathbf{v}_{i-\frac{1}{2},j}^2) + (\mathbf{v}_{i,j+\frac{1}{2}} \mathbf{v}_{i,j+\frac{1}{2}} - \mathbf{v}_{i,j-\frac{1}{2}} \mathbf{v}_{i,j-\frac{1}{2}}) \right) \\
&= -\frac{1}{\rho_{i,j}} (p_{i+1,j} - p_{i,j}) + \nu \nabla^2 \mathbf{v}_{i,j} + k_{\text{obs}} \frac{\mathbf{r}_{i,j}}{\|\mathbf{r}_{i,j}\|}
\end{aligned} \tag{3.3}$$

Applying the QUICK scheme to the convective term in the velocity to yield:

$$\begin{aligned}
& \frac{1}{\Delta x} \left((\mathbf{v}_{i+\frac{1}{2},j}^2 - \mathbf{v}_{i-\frac{1}{2},j}^2) + (\mathbf{v}_{i,j+\frac{1}{2}} \mathbf{v}_{i,j+\frac{1}{2}} - \mathbf{v}_{i,j-\frac{1}{2}} \mathbf{v}_{i,j-\frac{1}{2}}) \right) \\
&= \frac{1}{\Delta x} \left(\frac{1}{2} (\mathbf{v}_{i,j} + \mathbf{v}_{i+1,j})^2 - \frac{1}{2} (\mathbf{v}_{i-1,j} + \mathbf{v}_{i,j})^2 + \frac{1}{2} (\mathbf{v}_{i,j} + \mathbf{v}_{i,j+1}) (\mathbf{v}_{i,j} + \mathbf{v}_{i,j+1}) \right. \\
&\quad \left. - \frac{1}{2} (\mathbf{v}_{i,j-1} + \mathbf{v}_{i,j}) (\mathbf{v}_{i,j-1} + \mathbf{v}_{i,j}) + \frac{1}{6} (\nabla (\mathbf{v}_{i,j}^2) - \nabla (\mathbf{v}_{i-1,j}^2)) \right)
\end{aligned} \tag{3.4}$$

3.2 Crank-Nicolson for Time Step

Applying Crank-Nicolson for the time step in Equation (3.1) yields:

$$\begin{aligned}
& \frac{\rho_{i,j}^{n+1} - \rho_{i,j}^n}{\Delta t} + \frac{1}{2\Delta x} \left[\mathbf{v}_{i+\frac{1}{2},j}^n (\rho_{i+\frac{1}{2},j}^{n+1} + \rho_{i+\frac{1}{2},j}^n) - \mathbf{v}_{i-\frac{1}{2},j}^n (\rho_{i-\frac{1}{2},j}^{n+1} + \rho_{i-\frac{1}{2},j}^n) \right] \\
&+ \frac{1}{2\Delta y} \left[\mathbf{v}_{i,j+\frac{1}{2}}^n (\rho_{i,j+\frac{1}{2}}^{n+1} + \rho_{i,j+\frac{1}{2}}^n) - \mathbf{v}_{i,j-\frac{1}{2}}^n (\rho_{i,j-\frac{1}{2}}^{n+1} + \rho_{i,j-\frac{1}{2}}^n) \right] \\
&= \frac{1}{2} [D\nabla^2 \rho_{i,j}^{n+1} + D\nabla^2 \rho_{i,j}^n] + \frac{1}{2} [S_{i,j}^{n+1} + S_{i,j}^n]
\end{aligned} \tag{3.5}$$

Applying Crank-Nicolson for the time step in Equation (3.2) yields:

$$\begin{aligned}
& \frac{1}{\Delta x} \left(\mathbf{v}_{i+\frac{1}{2},j}^n \rho_{i+\frac{1}{2},j}^n - \mathbf{v}_{i-\frac{1}{2},j}^n \rho_{i-\frac{1}{2},j}^n \right) + \frac{1}{\Delta y} \left(\mathbf{v}_{i,j+\frac{1}{2}}^n \rho_{i,j+\frac{1}{2}}^n - \mathbf{v}_{i,j-\frac{1}{2}}^n \rho_{i,j-\frac{1}{2}}^n \right) \\
&= \frac{1}{2\Delta x} \left(\mathbf{v}_{i,j}^n (\rho_{i,j}^{n+1} + \rho_{i,j}^n) + \mathbf{v}_{i+1,j}^n (\rho_{i+1,j}^{n+1} + \rho_{i+1,j}^n) \right) - \frac{1}{2\Delta x} \left(\mathbf{v}_{i-1,j}^n (\rho_{i-1,j}^{n+1} + \rho_{i-1,j}^n) \right) \\
&+ \frac{1}{6\Delta x} \left(\nabla \rho_{i,j}^{n+1} - \nabla \rho_{i,j}^n \right) \\
&+ \frac{1}{2\Delta y} \left(\mathbf{v}_{i,j}^n (\rho_{i,j}^{n+1} + \rho_{i,j}^n) + \mathbf{v}_{i,j+1}^n (\rho_{i,j+1}^{n+1} + \rho_{i,j+1}^n) \right) \\
&- \frac{1}{2\Delta y} \left(\mathbf{v}_{i,j-1}^n (\rho_{i,j-1}^{n+1} + \rho_{i,j-1}^n) \right) + \frac{1}{6\Delta y} \left(\nabla \rho_{i,j}^{n+1} - \nabla \rho_{i,j}^n \right)
\end{aligned} \tag{3.6}$$

Applying Crank-Nicolson for the time step in Equation (3.3) yields:

$$\begin{aligned}
& \frac{\mathbf{v}_{i,j}^{n+1} - \mathbf{v}_{i,j}^n}{\Delta t} + \frac{1}{2\Delta x} \left[\left((\mathbf{v}_{i+\frac{1}{2},j}^n)^2 + (\mathbf{v}_{i+1,j}^n)^2 \right) - \left((\mathbf{v}_{i-\frac{1}{2},j}^n)^2 + (\mathbf{v}_{i,j}^n)^2 \right) \right] \\
&+ \frac{1}{2\Delta y} \left[\left((\mathbf{v}_{i,j+\frac{1}{2}}^n)^2 + (\mathbf{v}_{i,j+1}^n)^2 \right) - \left((\mathbf{v}_{i,j-\frac{1}{2}}^n)^2 + (\mathbf{v}_{i,j-1}^n)^2 \right) \right] \\
&= -\frac{1}{2\rho_{i,j}^n} \left[(p_{i+1,j}^{n+1} - p_{i,j}^{n+1}) + (p_{i+1,j}^n - p_{i,j}^n) \right] \\
&+ \frac{\nu}{2} \left(\nabla^2 \mathbf{v}_{i,j}^{n+1} + \nabla^2 \mathbf{v}_{i,j}^n \right) + \frac{k_{\text{obs}}}{2} \cdot \frac{\mathbf{r}_{i,j}^{n+1} + \mathbf{r}_{i,j}^n}{\|\mathbf{r}_{i,j}^n\|}
\end{aligned} \tag{3.7}$$

Applying Crank-Nicolson for the time step in Equation (3.4) yields:

$$\begin{aligned}
& \frac{1}{\Delta x} \left[\left(\mathbf{v}_{i+\frac{1}{2},j}^n \right)^2 - \left(\mathbf{v}_{i-\frac{1}{2},j}^n \right)^2 + \left(\mathbf{v}_{i,j+\frac{1}{2}}^n \right)^2 - \left(\mathbf{v}_{i,j-\frac{1}{2}}^n \right)^2 \right] \\
&= \frac{1}{2\Delta x} \left[\left(\mathbf{v}_{i,j}^{n+1} \right)^2 + \left(\mathbf{v}_{i+1,j}^{n+1} \right)^2 - \left(\mathbf{v}_{i-1,j}^{n+1} \right)^2 - \left(\mathbf{v}_{i,j}^{n+1} \right)^2 \right] \\
&+ \frac{1}{2\Delta y} \left[\left(\mathbf{v}_{i,j}^{n+1} \right)^2 + \left(\mathbf{v}_{i,j+1}^{n+1} \right)^2 - \left(\mathbf{v}_{i,j-1}^{n+1} \right)^2 - \left(\mathbf{v}_{i,j}^{n+1} \right)^2 \right] \\
&+ \frac{1}{6\Delta x} \left[\nabla \left(\mathbf{v}_{i,j}^{n+1} \right)^2 - \nabla \left(\mathbf{v}_{i-1,j}^n \right)^2 \right]
\end{aligned} \tag{3.8}$$

Equations (3.5)–(3.8) define the discretized form of the proposed Navier–Advection–Diffusion system using Crank–Nicolson temporal integration. This approach ensures second-order accuracy in time while maintaining numerical stability under moderate CFL conditions. The resulting scheme is suitable for simulating nonlinear traffic behavior in high-gradient environments like roundabouts.

3.3 Non-dimensionalization and Proposed Equation

Summing Equations (3.5)-(3.8) yields,

$$\begin{aligned}
& \frac{\rho_{i,j}^{n+1} - \rho_{i,j}^n}{\Delta t} + \frac{\mathbf{v}_{i,j}^{n+1} - \mathbf{v}_{i,j}^n}{\Delta t} \\
& + \frac{1}{2\Delta x} \left[\mathbf{v}_{i+\frac{1}{2},j} (\rho_{i+\frac{1}{2},j}^{n+1} + \rho_{i+\frac{1}{2},j}^n) - \mathbf{v}_{i-\frac{1}{2},j} (\rho_{i-\frac{1}{2},j}^{n+1} + \rho_{i-\frac{1}{2},j}^n) \right] \\
& + \frac{1}{2\Delta y} \left[\mathbf{v}_{i,j+\frac{1}{2}} (\rho_{i,j+\frac{1}{2}}^{n+1} + \rho_{i,j+\frac{1}{2}}^n) - \mathbf{v}_{i,j-\frac{1}{2}} (\rho_{i,j-\frac{1}{2}}^{n+1} + \rho_{i,j-\frac{1}{2}}^n) \right] \\
& + \frac{1}{2\Delta x} (\mathbf{v}_{i,j} (\rho_{i,j}^{n+1} + \rho_{i,j}^n) + \mathbf{v}_{i+1,j} (\rho_{i+1,j}^{n+1} + \rho_{i+1,j}^n) - \mathbf{v}_{i-1,j} (\rho_{i-1,j}^{n+1} + \rho_{i-1,j}^n)) \\
& + \frac{1}{2\Delta y} (\mathbf{v}_{i,j} (\rho_{i,j}^{n+1} + \rho_{i,j}^n) + \mathbf{v}_{i,j+1} (\rho_{i,j+1}^{n+1} + \rho_{i,j+1}^n) - \mathbf{v}_{i,j-1} (\rho_{i,j-1}^{n+1} + \rho_{i,j-1}^n)) \\
& + \frac{1}{\Delta x} \left((\mathbf{v}_{i+\frac{1}{2},j}^2 - \mathbf{v}_{i-\frac{1}{2},j}^2) + (\mathbf{v}_{i,j+\frac{1}{2}} \mathbf{v}_{i,j+\frac{1}{2}} - \mathbf{v}_{i,j-\frac{1}{2}} \mathbf{v}_{i,j-\frac{1}{2}}) \right) \\
& = \frac{1}{2} [D\nabla^2 \rho_{i,j}^{n+1} + D\nabla^2 \rho_{i,j}^n] + \frac{1}{2} [S_{i,j}^{n+1} + S_{i,j}^n] - \frac{1}{2\rho_{i,j}} (p_{i+1,j} - p_{i,j} + p_{i+1,j-1} - p_{i,j-1}) \\
& + \frac{\nu}{2\Delta x} (\nabla^2 \mathbf{v}_{i,j}^{n+1} + \nabla^2 \mathbf{v}_{i,j}^n) + \frac{k_{\text{obs}}}{2} \frac{\mathbf{r}_{i,j}^{n+1} + \mathbf{r}_{i,j}^n}{\|\mathbf{r}_{i,j}\|} + \frac{1}{6\Delta x} (\nabla \rho_{i,j}^{n+1} - \nabla \rho_{i,j}^n) \\
& + \frac{1}{6\Delta y} (\nabla \rho_{i,j}^{n+1} - \nabla \rho_{i,j-1}^n).
\end{aligned} \tag{3.9}$$

The non-dimensionalization of variables is presented in Table 3.1

Table 3.1: Non-dimensionalization of variables

Variable	Non-dimensionalized form
Density	$\rho = \rho_0 \tilde{\rho}$
Velocity	$\mathbf{v} = U \tilde{\mathbf{v}}$
Length	$x = L \tilde{x}, y = L \tilde{y}$
Time	$t = \frac{L}{U} \tilde{t}$
Pressure	$p = \rho_0 U^2 \tilde{p}$
Reynolds number	$\text{Re} = \frac{UL}{\nu}$
Strouhal number	$\text{St} = \frac{U}{S_0 L}$

Substituting the variables in Table 3.1 in Equation (3.9) yields,

$$\begin{aligned}
& \frac{\tilde{\rho}_{i,j}^{n+1} - \tilde{\rho}_{i,j}^n}{\Delta \tilde{t}} + \frac{\tilde{\mathbf{v}}_{i,j}^{n+1} - \tilde{\mathbf{v}}_{i,j}^n}{\Delta \tilde{t}} \\
& + \frac{1}{2\Delta \tilde{x}} \left[\tilde{\mathbf{v}}_{i+\frac{1}{2},j} (\tilde{\rho}_{i+\frac{1}{2},j}^{n+1} + \tilde{\rho}_{i+\frac{1}{2},j}^n) - \tilde{\mathbf{v}}_{i-\frac{1}{2},j} (\tilde{\rho}_{i-\frac{1}{2},j}^{n+1} + \tilde{\rho}_{i-\frac{1}{2},j}^n) \right] \\
& + \frac{1}{2\Delta \tilde{y}} \left[\tilde{\mathbf{v}}_{i,j+\frac{1}{2}} (\tilde{\rho}_{i,j+\frac{1}{2}}^{n+1} + \tilde{\rho}_{i,j+\frac{1}{2}}^n) - \tilde{\mathbf{v}}_{i,j-\frac{1}{2}} (\tilde{\rho}_{i,j-\frac{1}{2}}^{n+1} + \tilde{\rho}_{i,j-\frac{1}{2}}^n) \right] \\
& = \frac{1}{2\text{Re}} (\nabla^2 \tilde{\mathbf{v}}_{i,j}^{n+1} + \nabla^2 \tilde{\mathbf{v}}_{i,j}^n) + \frac{\text{St}}{2} (\tilde{S}_{i,j}^{n+1} + \tilde{S}_{i,j}^n) - \frac{1}{2\tilde{\rho}_{i,j}} (\tilde{p}_{i+1,j} - \tilde{p}_{i,j}) \\
& + \frac{k_{\text{obs}}}{2} \frac{\tilde{\mathbf{r}}_{i,j}^{n+1} + \tilde{\mathbf{r}}_{i,j}^n}{\|\tilde{\mathbf{r}}_{i,j}\|} + \frac{1}{6\Delta \tilde{x}} (\nabla \tilde{\rho}_{i,j}^{n+1} - \nabla \tilde{\rho}_{i,j}^n) + \frac{1}{6\Delta \tilde{y}} (\nabla \tilde{\rho}_{i,j}^{n+1} - \nabla \tilde{\rho}_{i,j-1}^n).
\end{aligned} \tag{3.10}$$

.

Equations (3.10) is the proposed model equations for predicting the traffic flow before and after roundabout. These equations are subjected to the following boundary conditions:

$$\begin{aligned}
v(x, y, t) & > 0 \\
v(x, y, 0) & = 0 \\
\Delta x & > \frac{1}{2}
\end{aligned} \tag{3.11}$$

The boundary conditions for (ρ) was assumed to be no-flux at the roundabout exits. The initial condition for (ρ) was a uniform distribution across the domain.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

The study has formulated equations to model traffic dynamics around roundabouts, thus, Chapter 4 examines the influence of roundabouts on traffic performance, focusing on traffic flow patterns, vehicle speed, and congestion levels based on Equation (3.10) and Table 4.1. Values in Table Table 4.1 were assumed based on typical lane capacity limits used in urban modeling. It begins by analyzing how roundabouts impact traffic flow, emphasizing their role in facilitating smoother and more continuous movement compared to conventional intersections. The discussion then shifts to vehicle speed, exploring how roundabout design elements, such as geometry and lane configurations, regulate speed, balancing efficiency and safety. The chapter also evaluates the effects of roundabouts on congestion, highlighting their capacity to reduce bottlenecks and enhance traffic throughput in busy areas. A sensitivity analysis identifies key factors such as traffic volume and driver behavior, offering insights into optimizing roundabout performance for improved urban traffic management. Later sections of the chapter discusses the results linking to the existing studies.

Table 4.1: Optimal Values of Parameters used

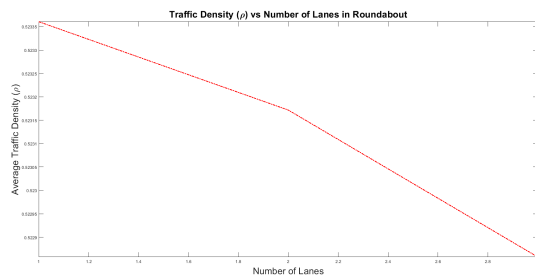
Parameter	Description	Value Range	Value used	Source
D	Diffusion coefficient	-	0.1	(Azam et al., 2014)
dt	Time step size	-	0.01	Assumed
dx	Spatial x-step size $> 1/2$	-	1	Assumed
dy	Spatial y-step size $> 1/2$	-	1	Assumed
t_{final}	Simulation time	-	1	Assumed
S	Source term for disruptions	-	0.05	Assumed

4.2 Numerical Simulation

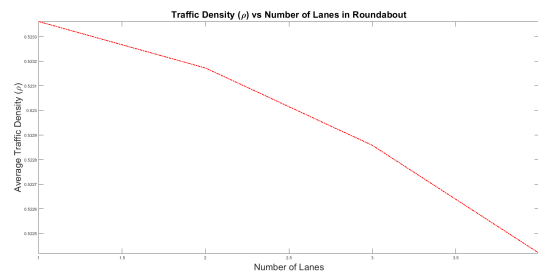
4.2.1 Traffic flow patterns and Vehicle Speed

4.2.1.1 Traffic Density against Number of Lanes

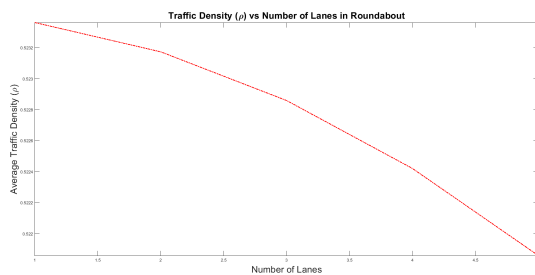
The study investigate the relationship between traffic density and the number of lanes in roundabout. The results are presented in Figure 4.1.



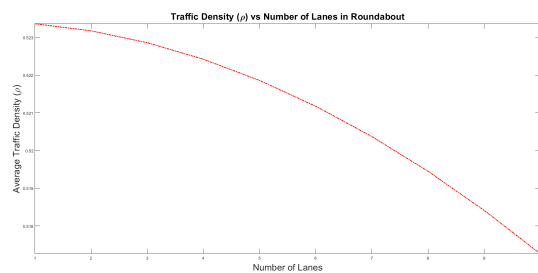
(a) Traffic density against 3 lanes in a roundabout



(b) Traffic density against 4 lanes in a roundabout



(c) Traffic density against 5 lanes in a roundabout



(d) Traffic density against 10 lanes in a roundabout

Figure 4.1: Traffic density against number of lanes in a roundabout

Figure 4.1 illustrate the relationship between traffic density (ρ) and the number of lanes in roundabouts, with specific cases shown for 3 (Figure 4.1a), 4(Figure 4.1b), 5(Figure 4.1c), and 10 (Figure 4.1d) lanes. Across all figures, a consistent decrease in traffic density is observed as the number of lanes increases, indicating that adding lanes reduces congestion within the roundabout. However, the rate of decrease in traffic density diminishes as the number of lanes increases, suggesting diminishing returns in terms of congestion reduction. For instance, the change in traffic density from 1 to 3 lanes is more articulated than the change from 8 to 10 lanes, highlighting a potential point of diminishing returns.

In comparing the different lane configurations, the graph for 3 lanes (Figure 4.1a) shows a sharp decrease in traffic density initially, indicating significant benefits of adding lanes at lower lane counts. Similarly, for 4 lanes (Figure 4.1a), the pattern persists, although the curve becomes slightly less steep, reflecting reduced baseline congestion. For 5 lanes (Figure 4.1a), the curve flattens further, showing that the marginal benefits of additional lanes decrease as the lane count increases. Finally, for 10 lanes (Figure 4.1a), the curve is the flattest, demonstrating minimal impact of adding further lanes beyond a certain threshold. This trend can be attributed to the increased capacity of the roundabout with additional lanes, allowing more vehicles to pass through simultaneously and reducing congestion. However, the diminishing returns suggest that beyond a certain number of lanes (likely between 5–6 lanes based on Figure 4.1), the improvements in traffic flow become negligible. This could be due to factors such as geometric constraints of the roundabout.

These results imply that there may be an optimal number of lanes for roundabout design, balancing the benefits of reduced congestion with the costs of construction and maintenance. Figure 4.1 suggest that roundabouts with 4-6 lanes may provide the most efficient balance between improving traffic flow and maintaining manageable infrastructure. Beyond this range, the additional lanes may not justify the associated costs. Figure 4.1 demonstrates a negative correlation between the number of lanes in a roundabout and the average traffic density, with diminishing returns evident as the number of lanes increases. Although Figure 4.1 shows optimal range around 5–6 lanes, further increase offers diminishing returns due to geometric constraints

Figure 4.3 and Figure 4.2 show the relationship between traffic density (ρ) and the number of lanes in a roundabout, considering variations in the x -direction (denoted as i) (how traffic flow changes with lanes) and the y -direction (denoted as j) (how lanes are added geometrically to a roundabout). Figure 4.2 the traffic density decreases consistently as the number of lanes increases in the j -direction. This indicates that expanding the roundabout in the y -direction effectively reduces traffic congestion. However, the rate of decrease in density is non-linear,

ρ against Number of Lanes (N) and j^{th} direction

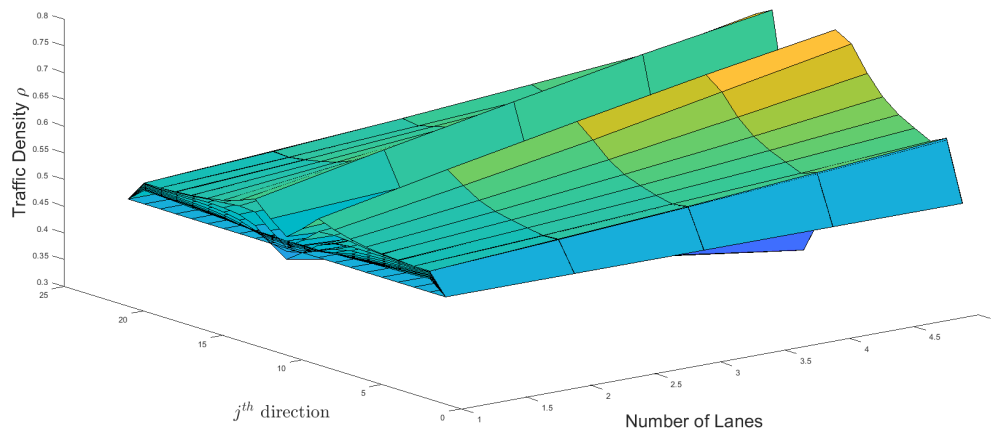


Figure 4.2: Variation of traffic density as number of lanes increases geometrically (j) to a roundabout.

suggesting diminishing returns beyond a certain threshold of lanes. For smaller values of j , the density is relatively higher, pointing to a more congested traffic scenario when the number of lanes is limited. As j increases, the surface flattens, indicating a point of saturation where further lane additions have minimal impact on reducing traffic density. This also indicate a more uniform flow.

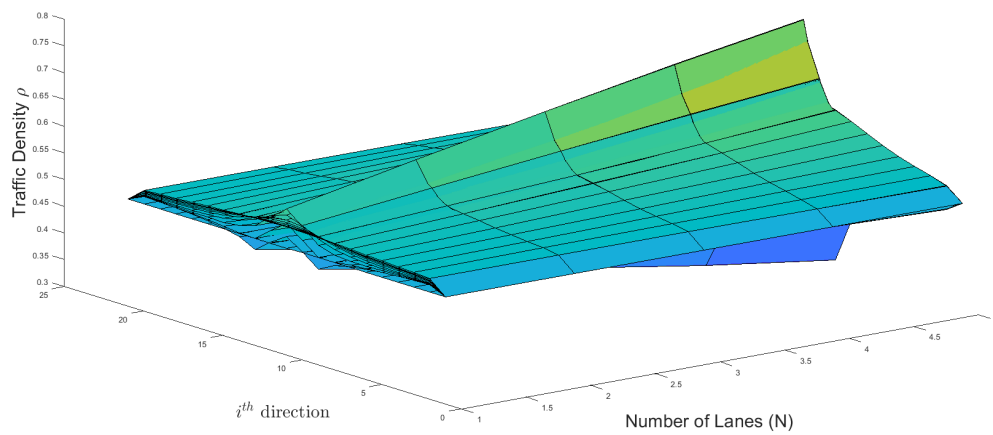
ρ vs Number of Lanes (N) in the i^{th} direction

Figure 4.3: Traffic density against number of lanes and how traffic flow changes with lanes (i) in a roundabout.

Figure 4.3 a similar pattern is observed, that is the traffic density decreases with an increasing number of lanes in the x -direction. The steep decline in ρ for lower values of i highlights the significant impact of initial lane expansions in reducing congestion. However, as the number of lanes increases, the slope of the surface becomes gentler, again emphasizing diminishing returns with higher lane counts. For large values of i , the traffic density stabilizes, indicating a limit to the improvements achievable through additional lanes.

Comparison of Figure 4.3 and Figure 4.2, it is evident that the trends are consistent across both the x - and y -directions. The symmetry suggests that the impact of lane additions is isotropic, meaning that expansions in either direction contribute similarly to reducing traffic density. This highlights the importance of balanced design in roundabout construction, where both the x - and y -directions are expanded to optimize traffic flow.

Figure 4.2 exhibits a prominent inverted wedge shape for moderate values of j . This region corresponds to an optimal balance between lane expansion and traffic density reduction, where congestion is effectively mitigated without over-expanding the roundabout. The sharp decline in density at lower values of j reflects the significant impact of initial lane additions.

However, as j increases beyond the wedge region, the surface flattens, indicating diminishing returns from additional lanes. This suggests that further expansion in the y -direction may not yield significant benefits in reducing traffic congestion. Figure 4.3 shows an inverted wedge-shaped region appearing for moderate values of i . This wedge signifies the optimal configuration where lane expansion in the x -direction achieves the greatest reduction in traffic density. Beyond this region, the slope of the surface decreases, showing that additional lanes provide limited improvements to traffic flow. The inverted wedge regions observed in both plots represent the most efficient configurations for lane expansion. In these regions, traffic density is minimized, and the benefits of adding lanes are maximized. Beyond the wedge-shaped regions, the diminishing returns indicate that over-expanding the roundabout in any direction would lead to unnecessary costs without substantial gains in reducing traffic congestion.

4.2.1.2 Traffic Density against Velocity

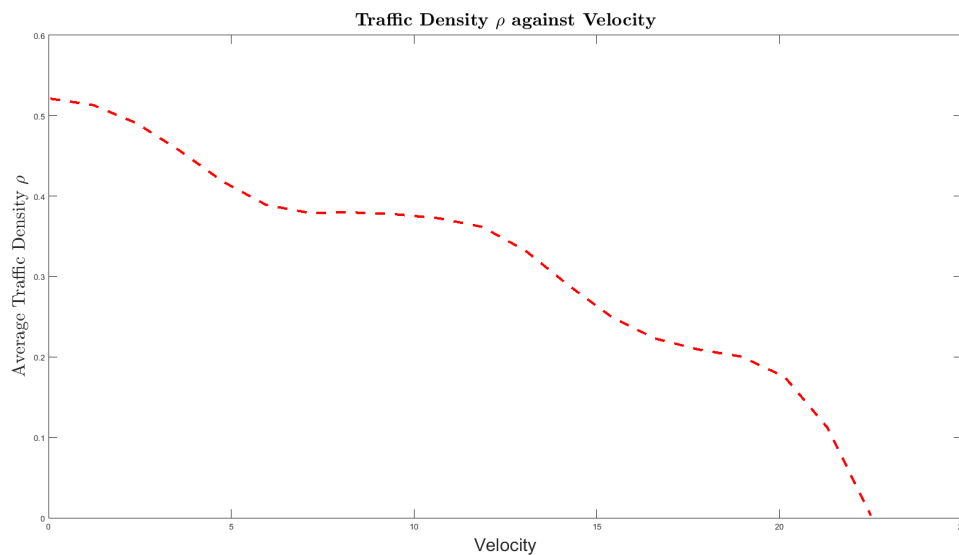


Figure 4.4: Traffic density against traffic velocity (m/s) in a roundabout.

Figure 4.4 represents the relationship between traffic density (ρ) and velocity (v) in a roundabout, revealing an inverse correlation between these variables. At low velocities ($v < 10$),

traffic density is high, indicating congestion where vehicles are closely packed, likely due to merging conflicts or insufficient gaps at the roundabout entrance. Conversely, at high velocities ($v > 15$), traffic density decreases significantly, representing free-flow conditions where vehicles move with minimal interaction. This behavior aligns with fundamental traffic flow theory, where increased speeds allow vehicles to spread out, reducing density.

The curve exhibits a nonlinear and continuous decline in density as velocity increases, with a critical transition zone observed between $v = 10$ and $v = 15$. In this region, the decrease in traffic density becomes clearer, indicating a shift from congested to free-flow traffic states. This transition likely reflects a critical velocity range where the roundabout's capacity to manage traffic shifts dynamically. The smooth nature of curve suggests that the relationship is governed by continuous traffic flow dynamics reminiscing that of Equation (3.10). The observations suggest that high traffic density and low velocity typically occur during congestion, potentially caused by high vehicle volumes or suboptimal roundabout geometry. On the other hand, low traffic density and high velocity correspond to efficient traffic flow, commonly observed during off-peak hours or with well-designed roundabout features. The findings imply that maintaining moderate velocities within the roundabout can help reduce congestion and improve overall flow efficiency.

4.2.1.3 Traffic Density against Velocity and Spatial Changes

Equation (3.10) is used to study the relationship between traffic density (ρ), velocity (v), and the variations in both spatial (Δx) and temporal (Δy) changes within a roundabout.

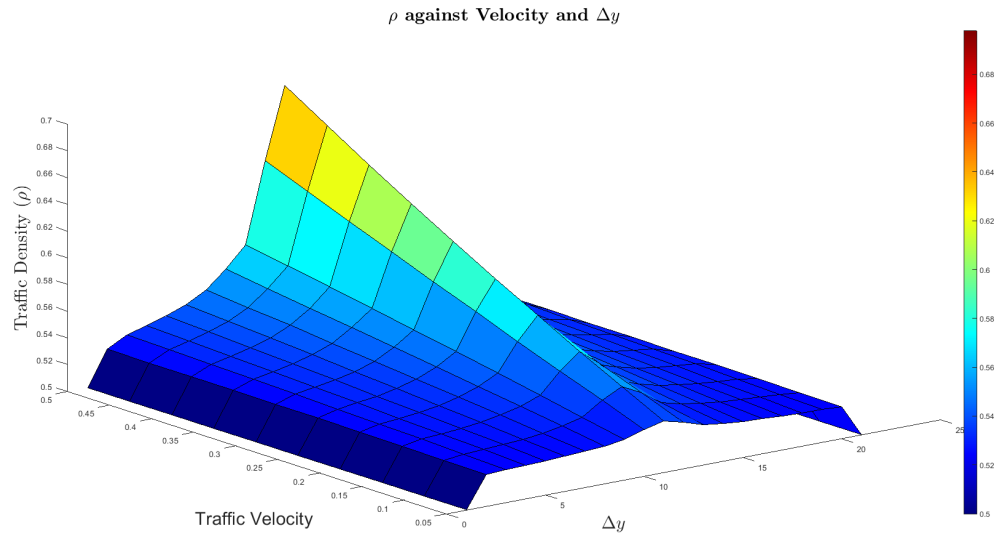


Figure 4.5: Traffic density against traffic velocity (m/s) and temporal change Δy in a roundabout.

Figure 4.5 explores the relationship between traffic density, velocity, and temporal change (Δy). It reveals that traffic density decreases significantly as Δy increases, indicating that larger time intervals between vehicles allow for smoother transitions and reduced congestion. At lower velocities, traffic density is highest when Δy is minimal, corresponding to closely packed vehicles entering or navigating the roundabout. As velocity increases, traffic density drops sharply, particularly at higher values of Δy . This suggests that vehicles with larger temporal gaps can maintain higher velocities while reducing density. However, at very low velocities combined with small Δy , congestion remains evident, highlighting the need for improved time-based coordination, such as adjusting entry times into the roundabout.

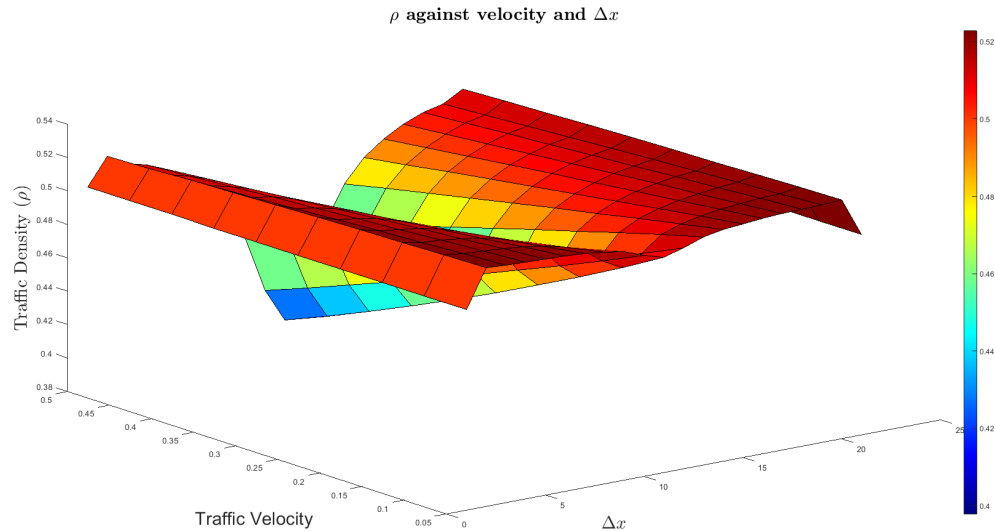


Figure 4.6: Traffic density against traffic velocity (m/s) and spatial change Δx in a roundabout.

Figure 4.6 shows traffic density as a function of velocity and spatial change (Δx). At lower velocities, traffic density (ρ) tends to be higher, especially when Δx is small. This behavior suggests that closely spaced vehicles result in greater congestion, as the spatial distance (Δx) between vehicles is reduced. As Δx increases, traffic density decreases gradually, reflecting improved spacing between vehicles that facilitates smoother flow. There is a pronounced decline in traffic density as velocity increases, regardless of Δx . This aligns with the expectation that higher velocities correlate with lower densities due to reduced interactions among vehicles. The gradual reduction of ρ with increasing Δx indicates that managing the spacing between vehicles in a roundabout is critical for alleviating congestion and maintaining efficient traffic flow.

Figures 4.5 and 4.6 reinforce the inverse relationship between traffic density and velocity, a fundamental principle of traffic dynamics. They also emphasize the importance of both spatial and temporal changes in mitigating congestion. Sufficient spacing between vehicles enhances the roundabout's efficiency by allowing for better maneuvering. Increasing time gaps between vehicles reduces the likelihood of congestion by minimizing vehicle interac-

tions. Figures 4.5 and 4.6 highlight that maintaining an optimal balance of velocity, spatial spacing, and temporal intervals is essential for efficient roundabout operations.

4.2.2 Traffic flow on Disruption and Diffusion

Figures 4.8 and 4.7 illustrate the relationship between velocity (V) and the number of lanes (N) in roundabouts, alongside two critical factors: disruption (S) and diffusion (D).

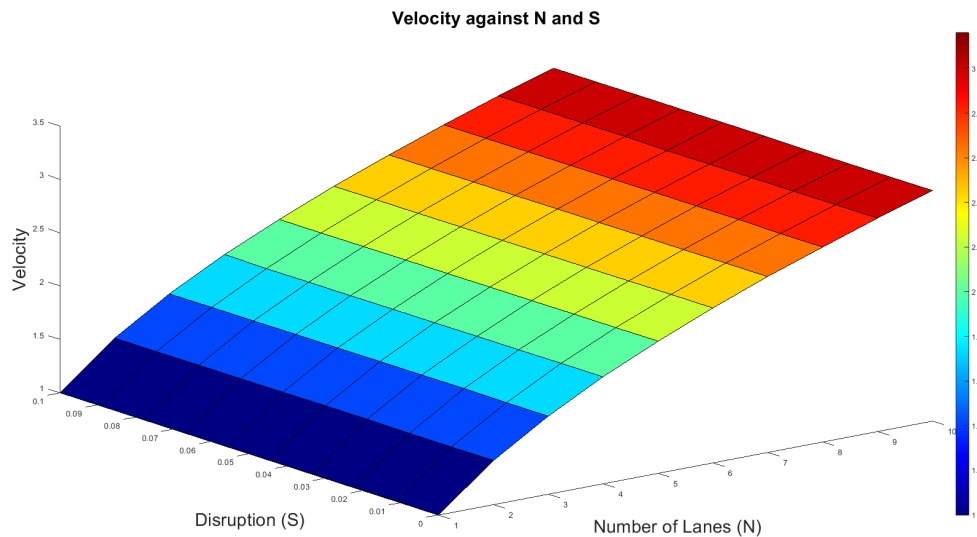


Figure 4.7: Traffic velocity (m/s) against number of lanes and traffic disruption (S).

Figure 4.7 which examines velocity as a function of the number of lanes and disruption, it is evident that velocity increases as the number of lanes rises. This trend suggests that more lanes provide greater capacity and maneuvering space, reducing congestion and enhancing traffic flow (see also Figure 4.1a). However, the level of disruption significantly impacts velocity. Higher disruption values, potentially caused by factors such as lane-changing conflicts, external interferences, or abrupt stops, lead to reduced velocities. Conversely, as disruption decreases, the velocity improves across all lane configurations. Thus, the importance of minimizing disruptions in roundabout design to maintain efficient traffic flow.

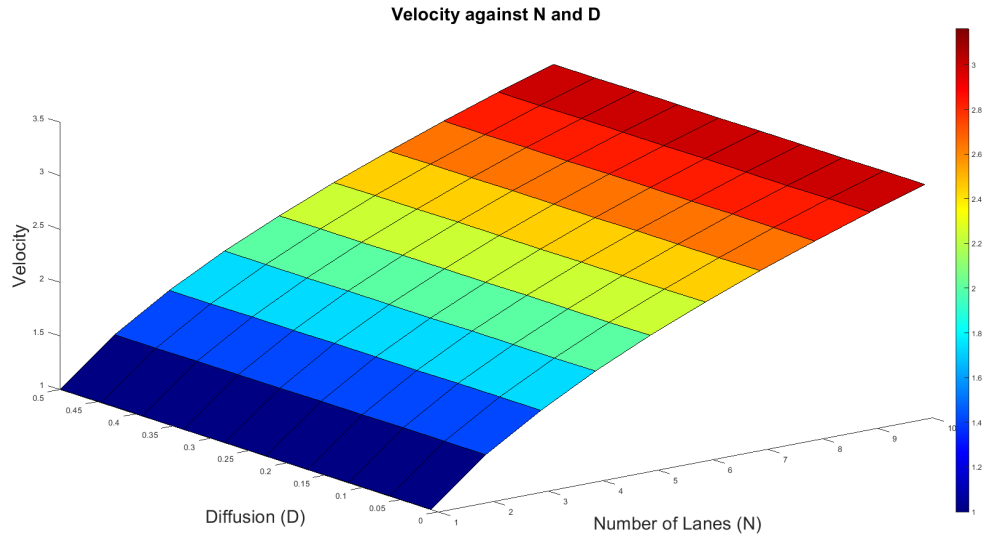


Figure 4.8: Traffic velocity (m/s) against number of lanes and traffic diffusion (D).

Figure 4.8 shows the relationship between velocity, the number of lanes, and diffusion. Similar to Figure 4.7, velocity increases with the number of lanes, indicating that higher lane capacity positively influences traffic movement. Diffusion, representing randomness in vehicle behavior or spacing, negatively impacts velocity. As diffusion increases, traffic becomes more dispersed or erratic, leading to reduced velocities. Lower diffusion values correspond to more orderly traffic patterns, which enhance flow efficiency. This suggests that managing driver behavior and promoting uniform lane usage through measures such as clear signage and lane guidance could mitigate the effects of diffusion. Figures 4.8 and 4.7 show that both disruption (S) and diffusion (D) are shown to negatively affect velocity. However, their impact diminishes as the number of lanes increases, emphasizing the importance of adequate lane capacity in roundabout design. To optimize traffic flow, it is essential to reduce both disruption and diffusion through thoughtful design strategies, while ensuring that roundabouts are equipped with sufficient lanes to handle varying traffic conditions effectively. Figure 4.9 illustrates the relationship between traffic density (ρ), velocity (V), and disruption (S) in a roundabout.

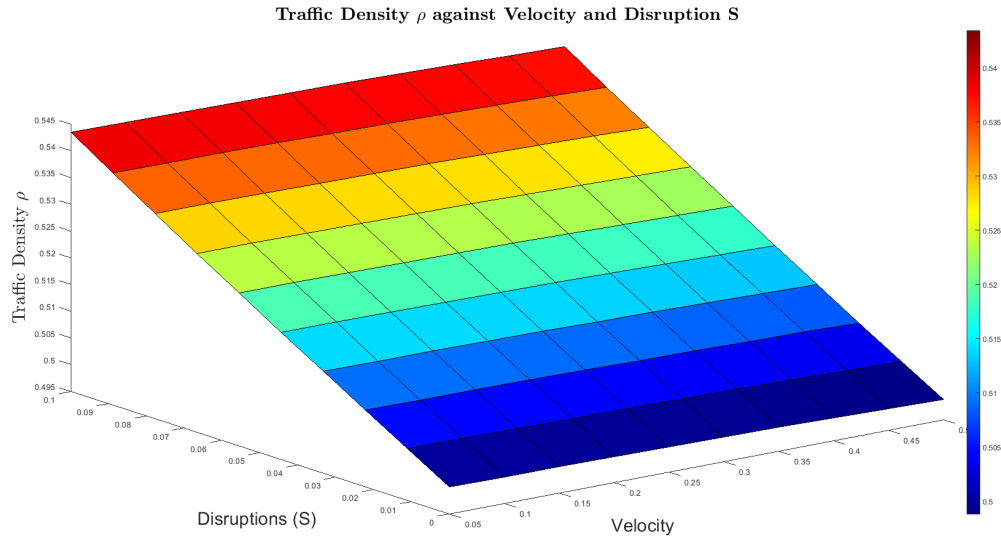


Figure 4.9: Traffic density against velocity and traffic disruption (S).

Figure 4.9 shows that traffic density decreases as velocity increases, indicating an inverse relationship. When vehicles move at higher speeds, they occupy less space per unit length of the roundabout, resulting in reduced density. Conversely, at lower velocities, vehicles are spaced closer together, leading to higher density. This trend suggests that maintaining optimal velocity is crucial for managing traffic density and ensuring smoother movement within the roundabout. Disruption (S), on the other hand, has a direct relationship with traffic density. Greater disruptions (because of conflicts in changing lanes, external disruptions, sudden braking, etc.), result in decreased vehicle velocity and higher traffic density. With disruptions growing, the vehicles are likely to occupy a small area which leads to congestion and reduces the efficiency of the roundabout. Conversely, at low levels of disruption, the level of traffic density will be manageable at different velocities, and it is important to minimize levels of disruption to enhance efficiency of traffic flow. The interplay between speed and disturbance on the traffic density is also visible. The level of disruption is relatively low at low levels, despite different velocities in the traffic. As the level of disruption increases however, there is a significant increase in traffic density particularly at low velocities. This implies that slow speed and high disorder aggravates congestion in the roundabout and it is

of utmost importance to consider both in ensuring that there is optimum traffic flow.

4.3 Discussion

4.3.1 Traffic Flow Patterns and Vehicle Speed

4.3.1.1 Traffic Density against Number of Lanes

The study investigated the relationship between traffic density (ρ) and the number of lanes in roundabouts. Figures 4.1, 4.3, and 4.2 illustrate a consistent trend: as the number of lanes increases, traffic density decreases. This aligns with Deluka Tibljaš et al. (2018) and Davidović et al. (2021), who emphasized the benefits of increased lane counts in reducing congestion at roundabouts. Similarly, Mohebifard and Hajbabaie (2021) and Zakeri and Choupani (2021) observed significant reductions in traffic delays and congestion with additional lanes in roundabouts.

However, the results from this study highlight a diminishing rate of reduction in traffic density as lanes increase. For example, while initial lane expansions (e.g., from 1 to 3 lanes) result in steep declines in density, further additions (e.g., beyond 8 lanes) show minimal improvements. This phenomenon is supported by Kabanga et al. (2022), who noted that geometric constraints and other parameters influence roundabout efficiency. Similarly, Damaskou et al. (2022) cautioned that the effectiveness of roundabouts is not solely a function of lane count but also depends on design considerations and driver behavior.

The inverted wedge-shaped regions observed in Figures 4.3 and 4.2 further emphasize optimal configurations for lane expansion. These results are consistent with Kabanga et al. (2022) and Cheah and Yeak (2022), who identified similar optimal designs for maximizing throughput while minimizing costs.

4.3.1.2 Traffic Density against Velocity

Figure 4.4 demonstrates an inverse relationship between traffic density (ρ) and velocity (v), with high density corresponding to low velocities and vice versa. This aligns with fundamental traffic flow theories as discussed by Shao (2023), who modeled traffic wave dynamics to reveal similar behavior. The critical velocity range ($10 \leq v \leq 15$) observed in the results represents a transition zone from congested to free-flow conditions, a finding consistent with Sun et al. (2020).

The study also suggests that maintaining moderate velocities can mitigate congestion, a conclusion supported by Damaskou et al. (2022), who emphasized the role of proper design and speed regulation in enhancing roundabout efficiency.

4.3.1.3 Traffic Density against Velocity and Spatial Changes

Figures 4.5 and 4.6 highlight the combined effects of velocity (v), spatial changes (Δx), and temporal changes (Δy) on traffic density (ρ). The results show that both larger spatial gaps (Δx) and time intervals (Δy) correlate with lower traffic densities, reinforcing the findings of Storani et al. (2021), who used hybrid models to demonstrate that adequate spacing reduces congestion.

The results also echo Kang et al. (2022), who argued that traditional constant-coefficient models fail to capture the dynamic interplay of spatial and temporal factors. The gradual decline in density with increasing Δx and Δy underscores the importance of managing these parameters for efficient traffic flow, as observed in both macroscopic and hybrid modeling approaches.

The findings of this study align with the broader body of literature, which emphasizes the importance of lane count, velocity management, and spatial-temporal dynamics in optimizing roundabout performance. Studies by Cheah and Yeak (2022) and Kabanga et al. (2022) have shown that modeling traffic flow in roundabouts requires incorporating real-world complexities, such as driver behavior, geometric constraints, and entry/exit dynamics. These aspects

are critical in achieving realistic and practical insights into traffic management.

Additionally, the diminishing returns observed in traffic density reduction with increased lanes highlight the importance of balancing construction costs and traffic benefits, as suggested by Zakeri and Choupani (2021). The transition zones in Figures 4.4, 4.6, and 4.5 align with Sun et al. (2020)'s emphasis on critical velocities and optimal configurations.

The results of this study suggest that roundabouts with 4–6 lanes may offer the most efficient balance between reducing congestion and minimizing costs. Beyond this range, the benefits diminish, as also noted by Kabanga et al. (2022). The critical transition zones observed in velocity and density relationships emphasize the need for proper design and regulation to optimize flow conditions, corroborating Damaskou et al. (2022).

4.3.2 Traffic flow on Disruption and Diffusion

Figures 4.8 and 4.7 illustrate the complex interactions between velocity (V), the number of lanes (N), disruption (S), and diffusion (D) within roundabout systems. The observed trends align with established findings that roundabouts promote improved traffic flow and reduced delays compared to traditional intersections. For example, Deluka Tibljaš et al. (2018) and Davidović et al. (2021) highlight the role of roundabouts in enhancing traffic efficiency through their continuous circular flow design.

In Figure 4.7, velocity increases with the number of lanes, emphasizing the benefits of increased capacity and maneuverability in reducing congestion. These results echo findings by Mohebifard and Hajbabaie (2021) and Zakeri and Choupani (2021), who demonstrated that adding lanes reduces average and maximum delays while significantly improving traffic safety. This improvement in velocity is critical as increased lane capacity mitigates the negative impacts of disruption (S), aligning with Damaskou et al. (2022)'s emphasis on minimizing driver confusion and external interferences to enhance roundabout performance.

Higher disruption values lead to reduced velocities across all lane configurations, as shown in Figures 4.7 and 4.8. This finding resonates with Cheah and Yeak (2022), who modeled

roundabouts under non-linear and congested traffic conditions, emphasizing the importance of addressing disruptions. Furthermore, the negative relationship between disruption and traffic flow efficiency suggests the critical need for design strategies that minimize abrupt stops and lane-changing conflicts, as corroborated by Kabanga et al. (2022).

Figure 4.8 highlights that higher diffusion—reflecting randomness in vehicle behavior—leads to reduced velocities. This underscores the need for measures that promote uniform lane usage and orderly driver behavior, as identified by Sun et al. (2020). Such strategies may include implementing clear signage and lane guidance to enhance traffic flow efficiency and mitigate the effects of diffusion.

The inverse relationship between traffic density and velocity in Figure 4.9 aligns with fundamental traffic flow principles, as modeled by Shao (2023). Lower velocities lead to higher densities due to closer vehicle spacing, while higher velocities reduce density as vehicles occupy less space per unit length. This relationship underscores the necessity of optimizing velocity to manage traffic density effectively, a concept also supported by Storani et al. (2021), whose hybrid traffic flow model demonstrated similar dynamics.

The combined analysis of Figures 4.8, 4.7, and 4.9 reveals the critical role of minimizing both disruption and diffusion while ensuring adequate lane capacity in roundabout design. These findings align with Kang et al. (2022)'s assertion that traditional models often fail to adapt to dynamic traffic changes and require extensions to capture real-world complexities. Although this study provides valuable insights, there remains a gap in empirical validation and real-world application scenarios, as identified by Cheah and Yeak (2022) and Storani et al. (2021). Future work should focus on validating these models using diverse traffic geometries and realistic driver behaviors to ensure their practical applicability.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

The chapter presents a summary of the study findings. Conclusion are also provided. Recommendations form the later part of the chapter.

5.2 Summary

5.2.1 Traffic Flow Patterns and Vehicle Speed

The study explores the relationship between traffic density (ρ) and lane count in roundabouts, highlighting critical insights into optimizing traffic flow and roundabout design. Figures 4.1, 4.3, and 4.2 reveal that increasing lane numbers reduces traffic density, aligning with Deluka Tibljaš et al. (2018) and Davidović et al. (2021), who emphasize roundabout efficiency in minimizing congestion. This trend is particularly pronounced in initial expansions (e.g., 1 to 3 lanes), but the rate of density reduction diminishes with further lane additions, consistent with Kabanga et al. (2022) and Damaskou et al. (2022), who highlight geometric and behavioral constraints.

The inverted wedge-shaped regions in Figures 4.3 and 4.2 indicate optimal lane configurations for balancing efficiency and costs, echoing findings from Cheah and Yeak (2022). Figure 4.4 demonstrates an inverse relationship between traffic density and velocity (v), with critical velocity ($10 \leq v \leq 15$) marking the transition from congested to free-flow conditions, supporting fundamental traffic flow theories and findings by Shao (2023) and Sun et al.

(2020).

Figures 4.6 and 4.5 illustrate the impact of spatial (Δx) and temporal (Δy) factors on density, showing that larger gaps correlate with reduced congestion. This aligns with Storani et al. (2021) and Kang et al. (2022), who emphasize the importance of capturing dynamic spatial-temporal interactions in traffic models.

The study concludes that roundabouts with 4–6 lanes achieve an efficient balance between reduced congestion and construction costs, consistent with Zakeri and Choupani (2021). Transition zones and critical velocities emphasize the importance of proper design, regulation, and modeling to optimize traffic flow and manage roundabout efficiency effectively.

5.2.2 Traffic flow on Disruption and Diffusion

The study examines the relationship between velocity (V), the number of lanes (N), disruption (S), and diffusion (D) in roundabouts, highlighting their impact on traffic flow. Figures 4.7 and 4.8 show that increasing the number of lanes enhances velocity, aligning with findings by Mohebifard and Hajbabaie (2021) and Zakeri and Choupani (2021), who observed reduced delays and improved safety with greater lane capacity. Disruption negatively affects velocity across all lane configurations, emphasizing the need to minimize lane-changing conflicts and external interferences, as noted by Damaskou et al. (2022) and Kabanga et al. (2022).

Diffusion, reflecting erratic vehicle behavior, also reduces velocity, reinforcing the importance of measures such as clear signage to promote orderly traffic, as highlighted by Sun et al. (2020). Figure 4.9 further illustrates an inverse relationship between traffic density and velocity, consistent with fundamental traffic flow principles and Shao (2023)'s hybrid model dynamics.

The combined findings stress the importance of addressing both disruption and diffusion while ensuring sufficient lane capacity to optimize roundabout performance. Despite these insights, gaps in empirical validation and real-world application remain, as noted by Cheah and Yeak (2022) and Storani et al. (2021).

5.3 Conclusion

This study successfully addressed the challenges of modeling traffic flow around roundabouts by employing the Navier-Stokes and advection-diffusion equations. Through advanced fluid mechanics principles, the research explored the complex dynamics of traffic density, velocity, and diffusion before and after roundabout implementation.

The findings align with the study's objectives, which aimed to develop mathematical models, evaluate the effects of roundabout geometry, and analyze the role of disruptions on traffic flow. The results demonstrate that roundabouts enhance efficiency and safety by reducing congestion and enabling smoother vehicle movement. Sensitivity analyses highlighted critical factors such as geometric design, traffic volume, and external disruptions. These elements play a decisive role in determining the overall performance of roundabouts, offering actionable insights for improving urban infrastructure.

The study contributes significantly to the field by bridging the gap between traditional models and real-world complexities. It underscores the importance of incorporating advanced mathematical frameworks in urban planning to optimize traffic flow. The proposed models proved effective in simulating traffic patterns and identifying parameters critical for designing efficient roundabouts. While cost reduction is a possible benefit, this was not explicitly studied in this research.

5.4 Recommendations

Policymakers, urban planners, and engineers should integrate these findings into roundabout design and implementation processes. Emphasis should be placed on conducting scenario-based sensitivity analyses to tailor designs to specific traffic conditions. Future research should extend the models to incorporate driver behavior, environmental factors, and multi-modal traffic systems for enhanced applicability. It is also recommended to evaluate traffic behavior using real-time traffic sensor data. This would strengthen their practical utility,

ensuring that the designs are adaptable and effective across diverse urban settings.

REFERENCES

- Albi, G., Bellomo, N., Fermo, L., Ha, S.-Y., Kim, J., Pareschi, L., Poyato, D., and Soler, J. (2019). Vehicular traffic, crowds, and swarms: From kinetic theory and multiscale methods to applications and research perspectives. *Mathematical Models and Methods in Applied Sciences*, 29(10):1901–2005.
- Alghamdi, T., Mostafi, S., Abdelkader, G., and Elgazzar, K. (2022). A comparative study on traffic modeling techniques for predicting and simulating traffic behavior. *Future Internet*, 14(10):294.
- Azam, T., Pandit, M., and Andallah, L. (2014). Numerical simulation of a second order traffic flow model. *GANIT: Journal of Bangladesh Mathematical Society*, 34:101–110.
- Bellomo, N. and Dogbe, C. (2011). On the modeling of traffic and crowds: A survey of models, speculations, and perspectives. *SIAM review*, 53(3):409–463.
- Boukerche, A., Tao, Y., and Sun, P. (2020). Artificial intelligence-based vehicular traffic flow prediction methods for supporting intelligent transportation systems. *Computer networks*, 182:107484.
- Campi, E., Mastinu, G., Previati, G., Studer, L., and Uccello, L. (2023). Roundabouts: Traffic simulations of connected and automated vehicles—a state of the art. *IEEE Transactions on Intelligent Transportation Systems*.
- Cao, R. and Cao, R. (2021). Computer simulation of water flow animation based on two-dimensional navier-stokes equations. *Advances in Mathematical Physics*, 2021:1–9.

- Chaari, A., Mouhali, W., LOUAKED, M., Sellila, N., and Mechkour, H. (2024). Numerical simulation of pollutant concentration patterns of a two-dimensional congestion traffic. *Available at SSRN 4901482*.
- Cheah, Y. H. and Yeak, S. H. (2022). Modeling of traffic flow on roundabouts. *Malaysian Journal of Fundamental and Applied Sciences*, 18(3):343–366.
- Chock, D. P. (1991). A comparison of numerical methods for solving the advection equation—iii. *Atmospheric Environment. Part A. General Topics*, 25(5-6):853–871.
- Chowdhury, D., Santen, L., and Schadschneider, A. (2000). Statistical physics of vehicular traffic and some related systems. *Physics Reports*, 329(4-6):199–329.
- Damaskou, E., Kehagia, F., Karagiotas, I., Anagnostopoulos, A., and Pitsiava-Latinopoulou, M. (2022). Driver's perceived satisfaction at urban roundabouts—a structural equation-modeling approach. *Future transportation*, 2(3):675–687.
- Davidović, S., Bogdanović, V., Garunović, N., Papić, Z., and Pamučar, D. (2021). Research on speeds at roundabouts for the needs of sustainable traffic management. *Sustainability*, 13(1):399.
- Delle Monache, M. L. (2014). *Traffic flow modeling by conservation laws*. PhD thesis, Université Nice Sophia Antipolis.
- Deluka Tibljaš, A., Giuffrè, T., Surdonja, S., and Trubia, S. (2018). Introduction of autonomous vehicles: Roundabouts design and safety performance evaluation. *Sustainability*, 10(4):1060.
- Demir, H. G. and Demir, Y. K. (2020). A comparison of traffic flow performance of roundabouts and signalized intersections: a case study in nigde. *The Open Transportation Journal*, 14(1).

- Dwivedi, K. D., Rajeev, Das, S., and Baleanu, D. (2020). Numerical solution of nonlinear space–time fractional-order advection–reaction–diffusion equation. *Journal of Computational and Nonlinear Dynamics*, 15(6):061005.
- Ershkov, S. V., Prosviryakov, E. Y., Burmasheva, N. V., and Christianto, V. (2021). Towards understanding the algorithms for solving the navier–stokes equations. *Fluid Dynamics Research*, 53(4):044501.
- Gautam, P. N., Sapkota, B. P., and Uprety, K. N. (2022). A brief review on the solutions of advection-diffusion equation. *Scientific World*, 15(15):4–9.
- Gazis, D. C. (1972). Traffic flow and control: Theory and applications: The car increases man’s mobility, until all decide to exercise this mobility simultaneously in space and time; then we must call traffic science to the rescue. *American Scientist*, 60(4):414–424.
- Haque, M. M., Chin, H. C., and Debnath, A. K. (2013). Sustainable, safe, smart—three key elements of singapore’s evolving transport policies. *Transport policy*, 27:20–31.
- Kabanga, D., Theuri, D. M., and Kioi, D. (2022). Evaluating unsignalized circular roundabout capacity using macroscopic traffic flow model. *Global Journal of Pure and Applied Mathematics*, 18(1):205–219.
- Kachroo, P., Özbay, K. M., Kachroo, P., and Özbay, K. M. (2018). Traffic flow theory. *Feedback Control Theory for Dynamic Traffic Assignment*, pages 57–87.
- Kalašová, A., Skřivánek Kubíková, S., Čulík, K., and Palúch, J. (2020). Comparison of traffic flow characteristics of signal controlled intersection and turbo roundabout. *Archiwum Motoryzacji*, 88(2):19–36.
- Kanda, I., Yamao, Y., Ohara, T., and Uehara, K. (2013). An urban atmospheric diffusion model for traffic-related emission based on mass-conservation and advection-diffusion equations. *Environmental Modeling & Assessment*, 18:221–248.

- Kang, Y., Mao, S., and Zhang, Y. (2022). Fractional time-varying grey traffic flow model based on viscoelastic fluid and its application. *Transportation research part B: methodological*, 157:149–174.
- Kessels, F., Kessels, R., and Rauscher (2019). *Traffic flow modelling*. Springer.
- Khan, M. I. and Alzahrani, F. (2020). Cattaneo-christov double diffusion (ccdd) and magnetized stagnation point flow of non-newtonian fluid with internal resistance of particles. *Physica Scripta*, 95(12):125002.
- Konkina, A. S. (2019). Numerical research of the mathematical model for traffic flow. *Let's start with the story. Article: The book and the book*, 12(4):128–134.
- Kundu, S. (2018). Suspension concentration distribution in turbulent flows: an analytical study using fractional advection–diffusion equation. *Physica A: Statistical Mechanics and its Applications*, 506:135–155.
- Liu, Y. (2019). A progressive motion-planning algorithm and traffic flow analysis for high-density 2d traffic. *Transportation Science*, 53(6):1501–1525.
- Łukaszewicz, G. and Kalita, P. (2016). Navier–stokes equations. *Advances in Mechanics and Mathematics*, 34.
- Matin, H. N. Z., Do, D., and Delle Monache, M. L. (2023). Nonlinear advection-diffusion models of traffic flow: a numerical study. In *2023 IEEE 26th International Conference on Intelligent Transportation Systems (ITSC)*, pages 2078–2083. IEEE.
- Mohammadian, S., Zheng, Z., Haque, M. M., and Bhaskar, A. (2023). Continuum modeling of freeway traffic flows: State-of-the-art, challenges and future directions in the era of connected and automated vehicles. *Communications in Transportation Research*, 3:100107.
- Mohebifard, R. and Hajbabaie, A. (2021). Connected automated vehicle control in single lane roundabouts. *Transportation research part C: emerging technologies*, 131:103308.

- Mollier, S., Monache, M. L. D., and Canudas-de Wit, C. (2018). A simple example of a two-dimensional model for traffic: Discussion about assumptions and numerical methods. *Transportation research record*, 2672(20):249–261.
- Pérez, A. P., de Navarra, E. O. I., and Sardà, I. d.-P. (2021). *Finite element methods for advection-diffusion-absorption and fluid flow problems*. PhD thesis, Universitat Politècnica de Catalunya (UPC).
- Perumal, P. S., Sujasree, M., Chavhan, S., Gupta, D., Mukthineni, V., Shingekar, S. R., Khanna, A., and Fortino, G. (2021). An insight into crash avoidance and overtaking advice systems for autonomous vehicles: A review, challenges and solutions. *Engineering applications of artificial intelligence*, 104:104406.
- Pinto, J. A., Kumar, P., Alonso, M. F., Andreão, W. L., Pedruzzi, R., dos Santos, F. S., Moreira, D. M., and de Almeida Albuquerque, T. T. (2020). Traffic data in air quality modeling: a review of key variables, improvements in results, open problems and challenges in current research. *Atmospheric Pollution Research*, 11(3):454–468.
- Qiu, Y. (2013). On nonlocal hydrodynamic model explaining synchronized traffic flow. *Journal of Non-Newtonian Fluid Mechanics*, 197:1–4.
- Ravish, R. and Swamy, S. R. (2021). Intelligent traffic management: A review of challenges, solutions, and future perspectives. *Transport and Telecommunication Journal*, 22(2):163–182.
- Rios-Torres, J. and Malikopoulos, A. A. (2016). A survey on the coordination of connected and automated vehicles at intersections and merging at highway on-ramps. *IEEE Transactions on Intelligent Transportation Systems*, 18(5):1066–1077.
- Roy, D., Ishizaka, T., Mohan, C. K., and Fukuda, A. (2019). Vehicle trajectory prediction at intersections using interaction based generative adversarial networks. In *2019 IEEE Intelligent transportation systems conference (ITSC)*, pages 2318–2323. IEEE.

- Saxena, N. and Jain, N. (2017). Evaluation of road traffic congestion by shock wave theory and reduction strategies. *International Journal on Recent and Innovation Trends in Computing and Communication*, 5(5):125–129.
- Sener, M. Z. and Aksu, E. (2022). The effects of head form on resistance performance and flow characteristics for a streamlined auv hull design. *Ocean Engineering*, 257:111630.
- Shao, Z. (2023). The riemann problem for a traffic flow model. *Physics of Fluids*, 35(3).
- Sreenivasan, K. R. and Yakhov, V. (2021). Dynamics of three-dimensional turbulence from navier-stokes equations. *Physical Review Fluids*, 6(10):104604.
- Stogios, C., Kasraian, D., Roorda, M. J., and Hatzopoulou, M. (2019). Simulating impacts of automated driving behavior and traffic conditions on vehicle emissions. *Transportation Research Part D: Transport and Environment*, 76:176–192.
- Storani, F., Di Pace, R., Bruno, F., and Fiori, C. (2021). Analysis and comparison of traffic flow models: a new hybrid traffic flow model vs benchmark models. *European transport research review*, 13(1):1–16.
- Suh, J. and Yeo, H. (2016). An empirical study on the traffic state evolution and stop-and-go traffic development on freeways. *Transportmetrica A: Transport Science*, 12(1):80–97.
- Sun, J., Zheng, Z., and Sun, J. (2018). Stability analysis methods and their applicability to car-following models in conventional and connected environments. *Transportation research part B: methodological*, 109:212–237.
- Sun, L., Jafaripournimchahi, A., Kornhauser, A., and Hu, W. (2020). A new higher-order viscous continuum traffic flow model considering driver memory in the era of autonomous and connected vehicles. *Physica A: Statistical Mechanics and Its Applications*, 547:123829.
- Sun, Y., Jayaraman, A. S., and Chirikjian, G. S. (2022). Approximate solutions of the advection–diffusion equation for spatially variable flows. *Physics of Fluids*, 34(3).

- Suzuki, H. and Marumo, Y. (2018). A new approach to green light optimal speed advisory (glosa) systems for high-density traffic flow. In *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*, pages 362–367. IEEE.
- Szymkiewicz, R. and Gkasiowski, D. (2021). Adaptive method for the solution of 1d and 2d advection–diffusion equations used in environmental engineering. *Journal of Hydroinformatics*, 23(6):1290–1311.
- Tao, W., Wang, Z.-Z., Liu, C.-J., Lu, Y.-N., Zhang, Y., and Jiang, Z.-H. (2024). Resilience assessment and enhancement of urban road networks subject to traffic accidents: A network-scale optimization strategy. *Journal of Intelligent Transportation Systems*, 28(4):494–510.
- Woodward, H., Schroeder, A. K., Le Cornec, C. M., Stettler, M. E., ApSimon, H., Robins, A., Pain, C., and Linden, P. F. (2022). High resolution modelling of traffic emissions using the large eddy simulation code fluidity. *Atmosphere*, 13(8):1203.
- Xiao, X. and Duan, H. (2020). A new grey model for traffic flow mechanics. *Engineering Applications of Artificial Intelligence*, 88:103350.
- Xue, Y., Zhang, Y., Fan, D., Zhang, P., and He, H.-d. (2019). An extended macroscopic model for traffic flow on curved road and its numerical simulation. *Nonlinear Dynamics*, 95:3295–3307.
- Yuan, Z. and Zeng, X. (2024). Dynamic network-level traffic speed and signal control in connected vehicle environment. *Sensors*, 24(20):6597.
- Zakeri, S. and Choupani, A.-A. (2021). Operational evaluation of a throughabout to give priority to public transport at standard roundabouts. *Journal of Advanced Transportation*, 2021:1–13.
- Zhang, Y., Smirnova, M., Bogdanova, A., Zhu, Z., and Smirnov, N. (2018). Travel time estimation by urgent-gentle class traffic flow model. *Transportation Research Part B: Methodological*, 113:121–142.

Zhou, L., Zhang, L., and Liu, C. S. (2021). Comparing roundabouts and signalized intersections through multiple-model simulation. *IEEE Transactions on Intelligent Transportation Systems*, 23(7):7931–7940.

Zhu, L., Li, X., Lu, W., and Zhang, Y. J. (2018). A field-based representation of surrounding vehicle motion from a monocular camera. In *2018 IEEE Intelligent Vehicles Symposium (IV)*, pages 1761–1766. IEEE.

Žura, M. (2022). Roundabout capacity estimation model considering driver behaviour on the exiting and entry flows. *Promet-Traffic&Transportation*, 34(3):397–405.

APPENDICES

Appendix I: Similarity Report

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