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The Role of Shell Model in Determining Pairing Interaction in Nuclei

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Abstract- **The role of the shell model in pairing interaction in nuclei is investigated by calculating the pairing energies of O–O (odd–odd), O–E (odd–even), E–E (even–even) and E–O (even–odd) isotopes of four elements namely; $_{15}$P, $_{25}$Mn, $_{40}$Zr and $_{60}$Nd. The pairing energies were computed using the values of binding energy (B) obtained from AME2016 atomic mass evaluation. The graphs of the pairing energies against the mass numbers revealed an increase of the pairing energies with the occurrence of periodic humps adjacent to the neutron magic numbers. The rise in the pairing energies is attributed to the bound states of heavy nuclei arising from the neutron-neutron pairs in the shell structures beyond the Fermi-surface, while in the light and intermediate nuclei, the rise in the pairing energies is due to the increase in the number of nucleons occupying the surface region. These neutron-neutron pairs formed beyond the Fermi-surface of nuclei of heavy elements have greater pairing energies, which contribute to greater binding energies associated with the heavy elements found in the neutron stars. It is concluded that the shell model can predict the existence of isotopes of heavy elements in the neutron stars and the criterion to ascertain their existence lies on the pairing energies. Calculations show that the isotopes with highest peaks among the heavy elements can predict the most abundant isotopes of the heavy elements in the neutron stars, for instance, ^{90}Zr, ^{91}Zr, ^{142}Nd, ^{145}Nd, ^{146}Nd, ^{148}Nd and ^{150}Nd nuclei.**

Keywords— binding energy, Pairing energy, shell model, isotopes

I. INTRODUCTION

The shell model [1] has made enormous contributions in describing the structure of the nucleus. This model is credited for its description of the well-known magic numbers, the spin-orbit interaction, spin-parity, the magnetic moments of nuclei and now the pairing interaction. The pairing of nucleons in the shell structures is one of the long-studied topics in nuclear physics since it explains many experimental facts.

Several scientists [2-7] have carried out investigations, spread over decades, on the nucleus of an atom with a view of understanding the nature of inter-nucleon interaction that exists in the nucleus. This includes the application of symmetry concepts to spin and iso-spin degrees of freedom [2-3] and the linking of the shell model with collective structure [4] using SU (3) model whose collective effects in atomic nuclei are described as elementary modes [5]. The backbone of these advances lies solidly on the nuclear models that have been proposed and confirmed experimentally. One of them is the liquid drop model [5], which despite explaining the spherical shape of most nuclei and predicting the binding energy of the nuclei, is not very successful in describing the low lying excited states of the nucleus due to the collective motions of the large number of the nucleons involved [6]. The Fermi-gas model is another model, which is quite simple and provides invaluable insights into the nuclear structure by assuming the nucleus as a degenerate gas of protons and neutrons [7]. Fermi gas model incorporates the concepts of quantum mechanics thus enriching the prediction of the binding energy proposed in the liquid drop model, by adding the volume energy term and the asymmetry energy in the Bethe-Weizsäcker mass formula. The limitation facing the Fermi gas model is that, it cannot predict the detailed properties of the low-lying states of nuclei observed in the radioactive decay processes [8]. However, the Fermi gas model successfully explains the properties of nuclei in the excited states and the unbound states of heavy and medium nuclei thus providing the basic understanding of the fermion system. The failure of the liquid drop model to explain the low-lying excited states in nuclei that are not closely spaced energy levels gave rise to the shell model [6].

The shell model considers the motion of individual nucleons in a potential well thus resulting in the existence of nuclear energy shells of varying energies at respective principal quantum numbers. It was observed that the occurrence of the so-called magic numbers; 2, 8, 28, 50, 82 and 126 from experimental point of view has been one of the strongest motivations for the formulation of the nuclear shell model where the magic numbers correspond to the shell closures of the nuclear energy shells in the atoms [9]. The shell model also succeeded in describing the spin-parity of the nuclei and the magnetic moments even though it has a limitation over the unknown underlying form of the potential.

Another model is the collective model, which is a merger of the shell model and the liquid drop model. This model considers the collective properties of the nucleon as a whole by considering the oscillations and the nuclear excitations of the liquid drop vibrating at high frequency. Even though the liquid drop model assumes the spherical shape of the nucleus, the collective model introduces the non-sphericity resulting from the rotation and vibration of nucleus that is necessary in explaining the many shortcomings in the pure shell model and for predicting additional rotational and vibration energy levels [12].

A closer review of all the nuclear models shows gradual advances in understanding the structure of the nuclei. However, the shell model stands out to be unique and sophisticated model that is the most successful in predicting the structure of most of the nuclei. It is in this regard that we use this model to study its role in determining the pairing interaction in the nuclei.

II. PAIRING INTERACTION OF NUCLEI

The Pairing interaction is one of the long-standing problems of nuclear structure, which was first investigated in even-odd staggering of binding energies by [11] and it remains to be one of the long-studied topics in nuclei and nuclear matter [12]. Accordingly, particle pairing [13] is an important interaction that is widely used in nuclear physics and other branches of physics since it allows us to understand many important experimental facts such as the energy gap, the level density, odd-even effect, moment of inertia, low-lying 2^+ states and deformations [9].

Experiments [6] have shown that the pairing energy depends on the mass number (A). A comparison between even A nuclei (even Z(proton number) and even N (neutron number)) and odd A nuclei (even Z and odd N or odd Z and even N) shows that the odd A nuclei are more strongly bound than odd-odd nuclei but they are less strongly bound when compared with even-even nuclei. This is because of the pairing of the nucleons of the same type with opposite spin in the shell structure. The pairing energy term increases the binding energy of the nucleons in that it is maximum for even-even nuclei since all nucleons with opposite spin form a pair, while for odd-even and even-odd nuclei there is one unpaired nucleon. For odd-odd nuclei, there are two unpaired nucleons which results in further weakening of the binding energies [6]. In our calculations, we investigated the role of the shell model in determining the pairing interaction in the nuclei $_{15}P$, $_{25}Mn$, $_{40}Zr$ and $_{60}Nd$. We propose that, the nuclei with large pairing energy may be in abundance and this may be the criteria, which can explain the existence of the heavy elements in the neutron stars.

III. THEORETICAL DERIVATIONS

The existence of odd-odd, odd-even, even-even and even-odd nuclei has influenced our choice of the following four nuclei for our study; $_{15}P$, $_{25}Mn$, $_{40}Zr$ and $_{60}Nd$. For $_{15}P$ and $_{25}Mn$, they constitute odd-odd and odd-even nuclei with two and one unpaired nucleon(s) respectively since their atomic number (Z) is odd, whereas $_{40}Zr$ and $_{60}Nd$ are even-even and even-odd nuclei with paired and one unpaired nucleon, respectively, since the atomic number (Z) is even. As the protons and neutrons pair up in the nuclear shell structure, the pairing energy affects the binding energy of the nucleons which is given by the Bethe-Weizsäcker mass formula [14] which is written as;

$$B(A, Z) = a_V A - a_S A^{2/3} - a_C Z(Z-1) A^{-1/3} - a_{Sym}(A-2Z)^2 A^{-1} + \delta \qquad (1)$$

Where $a_V = 15.99$ Mev, $a_S = 18.34$ Mev, $a_C = 0.71$ Mev, $a_{Sym} = 23.21$ Mev and δ is the pairing energy correction.

The aim is to study the role of shell model in determining the pairing interaction in Phosphorus (P), Manganese (Mn), Zirconium (Zr) and Neodymium (Nd), isotope wise, on the earth, and to establish a criterion for their existence in the collapsed neutron star merger. The pairing energy can sometimes be written as shown in Eq. (2) below [15];

$$P_n(A, Z) = \tfrac{1}{4}(-1)^{A-Z+1}[-M(A+1, Z) + 3M(A, Z) - 3M(A-1, Z) + M(A-2, Z)] \qquad (2)$$

Where P_n (A, Z) is the pairing energy, A is the mass number of the element, Z is the atomic number, and the right hand side gives the combination of the involved masses. However, the mass excess free of the bound nucleus can be written as ΔM, defined as the difference between the total mass of nucleons and the combined mass of the nucleus [6] such that,

$$\Delta M = ZM_P + NM_N - M(A, Z) \qquad (3)$$

Where Z is the proton number, N is the neutron number, M_P is the proton mass, M_N is the neutron mass and A is the mass number given by A = N + Z.

The mass excess of any element can also be expressed as a function of binding energy per nucleon relative to the binding energy per nucleon of carbon-12. Therefore, Eq. (2) can be modified and we use the binding energy (B) instead of the mass (M) to calculate the pairing energies P_n (A, Z) such that,

$$P_n(A, Z) = \tfrac{1}{4}(-1)^{A-Z+1}[-B(A+1, Z) + 3B(A, Z) - 3B(A-1, Z) + B(A-2, Z)] \qquad (4)$$

To calculate P_n (A, Z), the values of binding energy (B), are taken from AME2016 that gives atomic mass evaluation [15], and they are also used in the final calculations.

IV. RESULTS AND DISCUSSION

The Table 1 shows the calculated pairing energies of O–O (odd–odd) and O–E (odd-even) isotopes of $_{15}P$ and $_{25}Mn$ and E–E (even–even) and E–O (even–odd) isotopes of $_{40}Zr$ and $_{60}Nd$ calculated using Eq.(4). Our calculations show that as the mass number (A) increases the pairing energies (P_N) also increase. However, some unusual rises in the values of (P_N) are obtained for some isotopes. For instance, $_{15}P$ and $_{25}Mn$ isotopes have unusual rise at A=28, A=36, A=42 and A=52, A=64 for O–O, respectively. Similarly, the same behavior is seen at A=29, A=35 and A=53, A=64, A=70 for O–E nuclei of Phosphorus and Manganese isotopes, respectively. The calculations also reveal that, Zirconium and Neodymium isotopes have an increased number of peaks at, A=82, A=90, A=95, A=100 and A=130, A=142, A=148, A=149, A=150, A=156, for E–E nuclei. They are also noted at A=81, A=91, A=99, A=103, A=109 and A=131, A=143, A=149, A=156 for E–O nuclei of Zirconium and Neodymium isotopes, respectively. The graphs shown in Fig.1 to Fig. 8 illustrate the curves for increasing pairing energies with increase in the mass numbers with occurrence of periodic humps in the isotopes having unusual rises in the pairing energies. We deduced that these periodic humps represent the existence of most stable isotopes and the presence of shell closures among the isotopes. The periodic humps can also predict the abundance of heavy elements, that is, the isotopes with highest peaks among the isotopes of Zirconium and Neodymium. For instance, ^{90}Zr, ^{91}Zr and ^{142}Nd, ^{143}Nd, ^{148}Nd and ^{150}Nd were found to be the most abundant isotopes when compared with the values obtained from the table of isotopic masses and natural abundance in Table 2. Two isotopes namely ^{81}Zr and ^{130}Nd, did not satisfy the criterion since they fall under isotopes with neutron magic numbers and the regions where we may experience shell closures. However, the criteria of peaks in the binding energy (pairing energy)-mass number diagrams can be used to predict the abundance of isotopes of rare heavy elements in the neutron stars.

TABLE1: The table of calculated Pairing Energies (P_N) for $_{15}P$, $_{25}Mn$, $_{40}Zr$ and $_{60}Nd$

PHOSPHORUS (P)			MANGANESE (Mn)			ZIRCONIUM (Zr)			NEODYMIUM (Nd)		
(A)	*O-O (Z-N)*	*P_N (keV)*	*(A)*	*O-O (Z-N)*	*P_N (keV)*	*(A)*	*E-E (Z-N)*	*P_N (keV)*	*(A)*	*E-E (Z-N)*	*P_N (keV)*
26	15-11	-77.85	46	25-21	-42.58	80	40-40	-21.15	126	60-66	-9.750
28	15-13	-80.25	48	25-23	-26.07	82	40-42	-21.36	128	60-68	-10.00
30	15-15	-66.72	50	25-25	-20.32	84	40-44	-20.88	130	60-70	-10.81
32	15-17	-52.86	52	25-27	-22.95	86	40-46	-18.85	132	60-72	-9.960
34	15-19	-44.75	54	25-29	-20.83	88	40-48	-16.96	134	60-74	-9.670
36	15-21	-58.62	56	25-31	-19.70	90	40-50	-20.64	136	60-76	-9.258
38	15-23	-37.75	58	25-33	-15.70	92	40-52	-9.110	138	60-78	-8.150
40	15-25	-28.48	60	25-35	-15.15	94	40-54	-8.657	140	60-80	-8.173
42	15-27	-31.50	62	25-37	-14.55	96	40-56	-9.556	142	60-82	-9.733
44	15-29	-17.75	64	25-39	-16.31	98	40-58	-7.270	144	60-84	-6.536
46	15-31	-16.00	66	25-41	-12.06	100	40-60	-11.06	146	60-86	-7.009
			68	25-43	-12.30	102	40-62	-9.410	148	60-88	-7.343
			70	25-45	-11.25	104	40-64	-9.290	150	60-90	-7.328
						106	40-66	-5.850	152	60-92	-6.552
						108	40-68	-7.500	154	60-94	-5.416
						110	40-70	-9.000	156	60-96	-6.335
									158	60-98	-5.305
									160	60-100	-6.000
(A)	*O-E (Z-N)*	*P_N (keV)*	*(A)*	*O-E (Z-N)*	*P_N (keV)*	*(A)*	*E-O (Z-N)*	*P_N (keV)*	*(A)*	*E-O (Z-N)*	*P_N (keV)*
27	15-12	-75.18	47	25-22	-31.93	79	40-39	-15.00	127	60-67	-9.750
29	15-14	-85.29	49	25-24	-24.96	81	40-41	-23.87	129	60-69	-10.38
31	15-16	-42.52	51	25-26	-18.21	83	40-43	-21.62	131	60-71	-10.86
33	15-18	-45.51	53	25-28	-21.84	85	40-45	-20.16	133	60-73	-9.768
35	15-20	-50.03	55	25-30	-19.25	87	40-47	-18.30	135	60-75	-9.620
37	15-22	-44.51	57	25-32	-15.88	89	40-49	-16.08	137	60-77	-8.528
39	15-24	-34.98	59	25-34	-15.35	91	40-51	-17.32	139	60-79	-8.466
41	15-26	-27.60	61	25-36	-13.68	93	40-53	-9.165	141	60-81	-7.368
43	15-28	-26.40	63	25-38	-15.32	95	40-55	-8.333	143	60-83	-9.508
45	15-30	-16.00	65	25-40	-15.77	97	40-57	-8.126	145	60-85	-6.702
			67	25-42	-10.32	99	40-59	-11.24	147	60-87	-7.364
			69	25-44	-11.75	101	40-61	-8.968	149	60-89	-7.795
			71	25-46	-11.25	103	40-63	-9.468	151	60-91	-6.621
						105	40-65	-8.318	153	60-93	-5.478
						107	40-67	-5.335	155	60-95	-6.080
						109	40-69	-9.250	157	60-97	-6.030
						111	40-71	-8.250	159	60-99	-5.510
									161	60-101	-6.000

TABLE2: The table of natural abundance of $_{40}Zr$ and $_{60}Nd$, adapted from [16].

ISOTOPES	**% ABUNDANCE**	**ISOTOPES WITH HIGHEST PEAKS AND THE MOST ABUNDANT ISOTOPE**
^{90}Zr	51.45	^{90}Zr ^{91}Zr ^{81}Zr
^{91}Zr	11.22	
^{92}Zr	17.15	
^{94}Zr	17.38	
^{96}Zr	2.8	
^{142}Nd	27.2	^{142}Nd ^{143}Nd ^{130}Nd ^{148}Nd ^{150}Nd
^{143}Nd	12.2	
^{144}Nd	23.8	
^{145}Nd	8.3	
^{146}Nd	17.2	
^{148}Nd	5.7	
^{150}Nd	5.6	

The graphs of $_{15}$P and $_{25}$Mn shown in Fig. 1 to Fig. 4, fall under the category of light nuclei and intermediate-mass. These nuclei have 2-3 periodic humps indicating greater stability compared to the graphs in Fig. 5 to Fig. 8 for $_{40}$Zr and $_{60}$Nd that have 4-5 periodic humps indicating the potential of instability. The neutron-neutron pairs formed beyond the Fermi-surface of nuclei studied are observed to have greater pairing energies, which contributes to the greater binding energy and stability, thus, explaining as to why the neutron stars are strongly bound with heavy elements that may include Zirconium, Neodymium among others. It is therefore suggested that the criterion to decide the existence of isotopes of these elements in the neutrons stars should be the values of pairing energies of the nucleons in the nuclei.

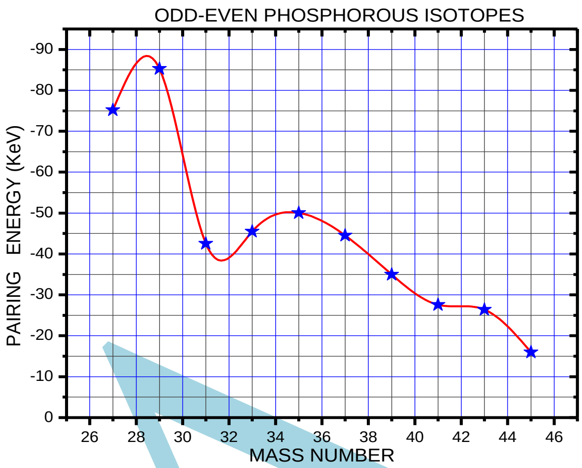

Fig.1. The graph of Odd-Even Phosphorus Isotopes

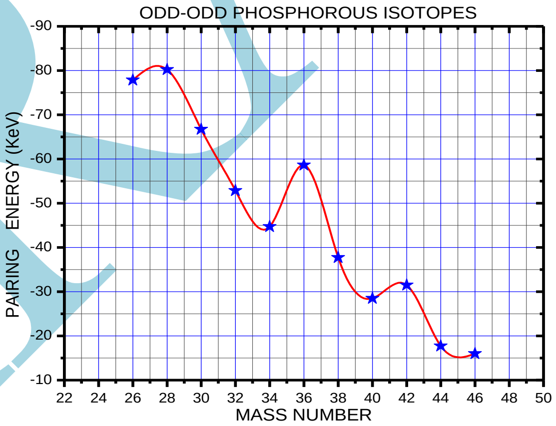

Fig.2. The graph of Odd-Odd Phosphorus Isotopes

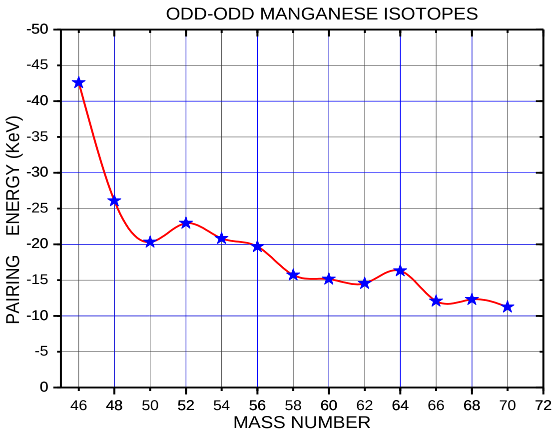

Fig.3. The graph of Odd-Odd Manganese Isotopes

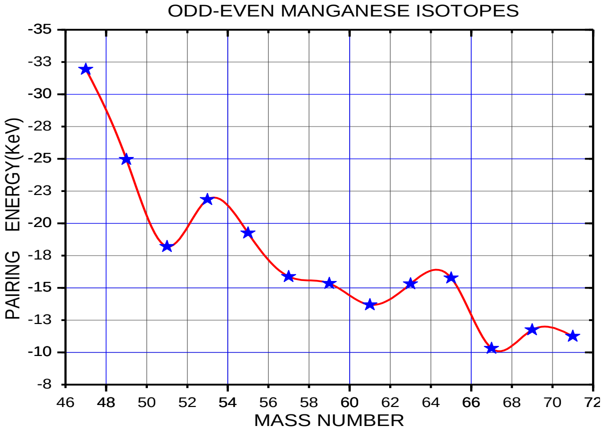

Fig.4. The graph of Odd-Even Manganese Isotopes

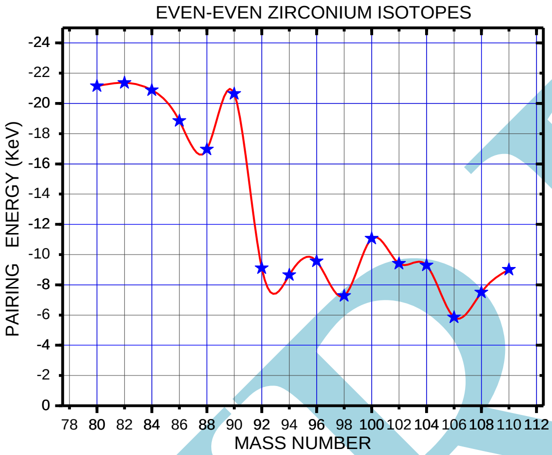

Fig.5. The graph of Even-Even Zirconium Isotopes

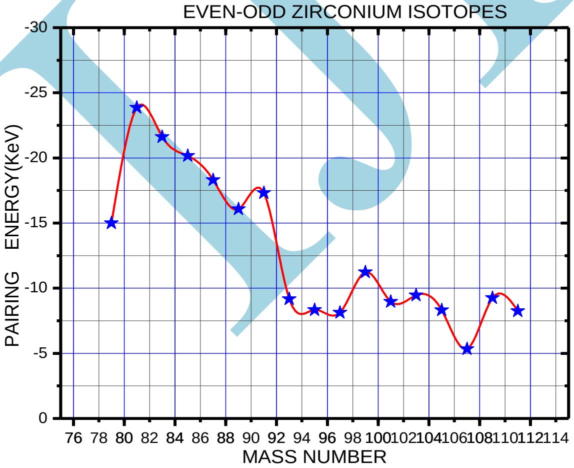

Fig.6. The graph of Even-Odd Zirconium Isotopes

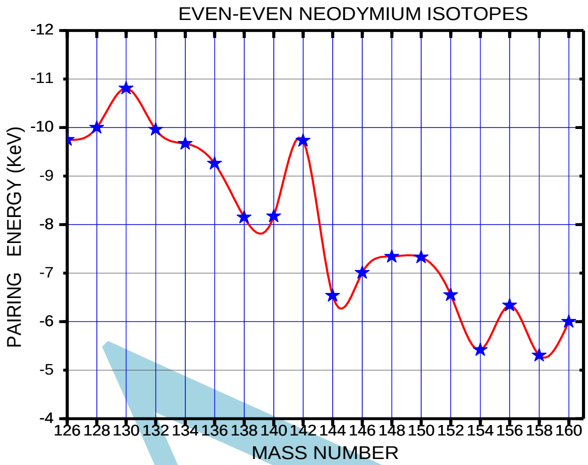

Fig.7. The graph of Even-Even Neodymium Isotopes

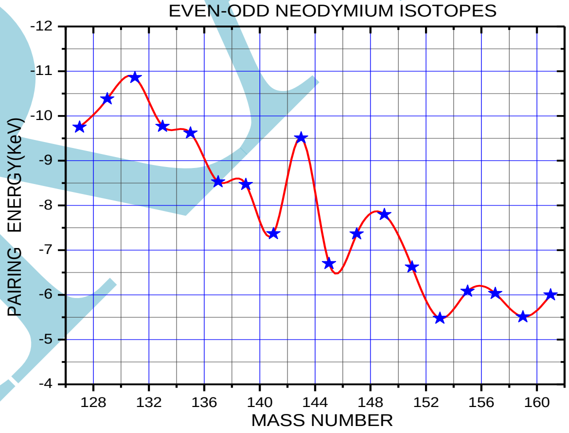

Fig.8. The graph of Even-odd Neodymium Isotopes

V. CONCLUSION

The calculations have shown that the shell model can explain the pairing interaction of O–O (odd–odd), O–E (odd–even), E–E (even–even) and E–O (even–odd) nuclei in the shell structure. It is noted that, the pairing interaction of the nucleons in the shell model predicts the occurrence of the neutron magic numbers as observed by the unusual rises of pairing energies (P_N) corresponding to periodic humps in the graphs of pairing energies against the mass numbers. These periodic humps can also show the stability of the isotopes of different elements. Finally, the shell model in conjunction with the pairing energy can predict the criterion for the existence of rare isotopes of different elements in the neutron stars. Calculations to ascertain the abundance of precious elements like, gold, platinum, uranium, etc. in the neutron stars are in progress and will be presented for publication in a separate communication.

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