CONSTRUCTION OF MODIFIED THIRD ORDER ROTATABLE DESIGNS THROUGH PAIRWISE BALANCED DESIGNS

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF MASTER OF SCIENCE IN MATHEMATICS (BIOSTATISTICS) IN THE SCHOOL OF SCIENCE, UNIVERSITY OF ELDORET, KENYA

OCTOBER, 2019

DECLARATION

DECLARATION BY THE STUDENT

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This thesis has been submitted with our approval as university supervisors.

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DEDICATION

I would like to dedicate this thesis to my dear parents Mr. Peter Kimaiyo and Mrs. Benedister Kimaiyo who gave me a lot of support throughout my studies and to my loving husband Kenneth Toroitich, children Ryan Kipkemboi and Renee Jeruto, brothers and sisters Edwin, Harrison, Donald, Collins, Evalyne and mercy who all they gave me invaluable support throughout my studies.

ABSTRACT

Rotatable designs are designed for use of response surfaces. Rotatability is evidently a greatly popular property for Response Surface Methodology (RSM). RSM is a group of statistical and mathematical technique valuable for developing, improving and optimizing models and process. Experimentation of any kind usually requires resources of which they may be limited due to either in availability or high cost of acquiring. To reduce on expenses of an experiment one has to make a better preference of the experimental design prior to the experiment. An appropriate design that would provide relatively less number of the design points of the response at particular points of significance is essential. The aim of this study was to construct a modified third order rotatable designs (MTORD) by use of Pairwise Balanced designs in order to address the above problem. The objectives of the study were to construct a three-level and five-level v-dimensional modified third order rotatable Designs (TORD) using Pairwise Balanced Designs (PBD). The three-level and five-level v-dimensional modified order three rotatable designs were constructed by use of a suitably chosen pair off Pairwise Balanced Designs by repeating the set of the design points generated from every one of the designs a constant numeral times. These points were combined together with a number of central points without any additional set of points. In this study, some modified third order rotatable design constructed through pairwise balanced designs were obtained. In conclusion, the modified TORD constructed using PBD yield relatively fewer numeral of the design points as compared to the corresponding existing designs in the literature. Other studies that could possibly lead to designs with fewer numeral design points than what is obtained in the present study could be explored. The study recommends further studies on latest methods of construction of modified higher order rotatable designs and applications on this area.

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LISTS OF ABBREVIATIONS AND DEFINITIONS

BIBD Balanced Incomplete Block Design

Inc. Incidence Matrix

PBD Pairwise Balanced Designs

RSM Response surface Methodology

SORD Second Order Rotatable Designs

SOSRD Second Order Slope rotatable Designs

TORD Third Order Rotatable Design

MTORD Modified Third Order Rotatable Desig

ACKNOWLEDGEMENT

I hereby wish to heartily express my gratitude to God for His love and care and for giving me the energy throughout the time of my thesis writing, to my university lecturers for guidance and encouragement during the various periods of consultation, and to my fellow students especially the Biostatistics class for continuously encouraging me throughout this time, not forgetting the staff at the department of mathematics and computer science.

More appreciation to my supervisors Prof. J. M. Mutiso and Dr. Mathew Kosgei for their guidance, suggestions, instructive discussions and assistance in going through this thesis and making necessary corrections. God bless you all.

Finally, my appreciation to my entire family for their efforts in ensuring that my thesis was successful and also for their moral and financial support thanks a lot.

CHAPTER ONE

INTRODUCTION

1.1 Background Information

Rotatable designs are those designs whose variance of the predictable response at a spot is a function of the distance of that specific point from the source (central point of the design) and thus invariant in orthogonal rotations of the design. Rotatable designs are designed for use of response surfaces. RSM is a group of statistical and mathematical technique valuable for developing, improving and optimizing models and process. Given a response defined by linearity of a function of independent variables, then its approximating function will be first order model. If a curvature exits in the response, in that case a higher degree polynomial ought to be used, this leads to second order then to third order and so on till the outcome anticipated is obtained.

First order rotatable designs helps in fitting of a first order (i.e linear) surface, a second order rotatable designs aids in fitting of a second order (i.e quadratic) surface, and a third order rotatable designs aids in fitting of a third order (i.e cubic) surface. Third order rotatable designs were derived from second order rotatable designs while second order designs were derived from first order rotatable designs. These designs were derived due to the need of minimizing the cost of experimental design thus the need of having a reduced number of design points.

The purpose is to optimize the response (output) influenced by a number of independent variables (input variables). The property of rotatability is a highly desirable quality of an experiment design and was first advanced by Box and Hunter (1957). This property indicates that the variances of estimates of the response made from the least squares estimates of the Taylor sequence be constant on a circle, sphere or hyper spheres on the core of the design. Thus a rotatable design is a design which achieves this property, it can be rotated through any angle around its center and the variance of responses estimated from it will be unchanged. In these rotatable designs, the moments of independent variable are the same (Box and Hunter, 1957), through order 2d, as those of spherical distribution, or that these moments are invariant under a rotation of the design around the center.

Kosgei et al (2013) constructed a modified order three designs that are rotatable through Balanced Incomplete Block Designs (BIBD). In this study, we obtain a modified order III designs that are rotatable through pairwise balanced designs (PBD) which gives designs with fewer numeral design points than what is available in the existing designs.

1.2 Basic concepts

1.2.1 Rotatable designs

Allow there be v variates, each at s levels. Assuming a design to be formed with N of the s^v treatment combinations, we write this as $N\ x\ v$ matrix, referred to as a design matrix.

$$\begin{pmatrix} x_{11} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots \\ x_{1N} & \cdots & x_{vN} \end{pmatrix} \qquad (1.2.1.1)$$

For appropriateness, a variate x_i has been linked with the ith factor to symbolize its level. Treatments in this combination will be referred to as points of the design. From Box and

Hunter (1957), we see that a design of the form stated above will be a rotatable design of order *d* if a response polynomial surface

$Y = \beta_0 + \sum\beta_i x_i + \sum\beta_{ij} x_i x_j + \sum\beta_{ijk} x_i x_j x_k + \ldots$ (1.2.1.2)

of order *d* of the response *y as* obtained from other treatments, on the variates x_i, I = 1, 2, …, *v*, with some appropriate origin and scale, can be fitted so that the variance of the estimated response from any treatment is a function of the sum of squares of the levels of the factors in the treatment combinations.

A design is said to be rotatable if the variance of the response estimate is a function only of the distance of the point from the design center, i.e. if the variance of $\hat{Y}_u$, of the estimated response is a function only of the distance of the point $x_{0u}, x_{1u}, \ldots, x_{ku}$ from the centre of the experimental region. The study of the rotatable designs is mostly focused on the estimation of differences of the yields and its response. It is not dependent on the orientation of the design with respect to the true response surface. The aspect here has been greatly of use by Box and Draper (1963) in the construction of the design for response surface models of second and third order.

Therefore, for an arrangement of the design matrix X of order three to be rotatable, we should have,

$\mathrm{Var}(\hat{Y}_u) = X_u^{[3]'}(X'\underline{X})^{-1}X_u^{[3]}\sigma^2 = \phi(X_{0u}^2 + X_{1u}^2 + \ldots + X_{ku}^2)\sigma^2 = \phi(\rho^2)\sigma^2$ = constants,

(1.2.1.3)

u = 1, 2,…, N

Consider the estimated response at $\underline{Y}_u$, where,

$\underline{Y}_u = R\underline{X}_u$ (1.2.1.4) and R is any orthogonal matrix.

The variance of the estimated response at $\underline{Y}_u$ is given as,

$$Var(\underline{V}_u) = \underline{V}_u^{[3]'}(X'X)^{-1}\underline{V}_u^{[3]}\sigma^2 = \underline{X}_u^{[3]'}R_u^{[3]'}(X'X)^{-1}\underline{R}_u^{[3]}\underline{X}_u^{[3]}\sigma^2 \quad (1.2.1.5)$$

For this condition to be satisfied that the variance of the estimated response at any point on the sphere with centre (0… 0) is constant, we necessitate that,

$$X_u^{[3]'}(X'X)^{-1}X_u^{[3]}\sigma^2 = X_u^{[3]'}R_u^{[3]'}(X'X)^{-1}R_u^{[3]}X_u^{[3]}\sigma^2 \text{ for every } \underline{X}_u \text{ and R} \quad (1.2.1.6)$$

Therefore,

$$(X'X)^{-1} = R^{[3]'}(X'X)^{-1}R^{[3]} \quad (1.2.1.7)$$

For, every orthogononal matrix R. $R^{[3]}$ is also orthogonal since R is orthogonal

This imply that,

$$R^{[3]'} = R^{[3]^{-1}} \quad (1.2.1.8)$$

Hence we have,

$$R^{[3]'}(X'X)^{-1}R^{[3]} = (X'X) = R^{[3]^{-1}}(X'X)^{-1}R^{[3]} \quad (1.2.1.9)$$

Indicating that,

$$(X'X) = [R^{[3]'}(X'X)^{-1}R^{[3]}]^{-1} = R^{[3]^{-1}}X'XR^{[3]} = R^{[3]'}X'XR^{[3]} \quad (1.2.1.10)$$

We now need to find the form of the moment matrix X'X for which the equation (1.2.1.9) is satisfied.

1.2.2 Incidence Matrix

For a dual design, the incidence matrix N = (n_{ij}) has elements

$$n_{ij} = \begin{cases} 1 \text{ if treatments I occursin block j} \\ 0 \text{ otherwise} \end{cases} \quad (1.2.2.1)$$

In addition, for an equireplicate design,

$$\sum_{j=i}^{n} n_{ij} = r, \; for\, all\, i \tag{1.2.2.2}$$

And for suitable design

$$\sum_{i=1}^{t} n_{ij} = k, \; for\, all\; j \tag{1.2.2.3}$$

Here r represents the number of replication for every treatment and k represents the block size.

1.2.3 Pairwise Balanced Designs (PBD)

Pairwise Balanced Design (PBD) is a generalization of a BIBD, in which the blocks may be of different sizes. Take arrangements of *v* treatment with blocks b which we called a Pairwise Balanced Design of index λ and form (v, k_1, k_2 . . . k_p) given that each block has k_1,k_2…,k_p treatments where ($k_i \leq v$, $k_i \neq k_j$) and every set of two of distinctive treatments appears in precisely λ blocks of the design. Given that k={k_1,k_2…,k_p} is a set of positive integers, a PBD B[k, λ, v] is a duo (V, B) so as B becomes a group of Blocks as of v-set of elements so as every pair of elements appears in precisely λ blocks of B and each block B is with cardinality of the set K.

A Pairwise Balanced Design is a design (X, A) where each two of a kind of distinctive points is linked in precisely λ blocks, where λ is a positive digit Moreover, (X, A) is a usual pairwise balanced design if each point $x \epsilon X$ appears in precisely r blocks $A \epsilon \mathcal{A}$, whereas r is a positive digit. A Pairwise Balanced Design (X, A) is permitted to include blocks of size |x| (i.e; whole blocks) if (X, A) consists merely of total blocks, it is considered a trivial pairwise balanced design. If (X, A) contains no complete blocks, it is considered a proper pairwise balanced design. A PBD of index λ is a method to select blocks as of a set of treatments in a way that any two treatments have covalence λ. Given that there are V treatment and if each block size is a element of some set of k of positive integers, the design is chosen a PBD (v; k; λ). The number of blocks is not normally treated as a parameter; one can have two pairwise balanced design with the same parameters but with different numbers of blocks.

For example, the two sets 1 2 3, 1 4 5, 2 4, 2 5, 3 4, 3 5 and 1 2 3, 1 4, 1 5, 2 4, 2 5, 3 4, 3 5, 4 5 are PBD- (5, {3;2}, 1) however they comprise of six and eight blocks respectively. It must not be that each and every member of k be a block size. Given K= {k} it follows that a Pairwise Balanced Design is reduced to that of a Balanced Incomplete Block Designs.

It is notable that pairwise balanced design has a wide application in construction of designs and it has been proved to be extremely valuable in the statistical design of experiments especially in agricultural experiments.

1.4 Statements of the Problem

Response surface methodology is a group of mathematical and statistical techniques valuable for developing, improving and optimizing models and processes. Experimentation of any kind usually requires resources of which they may be limited due to either unavailability or high costs of acquiring. An experimental design has to be chosen prior to carrying out tests (experimentation), this aid in reduction of cost of experiments. An appropriate design that would provide relatively less number of the design points of the response at specific points of interest is required. Several authors came with different methods of construction of modified rotatable designs to reduce the number of design points and to cut on costs of experimentation.

A number of authors have explored some constructions of modified rotatable designs. Victorbabu (2009) studied the aspect of different methods of constructing modified Second Order Response Surface Designs(SORD), modified SORD with equispaced levels by use of Central Composite Designs (CCD), Balanced Incomplete Block Designs (BIBD), Pairwise Balanced Designs (PBD), Symmetrical Unequal Block Arrangements (SUBA) among other methods. Victorbabu (2011) explored a new method of construction of the second-order slope-rotatable designs by use of Pairwise Balanced Designs (PBD). Kosgei et al (2013) examined constructions of five-level modified third order rotatable designs by use of a pair of Balanced Incomplete Block Design (BIBD) and reviewed the moment conditions for order III arrangement to be rotatable.

No work has been done in regard to the construction of modified order III rotatable designs through pairwise balanced designs. In order to obtain fewer number of design points and reduce the cost of experimentation of the third order rotatable designs through pairwise balanced designs, this study explored the method of construction of modified order III rotatable designs by use of Pairwise Balanced Designs (PBD).

1.5 Objectives of the study

1.5.1 General objective

The general objective of this study is to construct a modified third order rotatable designs through pairwise balanced designs (PBD)

1.5.2 Specific objective

In order to achieve the above general objective the following specific objectives have been set.

i. To construct three-level v-dimensional modified third order rotatable designs using Pairwise Balanced Designs (PBD)

ii. To construct five-level v-dimensional modified third order rotatable designs using Pairwise Balanced Designs (PBD)

1.6 Significance of the Study

The construction of modified third order rotatable designs provides an easier way to approximate the response at particular points of interest than what is obtainable from the corresponding accessible designs. This is best achieved especially when blocks are not the same. The most important aspect of rotatability is to minimize the cost of experimentation. To minimize on costs, an experimenter needs to choose a preferred experimental design prior to experimentation. In this study it is observed that the design obtained occasionally leads to designs with lesser number of designs points than those presented in the literature. This study can be useful in agricultural experiments for example change of yield of a crop in response to various fertilizer doses, and in chemical industries the rate of reaction in chemical experiments among other areas where maximization of a process is essential.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

The paper by Box and Wilson (1951) generated initial interest in use of Response Surface Methodology. Box and Hunter (1957) explored the idea and suggested the use of rotatable designs and also gave the conditions that were necessary and sufficient for a design to be rotatable. Box and Draper (1963) employed the same aspect to construct the designs for second and third order response models.

Response Surface Methodology (RSM) was first developed by Box together with his colleagues at Imperial Chemical Industries that is Box and Wilson (1951) and Box and Youle (1955). The interest in Response Surface Methodology has increased and books on this subject have been written by several authors such as Box and Draper (1987), Myers et al. (1989) and Myers et al. (1989) and Myers and Montgomery (1995). The standard texts on Response Surface Methodology by Box and Draper (1987), Khuri and Cornell (1996), and Myers and Montgomery (2002), give examples from many different applications, but still emphasize the chemical industry.

The most fruitful applications of Box and Wilson's methods have been in the fields of chemistry and chemical engineering where both the experimental designs and steepest ascent techniques have been used. Industrial and laboratory-based experiments on biotechnological processes are apparently alike to those in chemical engineering.

Response surface methodology explores the relationships between several explanatory variables and one or more response variables.

The concept of rotatability was first introduced by Box and Hunter (1957). Since its introduction, it has turn out to be an essential design criterion. The most widespread uses of Response Surface Methodology are in the meticulous situations where a number of input variables potentially control some performance measure or quality feature of the process. The field of Response Surface Methodology consists of the experimental approach for discovering the space of the process or independent variables, experiential statistical modeling to expand a suitable relationship between the yield and the process variable, and optimization method for obtaining values of the process variables that generate wanted values of the response.

From the start, RSM was developed to model experimental responses (Box and Draper, 1987), it then migrated into the modeling of numerical experiments. The difference is in the type of error generated by the response. Rotatable designs were introduced by Box and Hunter (1957) and are such that the variance of the estimated response at a point is a function of the distance of that point from the origin (centre of the design) and hence invariant under orthogonal rotations of the design. Rotatability is clearly a highly desirable property for Response Surface Methodology.

Box and Hunter (1957) constructed such designs through geometrical configuration. In their seminal paper they also derived the moment requirement of a d-th order rotatable design. Subsequently, several authors including Box & Behnken (1960), Gardiner et al, (1959), Bose and Draper (1959) gave methods of construction of second order and third order rotatable designs. Das (1961), Das and Narasimham (1962) and Das (1963) constructed these designs through factorial and incomplete block designs.

In many of such experiments the objective is to explain different aspects of the functional relationship $y = f(x_1, x_2) + e$, where y is the response, x_1, x_2 are the v factors and e represents the noise or error observed in the response y. The surface represented by $f(x_1, x_2)$ is called a *response surface*. The v factors are assumed to be independent. The errors in the prediction are assumed to be uncorrelated with a zero mean and variance σ^2.

To cut on costs, an experimenter has to make a choice of the experimental design prior to experimentation. The study of rotatable designs mainly emphasizes on estimation of output and obtaining of fewer design points in order to reduce on the cost of experimentation. Estimation of differences in responses at different points in the factor space will always be of importance. If a difference in responses at two points close together is of interest, then estimation of local slope (rate of change) of the response is required. Estimation of slopes occurs frequently in many practical situations i.e. in agriculture change of yield of a crop in response to various fertilizer doses, in chemical industries rate of reaction in chemical experiments among other areas where maximization of a process is essential.

The fitting of the response surface can be difficult and tiresome if done randomly. A number of authors suggested the use of rotatable designs. These designs ensured equivalent accuracy of the response estimates. Box and Draper (1963) have applied this idea in the construction of the designs for second and third order response surface models. Many second order rotatable designs have been studied. Das and Narasimham

(1962) constructed a rotatable second order designs through Balanced Incomplete Block Designs (BIBD). Victorbabu and Narasimham (1991, 1993, 2002) studied Second Order Slope Rotatable Designs (SOSRD) and their constructions.

Tyagi (1964) worked on construction of second and third order rotatable designs through Pairwise Balanced Designs (PBD) and Doubly Balanced Designs. Wilson (1972) reviewed an Existence Theory for Pairwise Balanced Designs I; composition theorems and morphisms. Wilson. (1974) investigated construction and uses of Pairwise Balanced Designs, mathematical centre tracts. Wilson (1975) explored an Existence Theory for Pairwise Balanced Designs III; Proof of the Existence Conjectures.

Bennett (1987) explored Pairwise Balanced Designs with Prime Power Block Sizes exceeding 7. Mullin (1989) studied Finite Bases for some PBD-closed sets and Bennett, Colbourn and Mullin (1998) examined Quintessential Pairwise Balanced Designs. Ling and Colbourn (1997) constructed Pairwise Balanced Designs with consecutive block sizes. Ling, Zhu, Colbourn and Mullin (1997) obtained Pairwise Balanced Designs with block sizes 8, 9 and 10.

Victorbabu and Narasimham (1993) extended the concept of construction of modified Second Order Slope Rotatable Designs using Pairwise Balanced Designs (PBD). Victorbabu and Vasundharadevi (2004b) investigated the performance of second order response surface designs for estimation of responses and slopes using Pairwise Balanced Designs. Victorbabu(2005) worked on Modified Second Order Slope-Rotatable Designs using Pairwise Balanced Designs. Victorbabu (2008c) reviewed Modified Second Order Slope-Rotatable Designs with equispaced levels using Pairwise Balanced Designs.

Victorbabu (2009) examined in detail different methods of construction of modified second order response surface designs, Modified Second Order Rotatable Designs (SORD), modified SORD with equispaced levels using Central Composite Designs, Balanced Incomplete Block Designs (BIBD), Pairwise Balanced Designs (PBD), and Symmetrical Unequal Block Arrangements (SUBA) among other methods. Victorbabu (2011) explored a new method of construction of the Second-Order Slope-Rotatable designs using Pairwise Balanced Designs (PBD). Victorbabu and Rajyalakshmi (2012) gave A new method of construction of Robust Second Order Slope Rotatable Designs using Pairwise Balanced Designs. Victorbabu and Surekha (2012) introduced Construction of measure of Second Order Slope Rotatable Designs using Pairwise Balanced Designs. Dukes and Ling (2014) explored Pairwise Balanced Designs with prescribed minimum dimension. Dukes and Niezen (2015) constructed pairwise balanced designs of dimension three.

Gardner et al (1959), Draper (1960a, 1960b, 1961, Thaker and Das (1961), Herzberg (1964), Huda (1982b, 1983), Mutiso and Koske (2005, 2007) and Kosgei et al (2011) among other authors reviewed many third order rotatable designs. Kosgei et al (2013) examined constructions of five-level modified third order rotatable designs using a pair of Balanced Incomplete Block Design and reviewed the moment conditions for third order arrangement to be rotatable.

A need to have fewer points in the third order rotatable designs led to different modifications of the existing designs. In the previous papers, a method has been given by using the properties of Pairwise Balanced Designs through which second order rotatable designs with any number of factors, with a reasonably small number of points, can be obtained. In this thesis, a modified third order rotatable designs using Pairwise Balanced Designs was obtained.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

The three-level and five-level v-dimensional modified third order rotatable designs were constructed using a suitably chosen pair of Pairwise Balanced Designs by repeating the set of the design points generated from each of the designs a constant number of times. These points were combined together with a number of central points without any additional set of points. This modified method was be obtained by considering the case of a pair of PBD is by taking the set of $b_1 2^{2(k_1)}$ design points generated from the first PBD design and repeating a constant number of times, say n_1. These points are augmented with the set of $b_2 2^{t(k_2)}$ design points generated from the second PBD design which are again repeated a constant number of times, say n_2. $b_i 2^{t(k_2)}$ denote the number of design points generated from the PBD designs (i = 1,2) by "multiplication", where $2^{t(k_1)}$ and $2^{t(k_2)}$ denote resolution V fractional replicates of $2^{(k_1)}$and $2^{(k_2)}$ factorials with levels ±1. The method of construction of third order rotatable design both sequential and non-sequential through PBD where $r \neq 3\lambda$ is actualized by taking the $a -$ combinations obtained through PBD referred to as a-combinations, together with one or more of the combinations of the type (b 0 . . . 0), (c c 0 . . . 0), (d d . . . d) involving fresh unknown levels b,c,d and then by 'multiplying' them with requisite number of associate combinations

The study adopted the approach by Kosgei et al (2013) in construction of modified third order rotatable design using BIBD and applied the same notation as that of Victorbabu in construction of modified second order rotatable design using pairwise balanced designs (PBD)

A review of the known results is given. These results were used to obtain the modified set of moment conditions for the set of points of the design matrix X to form a modified third order rotatable arrangement.

3.2 A Review of a Rotatable Arrangement of Order Three

A review of methods of obtaining moments conditions for a given set of points of a design matrix say X that satisfy a modified third order rotatable arrangement is undertaken. That is the conditions necessary for the moments of the coordinates of the points to be invariant under rotation will be set forth for a third order polynomial in x_{0u}, x_{1u}, …,x_{ku}, (u= 1, 2, ... , N). According to Kosgei et al (2013), specific restrictions are imposed other than the ordinary relations for the moments conditions for the third order rotatable arrangement.

3.3 Review of Moment Conditions for Third Order Arrangement to be Rotatable.

Supposing we want to use the third order response surface design $D = ((x_{iu}))$ to fit the surface,

$$Y_u = \beta_0 x_{0u} + \sum_{i=1}^{k} \beta_i x_{iu} + \sum_{i \le j=1}^{k} \beta_{ij} x_{iu} x_{ju} + \sum_{i \le j \le l=1}^{k} \beta_{ijl} x_{iu} x_{ju} x_{lu} + e_u \tag{3.3.1}$$

Where x_{ii}, denotes the level of the i^{th} factor (i = 1, 2,…, *k)* in the u^{th} run (u =1, 2, … , *N)* of the experiment, e_u's are uncorrelated random errors with mean zero and variance sigma squared. Here β_0, β_i, β_{ii}, β_{ij}, β_{iii}, β_{ijj}, and β_{ijl} are the parameters of the model and Y_u the response observed at the u^{th} design point. The parameters in the response relation are estimated using the least squares technique.

Further, we impose the following symmetry conditions on the design points to simplify the solutions of the normal equations.

$$\sum_{u=1}^{N}\prod_{i=1}^{k} x_{iu}^{\alpha i} = 0 \text{ if any } \alpha_i \text{ is odd for } \sum_{i=1}^{k}\alpha_i = 6 \quad (3.3.2)$$

i. $\sum_{u=1}^{N} x_{iu}^2 = N\lambda_2,\ \forall i$ (3.3.3)

ii. $\sum_{u=1}^{N} x_{iu}^4 = dN\lambda_4,\ \forall i$ (3.3.4)

iii. $\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = N\lambda_4,\quad i \neq j$ (3.3.5)

iv. $\sum_{u=1}^{N} x_{iu}^6 = hN\lambda_6,\ \forall i$ (3.3.6)

v. $\sum_{u=1}^{N} x_{iu}^2 x_{ju}^4 = hN\lambda_6,\quad i \neq j$ (3.3.7)

vi. $\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 x_{lu}^2 = N\lambda_6,\ i \neq j \neq l$ (3.3.8)

Where c, d, h, λ_2, λ_4 and λ_6 are constants, $x_{0u}=1$, $E(e_u)=0$, $var(e_u) = \delta^2$ (unknown),

$Cov(e_u, e_u^{'}) = 0$, $u \neq u' = 1,2,\ldots,N$. for third order rotatability, c=5, d=3, h=15.

The expectation of the response at the u^{th} run is given by;

$E(Y_u) = \underline{X}'_u\underline{\beta}$, (3.3.9)

Where X' denotes the level of the i^{th} factor (i = 1, 2,…, *k)* in the u^{th} run (u =1, 2, … , *N),* given as,

$\underline{X}'=$
$$(x_{0u}, x_{iu}, \ldots, x_{ku}, x_{1u}^2, \ldots, x_{ku}^2, x_{1u}x_{2u}, \ldots, x_{k-1u}x_{ku}, x_{1u}^3, \ldots, x_{ku}^3,$$
$$x_{1u}^2 x_{2u}, \ldots, x_{k-1u}^2 x_{ku}, x_{1u}x_{2u}^2, \ldots, x_{k-1u}x_{ku}^2, x_{1u}x_{2u}x_{3u}, \ldots, x_{k-2u}x_{k-1u}x_{ku})$$
(3.3.1)

Then $\underline{\beta}$ is the parameter of the model given as,

$\underline{\beta}$ = (β_0, β_1,…, β_k, β_{11},…, β_{kk}, β_{12},…, β_{k-1k}, β_{111},…, β_{kkk}, β_{112},…, $\beta_{k-1k-1k}$, β_{122},…, β_{k-1kk}, β_{123},…, $\beta_{k-2k-1k}$) (3.3.11)

3.4 Modified Moments of Rotatable Arrangement of Order Three

The common method of constructing a third order rotatable design is by putting some limitations indicating relationships among the order 3 moments,

$\sum x_{iu}^2$, $\sum x_{iu}^4$, $\sum x_{iu}^2 x_{ju}^2$, $\sum x_{iu}^6$, $\sum x_{iu}^4 x_{ju}^2$, and $\sum x_{iu}^2 x_{ju}^2 x_{lu}^2$ (3.4.1)

In third order rotatable designs, the restrictions used include;

$\sum x_{iu}^4 = 3 \sum x_{iu}^2 x_{ju}^2$ (3.4.2)

$\sum x_{iu}^6 = 5 \sum x_{iu}^4 x_{ju}^2 = 15 \sum x_{iu}^2 x_{ju}^2 x_{lu}^2$ (3.4.3)

Other limitations which may have not been exploited could also be possible. In this study we make use of the following restrictions obtained by Kosgei (2013):

$(\sum x_{iu}^2)^2 = N \sum x_{iu}^2 x_{ju}^2$, i.e. λ_2^2 (3.4.4)

and

$\left(\sum x_{iu}^2 x_{ju}^2\right)^3 = N \left(\sum x_{iu}^2 x_{ju}^2 x_{lu}^2\right)^2$ i.e $\lambda_2 \lambda_6 = \lambda_4^2$ (3.4.5)

These give another series of spherical third order response surface design which provides relatively smaller number of design points to the estimates of the response at specific points of interest than what is available from the corresponding literature.

3.5 Conditions for a Modified Rotatable Design

These are the conditions that an experimental design must satisfy in order to be a third order rotatable design. By utilizing the moment matrix of the third order polynomial, the non-singularity conditions were obtained through the determinant of this matrix. The expectation of the response at the u^{th} experimental point is given by $\widehat{Y}_u$. If it is assumed that the response surface may be approximated by a third order polynomial as follows

$$\hat{Y}_{u=}\ \beta_o + \sum_{i=1}^{k} \beta_o x_{iu} + \sum_{i\le j=1}^{k} \beta_{ij}\, x_{iu}x_{ju} + \sum_{i\le j\le l=1}^{k} \beta_{ij}\, x_{iu}x_{ju}x_{lu} \quad \text{, where u = 1,2,…,N} \tag{3.5.1}$$

From

$$X'X\beta = X'Y, \tag{3.5.2}$$

let M be the moment matrix, Where $M = \frac{I}{N}\ X'X$ (3.5.3)

Then we obtain the matrix

$$M_{\left(\binom{k+3}{3} X \binom{k+3}{3}\right)} = \begin{bmatrix} H & 0 & 0 & 0 & . & . & . & 0 & 0 \\ & I & 0 & 0 & . & . & . & 0 & 0 \\ & & G_1 & 0 & . & . & . & 0 & 0 \\ & & & G_2 & . & . & . & 0 & 0 \\ & & & & . & & & & . \\ & & & & & . & & & . \\ & (sym) & & & & & . & & . \\ & & & & & & & G_K & 0 \\ & & & & & & & & L \end{bmatrix} \tag{3.5.4}$$

Where

$$H_{((k+1)X(k+1))} = \begin{bmatrix} 1 & \lambda_2 & \lambda_2 & . & . & . & \lambda_2 \\ & 3\lambda_4 & \lambda_4 & . & . & . & \lambda_4 \\ & & 3\lambda_4 & . & . & . & \lambda_4 \\ & & & . & & & . \\ & (sym) & & & . & & . \\ & & & & & . & . \\ & & & & & & 3\lambda_4 \end{bmatrix} \tag{3.5.5}$$

$$I_{\left(\frac{k(k-1)}{2}X\frac{k(k-1)}{2}\right)} = \begin{bmatrix} \lambda_4 & 0 & . & . & . & 0 \\ & \lambda_4 & . & . & . & 0 \\ & & . & . & . & 0 \\ & (sym) & & . & & 0 \\ & & & & . & 0 \\ & & & & & \lambda_4 \end{bmatrix} \tag{3.5.6}$$

$$G_{i((k+1)X(k+1))} = \begin{bmatrix} \lambda_2 & \lambda_4 & \lambda_4 & . & . & . & \lambda_4 \\ & 15\lambda_6 & 3\lambda_6 & & & & 3\lambda_6 \\ & & 3\lambda_6 & . & . & . & \lambda_6 \\ & & & . & & & . \\ & (sym) & & & . & & . \\ & & & & & . & . \\ & & & & & & 3\lambda_6 \end{bmatrix} \quad \text{i}= 1,2, \ldots, \text{k} \tag{3.5.7}$$

And

$$L_{\left(\frac{k(k-1)(k-2)}{6}X\frac{k(k-1)(k-2)}{6}\right)} = \begin{bmatrix} \lambda_6 & 0 & . & . & . & 0 \\ & \lambda_6 & . & . & . & 0 \\ & & . & . & . & 0 \\ & (sym) & & . & & 0 \\ & & & & . & 0 \\ & & & & & \lambda_6 \end{bmatrix} \tag{3.5.8}$$

It is observed that the arrangement of the moment matrix in (3.5.4) to (3.5.8) differs from the arrangement of the second order moment matrix given by Box and Hunter (1957). The moment matrix is presented in this form to point out the orthogonality present.

3.6 Construction of the Designs

The method of construction of third order rotatable design both sequential and non-sequential through PBD where $r \neq 3\lambda$ is actualized by taking the a – combinations obtained through PBD referred to as a-combinations, together with one or more of the combinations of the type (b 0 . . . 0), (c c 0 . . . 0), (d d . . . d) Involving fresh unknown levels b, c, and d and then by 'multiplying' them with requisite number of associate combinations. The combinations taken are either the v-combinations obtained from the combination (b 0 . . . 0) by permuting over the different factors, or the combination (d d . . . d) accordingly as $r < 3\lambda$ or $r > 3\lambda$. The combinations (c c 0 0 . . . 0) give v(v-1)/2 combinations when permuted over all the v factors. The design points obtained by the combination of the type (b 0 . . . 0), (c c 0 . . . 0) and (d d . . . d) after "multiplication" with the requisite associate combinations are denoted respectively as (b 0 . . . 0) X 2^1, (c c 0 0 . . . 0) X 2^2 and (d d . . . d) X suitable fraction of 2^v. According to Das and Narasimham (1962), it becomes necessary sometimes to include the same design more than one set of the same type in order to obtain positive solutions for all the levels.

The current study adopted the method proposed by Victorbabu (2006) for constructing modified Second Order Rotatable Design (SORD) using a pair of Pairwise Balanced Designs and applied the conditions of modified third order rotatable designs as obtained by Kosgei et al, (2013). Specific methods for constructing modified TORD for the various levels using a pair of PBD with varied conditions for choosing appropriate designs are given independently while constructing individual designs.

The underlying principle behind this modified method considering the case of a pair of PBD is by taking the set of $b_1 2^{2(k_1)}$ design points generated from the first PBD design and repeating a constant number of times, say n_1. These points are augmented with the set of $b_2 2^{t(k_2)}$ design points generated from the second PBD design which are again repeated a constant number of times, say n_2. $b_i 2^{t(k_2)}$ denote the number of design points generated from the PBD designs (i = 1,2) by “multiplication”, where $2^{t(k_1)}$ and $2^{t(k_2)}$ denote resolution V fractional replicates of $2^{(k_1)}$and $2^{(k_2)}$ factorials with levels ±1. With the above design points together with n_0 central points, a modified TORD will be constructed.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

4.1 Introduction

Specific methods for constructing modified TORD for the various levels using a pair of PBD with varied conditions for choosing appropriate designs are given independently while constructing individual designs. The method of construction of third order rotatable design both sequential and non- sequential through PBD where $r \neq 3\lambda$ is actualized by taking the a – combinations obtained through PBD referred to as a-combinations, together with one or more of the combinations of the type (b 0 . . . 0), (c c 0 . . . 0), (d d . . . d) involving fresh unknown levels b,c,d and then by 'multiplying' them with requisite number of associate combinations

4.2 Three-Level Modified Third Order Rotatable Designs Using a Pair of PBD

The method of construction of three level modified TORD using suitably chosen pair of pairwise balanced designs (PBD) without any additional set of points was obtained. We utilize the notations of Das and Narasimham (1962) and Narasimham et al (1983).

Definition

Let $C_i = (v, b_i\ , r_i,\ k_{i1},\ k_{i2,}\ _{. . . ,}k_{ip},\ \lambda_i)$, for i=1,2 be an equi-replicated PBD and v-number of treatments in an experiment, b_i- number of blocks , r_i- number of times a treatment is replicated, k_1, k_2... K_p- block sizes and λ_i- number of times each pair is replicated. Let $2^{t(k_i)}$ denote the resolution V fractional replicates of $2^{(k_i)}$ factorials with +1 or -1 levels in treatments with $r_1 \leq 5\lambda_1$ and $r_2 \geq 5\lambda_2$ respectively.

Let $\left[a-(v, b_1, r_1, k_{11}, k_{12, \ldots,} k_{1p}, \lambda_1)\right]2^{t(k_1)}$ and $\left[a-(v, b_2, r_2, k_{21}, k_{22, \ldots,} k_{2p}, \lambda_2)\right]2^{t(k_2)}$ denote $b_1 2^{t(k_1)}$ and $b_2 2^{t(k_2)}$ design points generated from PBD by multiplication respectively. The set of $b_1 2^{t(k_1)}$design points generated from PBD-C_1 is repeated n_1-times and the set of $b_2 2^{t(k_2)}$design points generated from the PBD-C_2 is repeated n_2-times respectively.

Let n_o denote the number of central points. Then with the above design points, $n_i b_i 2^{t(k_i)}$, we construct a three level modified TORD as given in the following theorem.

Theorem 4.1

The design points

$n_1\left[a-(v, b_1, r_1, k_{11}, k_{12, \ldots,} k_{1p}, \lambda_1)\right]2^{t(k_1)} U\, n_2\left[a-(v, b_2, r_2, k_{21}, k_{22, \ldots,} k_{2p}, \lambda_2)\right]2^{t(k_2)} \mathrm{U}\, n_o$

Where v_i- number of treatments in an experiment

b_i- number of blocks

r_i- number of times a treatment is replicated

k_{i1}, k_{i2}... K_{ip}- block sizes

λ_i- number of times each pair is replicated

and i = 1,2

Give a three level v-dimensional modified TORD in,

$$N = \frac{\left[n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)}\right]^2}{\left[n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)}\right]} \tag{4.1.1}$$

Design points if;

$$(r_1 - 5\lambda_1)(r_2 - 5\lambda_2) \leq 0, \tag{4.1.2}$$

$$\frac{n_2}{n_1} = \frac{(5\lambda_1 - r_1)2^{t(k_1)-t(k_2)}}{(r_2 - 5\lambda_2)}, \tag{4.1.3}$$

$$n_o = \frac{\left[n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)}\right]^2}{\left[n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)}\right]} - n_1\, b_1 2^{t(k_1)} - \ n_2 b_2 2^{t(k_2)} \tag{4.1.4}$$

And n_o turns out to be an integer.

Proof

In support of the design points obtained, n_1-repetitions of points from PBD-C_1 and n_2-repetitions of points from PBD-C_2, for a modified TORD to be true, the conditions are as follows:

$$\sum x_{iu}^2 = \ n_1 r_1 2^{t(k_1)} a^2 + n_2 r_2 2^{t(k_2)} a^2 = X \tag{4.1.5}$$

$$\sum x_{iu}^4 = \ n_1 r_1 2^{t(k_1)} a^4 + n_2 r_2 2^{t(k_2)} a^4 = 3Y \tag{4.1.6}$$

$$\sum x_{iu}^2 x_{ju}^2 = \ n_1 \lambda_1 2^{t(k_1)} a^4 + n_2 \lambda_2 2^{t(k_2)} a^4 = Y \tag{4.1.7}$$

$$\sum x_{iu}^6 = \ n_1 r_1 2^{t(k_1)} a^6 + n_2 r_2 2^{t(k_2)} a^6 = 15Z \tag{4.1.8}$$

$$\sum x_{iu}^2 x_{ju}^4 = \ n_1 \lambda_1 2^{t(k_1)} a^6 + n_2 \lambda_2 2^{t(k_2)} a^6 = 3Z \tag{4.1.9}$$

Where $X = N\lambda_2$, $Y = \ N\lambda_4$ and $Z = N\lambda_6$

From (4.1.8) and (4.1.9), we have,

$$n_1 \lambda_1 2^{t(k_1)} a^6 + n_2 \lambda_2 2^{t(k_2)} a^6 = 5\left[n_1 \lambda_1 2^{t(k_1)} a^6 + n_2 \lambda_2 2^{t(k_2)} a^6\right] \tag{4.1.10}$$

This leads to,

$\frac{n_2}{n_1} = \frac{(r_1 - 5\lambda_1)2^{t(k_1)-t(k_2)}}{(5\lambda_2 - r_2)}$, given in equation (4.1.3).

The modified condition $(\sum x_{iu}^2)^2 = N \sum x_{iu}^2 x_{ju}^2$ leads to

$N = \frac{\left[n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)}\right]^2}{\left[n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)}\right]}$ given in equation (4.1.1).

Given n_o central points, N may be obtained directly as

$N = n_1\, b_1\, 2^{t(k1)} + n_2\, b_2\, 2^{t(k2)} + n_o.$ (4.1.11)

Example 4.2.1

From the design points

n_1 [a - (v=6, b_1=11, r_1=4, k_{11}=3, k_{12} =2, λ_1=1)] $2^{\,3}$U n_2 [a - (v=6, b_2=15, r_2=5, k_{21}=2, λ_2=1)2^2 U n_o

We obtain,

$$\frac{n_2}{n_1} = \frac{(4-5)2^{3-2}}{5-5} = \frac{2}{0} \qquad (4.1.12)$$

$$N = \frac{[0*4*2^3+2*5*2^2]^2}{[0*1*2^3+2*1*2^2]} = \frac{[0+40]^2}{0+8} = \frac{40^2}{8} = 200 \qquad (4.1.13)$$

$$n_0 = 200 - (0*11*2^3) - (2*15*2^2) = 200 - 0 - 120 = 80 \qquad (4.1.14)$$

Which gives a three-level 6-dimensional modified third order rotatable design in N= 200 design points with $n_1 = 0$, $n_2 = 2$ and n_o = 80.

Example 4.2.2

The design points

n_1 [a - (v=9, b_1=13, r_1=4, k_{11}=4, k_{12} =3, k_{13} =2, λ_1=1)] 2^4U n_2 [a - (v=9, b_2=30, r_2=7, k_{21}=3, k_{22}=2, λ_2=1)2^3 U n_o

Give a three-level 9-dimensional modified third order rotatable design in N= 1200 design points with$n_1 = 2$, $n_2 = 2$ and n_o = 304.

The values of n_1, n_2, N, and n_0 are as shown from workings on the equations (4.1.12), (4.1.13), and (4.1.14) respectively.

Consider two pairwise balanced designs, PBD-C$_1$ $(v = 6, b_1 = 11\,, r_1 = 4,\ k_{11} = 3, k_{12}\ = 2, \lambda_1 = 1)$ and PBD-C$_2$ $(v = 6, b_2 = 15\,, r_2 = 5,\ k_{21} = 2, \lambda_2 = 1)$

Then the design sets can be represented as incident matrix as follows:

Let *inc*. PBD-C$_1$ $(v = 6, b_1 = 11\,, r_1 = 4,\ k_{11} = 3, k_{12}\ = 2, \lambda_1 = 1)$be given as shown in the table below;

Table 1: Incidence Matrix of PBD-C1$(\boldsymbol{v = 6, b_1 = 11\,, r_1 = 4,\ k_{11} = 3, k_{12}\ = 2, \lambda_1 = 1})$

t_1	t_2	t_3	t_4	t_5	t_6
1	1	1	0	0	0
0	0	0	1	1	1
1	0	0	1	0	0
1	0	0	0	1	0
1	0	0	0	0	1
0	1	0	1	0	0
0	1	0	0	1	0
0	1	0	0	0	1
0	0	1	1	0	0
0	0	1	0	1	0
0	0	1	0	0	1

This can as well be written in block form as:

$$\begin{pmatrix} 1\ 2\ 3 \\ 4\ 5\ 6 \\ 1\ 4 \\ 1\ 5 \\ 1\ 6 \\ 2\ 4 \\ 2\ 5 \\ 2\ 6 \\ 3\ 4 \\ 3\ 5 \\ 3\ 6 \end{pmatrix}$$

Using the relation, $\sum x_{iu}^6 = 5 \sum x_{iu}^2 x_{ju}^4$, we obtain,

$4 = 5 \times 1]2^3$ design points.

$\Rightarrow 32 = 40$ *a(i)* (4.1.14)

Let *inc*. PBD-C_2($v = 6, b_2 = 15\ , r_2 = 5,\ k_{21} = 2, \lambda_2 = 1$) be given as follows;

Table 2: Incidence Matrix of PBD-C_2 ($\boldsymbol{v = 6, b_2 = 15\ , r_2 = 5,\ k_{21} = 2, \lambda_2 = 1}$)

t_1	t_2	t_3	t_4	t_5	t_6
1	1	0	0	0	0
1	0	1	0	0	0
1	0	0	1	0	0
1	0	0	0	1	0
1	0	0	0	0	1
0	1	1	0	0	0
0	1	0	1	0	0
0	1	0	0	1	0
0	1	0	0	0	1
0	0	1	1	0	0
0	0	1	0	1	0
0	0	1	0	0	1
0	0	0	1	1	0
0	0	0	1	0	1
0	0	0	0	1	1

This can as well be written in block form as:

$$\begin{pmatrix} 1\ 2 \\ 1\ 3 \\ 1\ 4 \\ 15 \\ 1\ 6 \\ 2\ 3 \\ 2\ 4 \\ 2\ 5 \\ 2\ 6 \\ 3\ 4 \\ 3\ 5 \\ 3\ 6 \\ 4\ 5 \\ 4\ 6 \\ 5\ 6 \end{pmatrix}$$

Using the relation, $\sum x_{iu}^{6} = 5\sum x_{iu}^{2} x_{ju}^{4}$, we obtain,

$5 = 5 \times 1]2^2$ design points.

$\Rightarrow 20 = 20$ *a(ii)* (4.1.15)

Repeating the set of points C_1, *a(i)*(4.1.14), zero times and in design C_2, *a(ii)*(4.1.15), two times (since n_1= 0 and n_2= 2), we obtain,

$$
\begin{array}{lcc}
C_1 - [32 = 40] \times 0 & & 0 \leftrightarrow 0 \\
+ C_2 - [20 = 20] \times 2 & \Rightarrow & 40 \leftrightarrow 40 \\
\hline
& & 40 = 40
\end{array}
\qquad (4.1.16)
$$

From this it is observed that the number of design points obtained is much less than the number of design points earlier obtained, i.e the design points earlier obtained is N=200 from equation (4.1.12) and the designs points obtained after is N=40 as seen from equation (4.1.16). This shows a great reduction in the number of design points thus it cuts on costs of experimentation.

4.3 Five-level modified third order Rotatable Designs Using a Pair of PBD

The method of constructing a modified TORD of five-level by use of a properly chosen pair of pairwise balanced designs (PBD) with no other additional set of points was obtained

Definition

Allow $C_I = (v, b_i\ , r_i,\ k_{i1},\ k_{i2,}\ \ldots\ ,k_{ip},\ \lambda_i)$, for i=1,2, be an equi-replicated PBD where *v*- number of treatments in an experiment, b_i- number of blocks , r_i- number of times a treatment is replicated k_1, k_2… K_p- block sizes and λ_i- number of times each pair is replicated. Let $2^{t(k_i)}$ denote the resolution V fractional replicates of $2^{(k_i)}$ factorials with +1 or -1 levels in treatments with $r_1 \leq 5\lambda_1$ and $r_2 \geq 5\lambda_2$ respectively.

Let $\left[1-(v, b_1, r_1, k_{11}, k_{12}, \ldots, k_{1p}, \lambda_1)\right]2^{t(k_1)}$ and $\left[a-(v, b_2, r_2, k_{21}, k_{22}, \ldots, k_{2p}, \lambda_2)\right]2^{t(k_2)}$ denote $b_1 2^{t(k_1)}$ and $b_2 2^{t(k_2)}$ design points generated from PBD by multiplication respectively. The set of $b_1 2^{t(k_1)}$design points generated from PBD-C_1 is repeated n_1-times and the set of $b_2 2^{t(k_2)}$design points generated from the PBD-C_2 is repeated n_2-times respectively.

Let n_o denote the number of central points. Then with the above design points, $n_i b_i 2^{t(k_i)}$, we construct a five level modified TORD as given in the theorem below.

Theorem 4.2

Let a set be given by the following design points

$n_1[1-$

$(v, b_1, r_1, k_{11}, k_{12}, \ldots, k_{1p}, \lambda_1)]2^{t(k_1)} U\, n_2\left[a(v, b_2, r_2, k_{21}, k_{22}, \ldots, k_{2p}, \lambda_2)\right]2^{t(k_2)}$U n_o

Where vi- number of treatments in an experiment

b_i- number of blocks

r_i- number of times a treatment is replicated

$k_{i1}, k_{i2} \ldots K_{ip}$- block sizes

λ_i- number of times each pair is replicated

and i = 1,2

Gives a five level v-dimensional modified TORD in,

$$N = \frac{\left[n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^2\right]^2}{\left[n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)} a^4\right]} \tag{4.2.1}$$

Design points if;

$$(r_1 - 5\lambda_1)(r_2 - 5\lambda_2) \leq 0, \tag{4.2.2}$$

$$a^6 = \frac{n_1(r_1 - 5\lambda_1)2^{t(k_1)-t(k_2)}}{n_2(5\lambda_2 - r_2)}, \tag{4.2.3}$$

$$n_o = \frac{\left[n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^2\right]^2}{\left[n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)} a^4\right]} - n_1\, b_1 2^{t(k_1)} - \; n_2 b_2 2^{t(k_2)} \tag{4.2.4}$$

And n_o turns out to be an integer.

Proof

In support of the design points obtained, n_1-repetitions of points from PBD-C_1 and n_2-repetitions of points from PBD-C_2, for a modified TORD to be true, the conditions are as follows:

$$\sum x_{iu}^2 = \; n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^2 = X \tag{4.2.5}$$

$$\sum x_{iu}^4 = \; n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^4 = 3Y \tag{4.2.6}$$

$$\sum x_{iu}^2 x_{ju}^2 = \; n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)} a^4 = Y \tag{4.2.7}$$

$$\sum x_{iu}^6 = \; n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^6 = 15Z \tag{4.2.8}$$

$$\sum x_{iu}^2 x_{ju}^4 = \; n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)} a^6 = 3Z \tag{4.2.9}$$

Where $X = N\lambda_2$, $Y = \; N\lambda_4$ and $Z = N\lambda_6$

From (4.2.8) and (4.2.9), we have,

$$n_1 \lambda_1 2^{t(k_1)} a^6 + n_2 \lambda_2 2^{t(k_2)} a^6 = 5\left[n_1 \lambda_1 2^{t(k_1)} a^6 + n_2 \lambda_2 2^{t(k_2)} a^6\right] \tag{4.2.10}$$

This leads to,

$a^6 = \frac{n_1(r_1 - 5\lambda_1)2^{t(k_1)-t(k_2)}}{n_2(5\lambda_2 - r_2)}$, given in equation (4.2.3).

The modified condition $(\sum x_{iu}^2)^2 = N \sum x_{iu}^2 x_{ju}^2$ leads to

$N = \frac{[n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^2]^2}{[n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)} a^4]}$, given in equation (4.2.1).

Given n_o central points, N may be obtained directly as

$$N = n_1 b_1 2^{t(k_1)} + n_2 b_2 2^{t(k_2)} + n_o. \qquad (4.2.11)$$

Example 4.3.1

Let a set of PBDs be given with the following design points

$n_1[a - (v = 9, b_1 = 13, r_1 = 4, k_{11} = 4, k_{12} = 3, k_{13} = 2, \lambda_1 = 1)]2^4 U\, n_2[a - (v = 9, b_2 = 30, r_2 = 7, k_{21} = 3, k_{22} = 2, \lambda_2 = 1)]2^3 \mathrm{U}\, n_o$

From the above PBDs we obtain

$$\frac{n_2}{n_1} = \frac{(4-5)2^{4-3}}{5-7} = \frac{2}{2} \qquad (4.2.12)$$

$$\mathrm{N} = \frac{[2*4*2^4+2*7*2^3]^2}{[2*1*2^4+2*1*2^3]} = \frac{[128+112]^2}{32+16} = \frac{240^2}{48} = 1200 \qquad (4.2.13)$$

$$n_0 = 1200 - (2*13*2^4) - (2*30*2^3) = 1200 - 416 - 480 = 304 \qquad (4.2.14)$$

$$a^6 = \frac{2(4-5)2^{4-3}}{2(5-7)} = \frac{-4}{-4} = 1 \qquad (4.2.15)$$

Which gives a five-level 9-dimensional modified third order rotatable design in N= 1200 design points with $n_1 = 2,\ n_2 = 2$. In this case (4.2.2) gives a^6=1 and (4.2.3) gives n_o = 304

The values of n_1, n_2, N, n_0 and a^6 are as shown from workings on the equations (4.2.12), (4.2.13), (4.2.14) and (4.2.15) respectively.

Consider two pairwise balanced designs, PBD-C_1 $(v = 9, b_1 = 13, r_1 = 4, k_{11} = 4, k_{12} = 3, k_{13} = 2, \lambda_1 = 1)$ and PBD-C_2 $(v = 9, b_2 = 30, r_2 = 7, k_{21} = 3, k_{22} = 2, \lambda_2 = 1)$. The design sets can be represented as incident matrix as follows:

Let *inc*. PBD-C_1 ($v = 9, b_1 = 13\ , r_1 = 4,\ k_{11} = 4, k_{12}\ = 3, k_{13}\ = 2, \lambda_1 = 1$)be given as follows;

Table 3: Incidence Matrix of PBD-C_1 ($\boldsymbol{v = 9, b_1 = 13\ , r_1 = 4,\ k_{11} = 4, k_{12}\ = 3, k_{13}\ = 2, \lambda_1 = 1}$)

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9
1	1	1	1	0	0	0	0	0
1	0	0	0	1	1	0	0	0
0	1	0	0	1	0	1	0	0
0	0	1	0	1	0	0	1	0
0	0	0	1	1	0	0	0	1
1	0	0	0	0	0	1	0	1
0	1	0	0	0	0	0	1	1
0	0	1	0	0	1	0	0	1
0	0	0	1	0	0	1	1	0
1	0	0	0	0	0	0	1	0
0	1	0	0	0	1	0	0	0
0	0	1	0	0	0	1	0	0
0	0	0	1	0	1	0	0	0

This can as well be written in block form as:

$$\begin{pmatrix} 1\ 2\ 3\ 4 \\ 1\ 5\ 6 \\ 2\ 5\ 7 \\ 3\ 5\ 8 \\ 4\ 5\ 9 \\ 1\ 7\ 9 \\ 2\ 8\ 9 \\ 3\ 6\ 9 \\ 4\ 7\ 8 \\ 1\ 8 \\ 2\ 6 \\ 3\ 7 \\ 4\ 6 \end{pmatrix}$$

Using the relation, $\sum x_{iu}^{6} = 5\sum x_{iu}^{2}x_{ju}^{4}$, we obtain,

$4 = 5 \times 1]2^{4}$ design points.

$\Rightarrow 64 = 80$ *a(i)* (4.2.16)

Let *inc*. PBD-C_2 ($v = 9, b_2 = 30, r_2 = 7,\ k_{21} = 3, k_{22} \ = 2, \lambda_2 = 1$)be given as follows;

Table 4: Incidence Matrix of PBD-C_2 ($\boldsymbol{v = 9, b_2 = 30, r_2 = 7,\ k_{21} = 3, k_{22} \ = 2, \lambda_2 = 1}$)

t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9
1	1	1	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	0	0	1	1	1
1	0	0	1	0	0	0	0	0
1	0	0	0	1	0	0	0	0
1	0	0	0	0	1	0	0	0
1	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	1	0
1	0	0	0	0	0	0	0	1
0	1	0	1	0	0	0	0	0
0	1	0	0	1	0	0	0	0
0	1	0	0	0	1	0	0	0
0	1	0	0	0	0	1	0	0
0	1	0	0	0	0	0	1	0
0	1	0	0	0	0	0	0	1
0	0	1	1	0	0	0	0	0
0	0	1	0	1	0	0	0	0
0	0	1	0	0	1	0	0	0
0	0	1	0	0	0	1	0	0
0	0	1	0	0	0	0	1	0
0	0	1	0	0	0	0	0	1
0	0	0	1	0	0	1	0	0
0	0	0	1	0	0	0	1	0
0	0	0	1	0	0	0	0	1
0	0	0	0	1	0	1	0	0
0	0	0	0	1	0	0	1	0
0	0	0	0	1	0	0	0	1
0	0	0	0	0	1	1	0	0
0	0	0	0	0	1	0	1	0
0	0	0	0	0	1	0	0	1

This can as well be written in block form as:

$$\begin{pmatrix} 1\ 2\ 3 \\ 4\ 5\ 6 \\ 7\ 8\ 9 \\ 14 \\ 1\ 5 \\ 1\ 6 \\ 1\ 7 \\ 1\ 8 \\ 1\ 9 \\ 2\ 4 \\ 2\ 5 \\ 2\ 6 \\ 2\ 7 \\ 2\ 8 \\ 2\ 9 \\ 3\ 4 \\ 3\ 5 \\ 3\ 6 \\ 3\ 7 \\ 3\ 8 \\ 3\ 9 \end{pmatrix}$$

$$\begin{pmatrix} 4\,7 \\ 4\,8 \\ 4\,9 \\ 5\,7 \\ 5\,8 \\ 5\,9 \\ 6\,7 \\ 6\,8 \\ 6\,9 \end{pmatrix}$$

Using the relation, $\sum x_{iu}^6 = 5\sum x_{iu}^2 x_{ju}^4$, we obtain,

$7 = 5 \times 1]2^3$ design points.

$\Rightarrow 56 = 40$ *a(ii)*(4.2.17)

Repeating the set of points C_1,(*a(i)*), zero times and in design C_2, (*a(ii)*), two times (since n1= 0 and n2= 2), we obtain,

C_{1} – $[64 = 80] \times 2$ $\quad$ $128 \leftrightarrow 160$

$+$ $\quad \Rightarrow$ $112 \leftrightarrow 80$

C_2 - $[56 = 40] \times 2$ $\quad$ $\underline{\underline{240 = 240}}$ (4.2.18)

From this it is crystal clear that the number of design points obtained is much less than the number of design points earlier obtained, i.e the design points earlier obtained is N=1200 from equation (4.2.13) and the designs points obtained after is N=240 as obtained from equation (4.2.18) . This shows a great reduction in the number of design points thus it cuts on costs of experimentation.

4.4 Summary of Results

A three level modified third order rotatable design was constructed using a suitably chosen pair of pairwise balanced designs (PBD) without any additional set of points. In example 4.2.1, the number of design points obtained is more less than the design points earlier obtained i.e final N=40 design points as obtained from the working of equation (4.1.16) after repeating a constant number of times n_1 and n_2 respectively, down from N=240 design points as obtained from equation (4.1.12). This showed that, the set of the design points;

$n_1[a - (v, b_1, r_1, k_{11}, k_{12}, \ldots, k_{1p}, \lambda_1)]2^{t(k_1)} U\, n_2[a - (v, b_2, r_2, k_{21}, k_{22}, \ldots, k_{2p}, \lambda_2)]2^{t(k_2)} \text{U}\, n_o$ gives a three level v-dimensional modified TORD in,

$N = \frac{[n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)}]^2}{[n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)}]}$ design points, as given in equation (4.1.1)

Or

$N = n_1 b_1 2^{t(k_1)} + n_2 b_2 2^{t(k_2)} + n_o$. When given n_o central points. This is also given in equation (4.1.11)

A five level modified third order rotatable design was constructed using a suitably chosen pair of pairwise balanced designs (PBD) without any additional set of points. In example 4.3.1, the number of design points obtained is much less than the design points obtained before i.e final N=240 design points as obtained from equation (4.2.18) after repeating when n_1= 2 times and n_2=2 times respectively, down from N=1200 design points as shown from the working from equation (4.2.13). This showed that, the set of the design points;

$n_1[1(v, b_1, r_1, k_{11}, k_{12}, \ldots, k_{1p}, \lambda_1)]2^{t(k_1)} U\, n_2[a(v, b_2, r_2, k_{21}, k_{22}, \ldots, k_{2p}, \lambda_2)]2^{t(k_2)}$ U n_o gives a five level v-dimensional modified TORD in,

$N = \frac{[n_1 r_1 2^{t(k_1)} + n_2 r_2 2^{t(k_2)} a^2]^2}{[n_1 \lambda_1 2^{t(k_1)} + n_2 \lambda_2 2^{t(k_2)} a^4]}$, design points, under specified restrictions as given in equation (4.2.1)

Or

$N = n_1 b_1 2^{t(k_1)} + n_2 b_2 2^{t(k_2)} + n_o$. When given n_o central points. This is given in equation (4.2.11)

Thus the method of construction of a three level and a five level modified third order rotatable design was constructed using a suitably chosen pair of pairwise balanced designs (PBD) without any additional set of points. This gave some modified third order rotatable designs with less number of design points constructed through pairwise balanced designs, as compared to existing designs of the same dimensions hence they are cost effective.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The construction of rotatable designs using pairwise balanced designs has been studied by a number of authors in the previous years. This study explored construction of a three level and a five level modified third order rotatable designs using pairwise balanced designs.

A three level modified third order rotatable design was constructed using a suitably chosen pair of pairwise balanced designs (PBD) without any additional set of points by repeating the set of the design points generated from each of the designs a constant number of times. These points were combined together with a number of central points without any additional set of points. This was shown in example 4.2.1, where the number of design points obtained were fewer than the design points earlier obtained i.e finally N=40 design points as obtained from the working of equation (4.1.16) after repeating a number of times that is when $n_1=0$ and $n_2=2$ respectively, this is down from N=240 design points as obtained earlier from equation (4.1.12).

Again, a five level modified third order rotatable design was constructed using a suitably chosen pair of pairwise balanced designs (PBD) without any additional set of points by repeating the set of the design points generated from each of the designs a constant number of times. These points were combined together with a number of central points without any additional set of points. This was worked in example 4.3.1, where the number of design points obtained were much fewer than the design points earlier obtained i.e finally N=240 design points as obtained from equation (4.2.18) after repeating when n_1= 2 times and n_2=2 times respectively, down from the earlier obtained N=1200 design points as shown from the working from equation (4.2.13).

Both three-level and five-level modified third order rotatable design was constructed using a suitably chosen pair of pairwise balanced designs (PBD) without any additional set of points. Designs with fewer number of design points relative to the existing corresponding designs were obtained.

The implications of fewer number of design points leads to effective and reduced cost of experimentation.

5.2 Recommendations

Other studies that could possibly lead to designs with fewer number of design points than what was obtained in the present study could be explored. One may consider studying construction of modified higher order of slope rotatable designs. Further investigations can still be explored in line with this study on modified fourth order and higher order designs. Not much work is available with regard to constructions of designs in this area. Another area in which one may be interested in is to study some new methods of construction of modified Group-Divisible third order Rotatable designs designs using pairwise balanced designs, central composite designs, balanced incomplete block designs, etc.

The study recommends practical applications of the designs obtained. This study can be useful in agricultural experiments for example change of yield of a crop in response to various fertilizer doses and in chemical industries the rate of reaction in chemical experiments among other areas where maximization of a process is essential.

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APPENDIX

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